Microeconomics

3. Information Economics

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Information Economics

We will apply the tools developed in game theory to analyse the effect of informational asymmetries in the models of:

1.a) adverse selection
1.b) signaling
1.c) screening.

1.a Adverse Selection

Definition

Adverse Selection arises when an informed individual's trading decision depends on privately held information in a way that adversely affects other uninformed market participants.

When adverse selection is present, uninformed traders will be wary of any informed trader who wishes to trade with them, and their willingness to pay for the product offered will be low.

Remark: This excludes first-best outcomes.

1.a Adverse Selection

Consider the following motivating example

- there is a large number of buyers and sellers
- each seller has one car
- each buyer is willing to buy at most one car
- suppose the quality of a car can be indexed by some number $q \in [0, 1]$
- buyers are willing to pay $\frac{3}{2}q$ for a car of quality q
- sellers are willing to sell a car of quality q for a price of q
- assume that $q \sim U_{[0,1]}$

1.a Adverse Selection

- under perfect information, since $\frac{3}{2}q \geq q$ for all q, all cars of any quality are sold
- if q were mutually unknown (but the beliefs are the same for buyers and seller) any price between $1/2$ and $3/4$ will lead to efficient allocation

Conclusion: Under symmetric information the outcome is efficient (first-best).

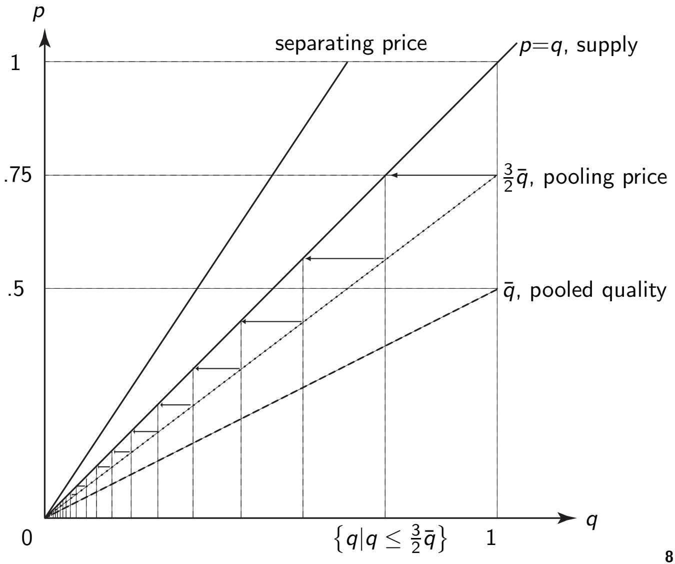
1.a Adverse Selection

- now $q \sim U_{[0,1]}$: since the expected quality of a car for the whole market is $\bar{q} \equiv \mathbb{E}[q] = \frac{1}{2}$, only a 'pooling' price of $p \leq \frac{3\bar{q}}{2} = \frac{3}{4}$ will be offered by the buyers
- but at this price, the top quarter of the whole market will not be supplied because their known valuation by the sellers is higher than the pooling price offered by the buyers
- therefore buyers need to re-calculate the expected market quality without the withdrawn top-quality quarter: The expected quality of the market is $\mathbb{E}[q'] = \bar{q}' = \frac{3}{8}$ and prices up to $p' \leq \frac{3\bar{q}'}{2} = \frac{9}{16}$ are offered
- at this price now only $\frac{9}{16} < \frac{3}{4}$ of the whole market will be offered by the sellers

1.a Adverse Selection

- therefore buyers again will re-calculate the expected quality and the market will shrink further
- as we can see below, this process is monotonic and will only stop at $q = p = 0$
- hence the complete market unravels and *no* car (the probability of $q = 0$ is zero) is sold.

1.a Adverse Selection



1.a Adverse Selection

More formally (without assuming uniform distribution), the supply of sellers with quality q at price p is

$$s(q,p) = \begin{cases} 1 & \text{if} \quad p \geq q \\ 0 & \text{if} \quad p < q \end{cases} .$$

Buyers know that at price p, only cars with $q \leq p$ are offered. Any buyer's expected utility is given by

$$\frac{3}{2}\mathbb{E}[q|q \leq p] - p.$$

Therefore, the buyers' demand at price p is given by

$$d(p) = \begin{cases} 1 & \text{if} \quad \frac{3}{2}\mathbb{E}[q|q \leq p] \geq p \\ 0 & \text{if} \quad \frac{3}{2}\mathbb{E}[q|q \leq p] < p. \end{cases}$$

1.b Signaling

Definition

In signaling models, agents can take actions to distinguish themselves from their lower-ability counterparts. The precondition for this action to be useful as a signaling device is that its marginal cost must depend on the agents' type.

In screening models, the uninformed firm tries to reduce the agents' informational rent (after contracting) by offering a menu of contracts.

In signaling models, the informed agent tries to convey private information to the uninformed firm (before contracting) through her signal. Hence the problems of learning and updating of beliefs arise.

1.b Signaling

Timing

1. Nature selects each agent's type; this is privately observed
2. agents choose a signalling activity
3. based on the observed signal, firms simultaneously make wage offers
4. agents select one preferred contract
5. outcomes & payoffs realize.

Since the informed party moves first, there is learning in the game. Hence the appropriate sol'n method is BPNE'qm (and not SGP NE'qm as before).

1.b Signaling: Spence

In the classic signaling model due to Spence (1974) we consider a competitive labor market where

- workers privately know their productivity $r \in \{r_L, r_H\}$ with $r_L < r_H$
- r_i depends on the private type $\theta_i,\ i \in \{L, H\}$; ass. $r'(\cdot) > 0$
- 2 firms hire workers on the basis of the expected productivity $r_L < \mathbb{E}[r] < r_H$ of a worker on the market.

Since competitive firms will compensate workers on the basis of the market expectation, it is in the interest of high ability workers to signal their high productivity and sign a better contract.

How can they do that?

1.b Signaling: Spence

The **Spence model**:

- Education is used as a perfectly useless but observable signal sent by the informed employee to let the employer infer her private type.
- The basic idea is that it is more costly for a low productivity worker to acquire education than for high productivity workers. Hence, by acquiring more education, the latter can distinguish themselves.
- Notice that the precondition for education to be useful as a signal is that its marginal cost must depend on the agent's type (as does her productivity).

1.b Signaling: Spence

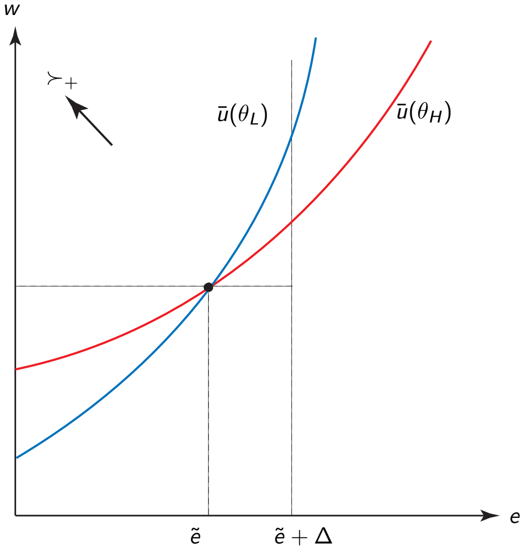
Thus the model is closed by assuming that

- workers select education e before contracting
- workers face a cost of acquiring education $c(e,\theta)$ with $c(0,\theta)=0$, $c_e(e,\theta)>0$, $c_{ee}(e,\theta)>0$, $c_\theta(e,\theta)<0$, $c_{e\theta}(e,\theta)<0$
- workers are willing to work at any wage where $u(w)=w-c(e,\theta)>\bar{u}=0$
- firms maximise profits $r-w$ and hire any workers accepting $w\leq\mathbb{E}[r]$
- the firms' prior beliefs are such that $\mu_i=\text{pr}(\theta=\theta_i),\ i\in\{L,H\}$
- there are 2 firms and one worker (population).

1.b Signaling: Spence

Negative cross derivative of the cost function implies that it costs the high ability type less to gain another unit of education.

Hence the low type has the steeper indifference curves.



1.b Signaling: Spence

More formally, **single-crossing** arises here because the worker's marginal rate of substitution between wages and education at any given pair (w, e) is $(dw/de)_{\overline{u}} = c_e(e, \theta)$, which is decreasing in θ since $c_{e\theta}(e, \theta) < 0$.

Let's start with the first-best set up:

- it is obvious that each worker type chooses $e = 0$ because education serves no purpose and is costly
- hence each worker type gets a wage $w_i = \theta_i, \; i \in \{L, H\}$.

1.b Signaling: Spence

Now suppose that productivity is not observable

- at $t = 1$, the worker chooses education level e; let the probability that type θ_i chooses education level e be given by $p_i(e) = \text{pr}(e(\theta_i) = e)$
- at $t = 2$ the firms simultaneously made a wage offers. The outcome of the game is determined entirely through the employer's beliefs of which type of worker she faces; let these beliefs be given by the conditional probability $\mu(\theta_i|e)$ which denotes the firm's revised beliefs about productivity after observing the signal e.

The e'qm wage is then given by the expected productivity

$$w(e) = \mu(\theta_H|e)r_H + \mu(\theta_L|e)r_L \in [r_L, r_H], \forall e \geq 0.$$

1.b Signaling: Spence

The key challenge in solving the problem outlined above is the evolution of the firms' beliefs given the observed event. There are many game theoretic solution concepts incorporating this and it is not clear which one to choose.

Let's begin with the simplest such solution concept, Perfect Bayesian NE'qm, and see if we need more powerful concepts as we go along.

(We will.)

1.b Signaling: Spence

Definition

A Perfect Bayesian NE'qm (PBE'qm) in Spence's signaling model is a strategy $p_i(e)$ for each possible worker type and a set of conditional beliefs $\mu(\theta_i|e)$ for the employer such that

- all education levels chosen with positive probability in e'qm must maximise the worker's expected payoff: that is, $\forall e^*$ su.t. $p_i(e^*) > 0$, we have

$$e^* \in \underset{e}{\operatorname{argmax}}\, \mu(\theta_H|e)w_H + \mu(\theta_L|e)w_L - c(e, \theta_i)$$

- if possible, the posteriors must satisfy Bayes' rule

$$\mu(\theta_i|e) = \frac{\operatorname{pr}(e|\theta_i)\operatorname{pr}(\theta_i)}{\operatorname{pr}(e)} = \frac{p_i(e)\mu(\theta_i)}{\sum_j p_j(e)\mu(\theta_j)}$$

1.b Signaling: Spence

- otherwise unrestricted posterior beliefs must be specified everywhere
- firms pay workers their expected productivity

$$w(e) = \mu(\theta_H|e)r_H + \mu(\theta_L|e)r_L.$$

Assume that the firms have common beliefs on and off the e'qm path.

1.b Signaling: Spence

Definition

A separating PBE in Spence's signaling model is an e'qm where each type of the worker chooses a different signal in e'qm: ie. $e_H \neq e_L$ such that $\mu(\theta_H|e_H) = 1$ and $\mu(\theta_L|e_L) = 1$. In a separating e'qm in Spence's signaling model we get $w_i = r_i$, $i \in \{L, H\}$.

Definition

A pooling PBE in Spence's signaling model is an e'qm where each type of the worker chooses the same signal in e'qm: ie. $e_H = e_L$ such that the offered wage is $w = \mu(\theta_H|e)r_H + \mu(\theta_L|e)r_L$.

(Semi-separating equilibria are defined in the obvious way.)

1.b Signaling: Spence

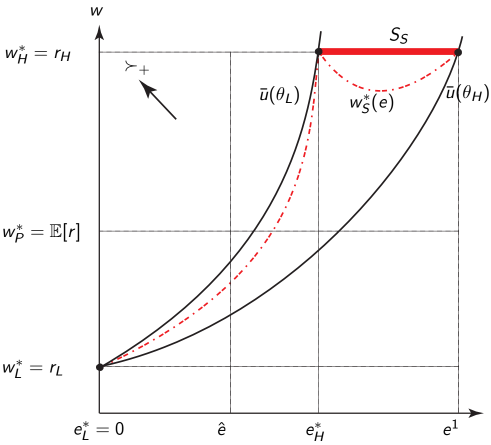
Case 1. Separating e'qa

Lemma 1:
In any separating PBNE $w^*(e_H) = r_H$ and $w^*(e_L) = r_L$.

Lemma 2:
In any separating PBNE $e_L = 0$

1.b Signaling: Spence

Case 1. Separating e'qa



The dotted line gives a possible wage schedule and thus represents off-e'qm-path beliefs. The problem is that off-e'qm we can draw it in whatever way we like—since Bayes' rule cannot be applied ($\text{pr}(\hat{e}) = 0$ in e'qm), we can specify anything we want.

Thus anything in the red stretch is a separating PBE'qm.

1.b Signaling: Spence

More formally, the set of possible e in **all** separating PBE'qa is

$$S_S = \left\{(e_L, e_H) | e_L = 0 \text{ and } e_H \in \left[e_H^*, e^1\right]\right\}.$$

To see this, notice that since the firms pay the workers their expected productivity, in **all** separating equilibria $w(e_L) = r_L$ and $w(e_H) = r_H$.

For the equilibria to be separating it must be the case that

$$w_H - c(e_H, \theta_H) \geq w_L, \quad w_H - c(e_H, \theta_L) \leq w_L$$

which is the above interval (in e'qm $r_L = w_L$ & $r_H = w_H$).

1.b Signaling: Spence

Notice that since

$$w^*(e) = \mu^*(\theta_H|e)w_H + (1 - \mu^*(\theta_H|e))w_L$$

there is a one-to-one mapping between wages and beliefs (on and off e'qm path)

$$\mu^*(\theta_H|e) = \frac{w^*(e) - r_L}{r_H - r_L}.$$

But since many schedules $w^*(e)$ are compatible with, for instance, the outcome on the previous slide, there is no way we can get a unique prediction from PBE'qm.

1.b Signaling: Spence

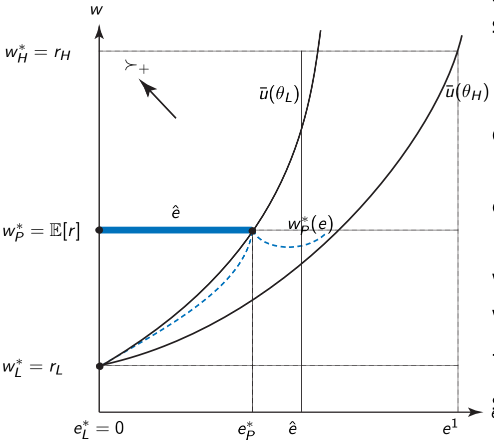
But clearly, there are better and worse separating e'qa (for both the employer and the worker)—the problem is that PBE'qm doesn't give us a handle on which to choose!

Notice that

- low-ability workers are worse off through the introduction of education: they would get $\mathbb{E}[r]$ if education was forbidden
- high-ability workers may profit or loose from the introduction of education (depending on whether $\mathbb{E}[r] \approx r_L$ or $\mathbb{E}[r] \approx r_H$ before).

1.b Signaling: Spence

Case 2. Pooling e'qa



Again the dotted line gives a possible wage schedule and thus represents off-e'qm-path beliefs. Again we can draw it in many ways because Bayes' rule cannot be applied ($\text{pr}(\hat{e}) = 0$ in e'qm) and we can specify anything we want.

Thus anything in the blue stretch is a pooling PBE'qm.

1.b Signaling: Spence

Similarly, the possible set of e in pooling PBE'qa is

$$S_P = \{(e_L, e_H) | e_L = e_H \in [0, e_P^*]\} .$$

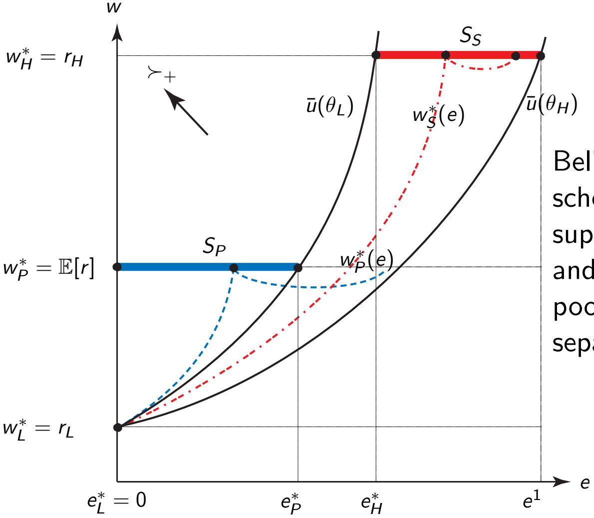
The set's upper bound derives directly from the low type's utility $w_L \leq w_P - c(e_P^*, \theta_L)$ which, for the e'qm wage w_P gives that e_P^* should satisfy

$$w_L = w_P - c(e_P^*, \theta_L) = \mu_L r_L + \mu_H r_H.$$

Clearly both agents accept anything better, too. Again there are better and worse such pooling e'qa from all player's point of view. The Pareto-dominant one is clearly $e_L^* = e_H^* = 0$. But PBE'qm fails to select among them.

1.b Signaling: Spence

Anything goes: possible PBE'qa



Belief (wage) schedules supporting multiple and suboptimal pooling and separating PBE'qm.

1.b Signaling: Refinements

Since PBE'qm doesn't give us *any* handle on what to believe, we'll look at stronger criteria:

Cho & Kreps' (1987) Intuitive Criterion (InC) is based on the idea that particular deviations from the e'qm path may be in the interest of some types but not in that of others. Hence we get a better idea of what to think about the deviator after observing a deviation than in PBE'qm.

1.b Cho & Kreps (1987)

Definition

(Cho & Kreps 1987) Let $u_i^* = w_i - c(e_i, \theta_i)$ denote type i's *e'qm* payoff. Then $\mu(\theta_j|e) = 0$ for an out of e'qm effort $e \neq (e_i^*; e_j^*)$, whenever $r_H - c(e, \theta_j) < u_j^*$ and $r_H - c(e, \theta_i) \geq u_i^*$, for $i \neq j \in \{L, H\}$.

In words: If a deviation is (in e'qm) *dominated* for one type but not for another, this deviation should not be attributed to the player for whom the deviation is dominated.

Because it is about what to believe after deviations, the InC is a restriction on off-e'qm path beliefs.

1.b Cho & Kreps (1987)

Let's apply the InC first to pooling e'qa of the Spence model. We observe that:

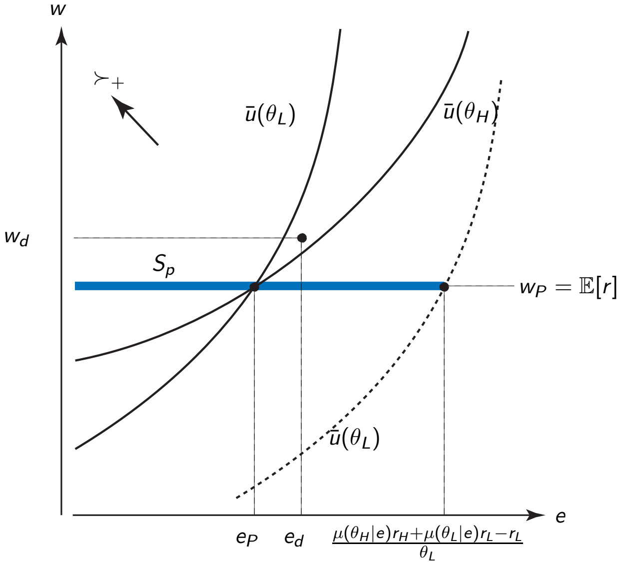
- in any pooling e'qm, the 2 types' indifference curves must cross as specified by the single crossing property: hence there is a 'wedge' separating high and low types
- but then, the high type θ_H can *always* find a profitable deviation (in the wedge) by choosing e_d
- at e_d, the firm offers a wage of $w(e_d) = r_H$ because, in e'qm, the deviation can only be profitable for a high type (while it is dominated for the low type θ_L).

This argument eliminates *all* pooling e'qa and cannot be made using PBE'qm!

1.b Cho & Kreps (1987)

The InC destroys *all* pooling e'qa!

Because $w_d = r_H$ attracts only the good guys.



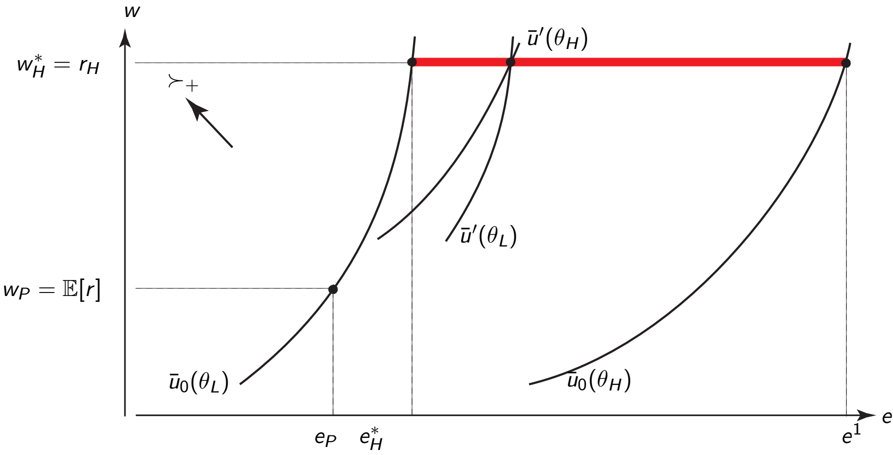
1.b Cho & Kreps (1987)

Let's see what the InC does to the set of separating e'qa. Recall that $r_L = w_L$ and $r_H = w_H$:

- for each $e > e_H^*$, the utility the low type θ_L gets, is below what he gets if he exerts $e = 0$
- but then all effort levels $e > e_H^*$ should be ascribed to the high type θ_H and the probability of coming from the low type should be set to zero
- since the same argument applies for any $e > e_H^*$, there is no point in wasting effort on the useless e and the only e'qm not eliminated is the one at the minimum cost e_H^* (the so called 'Riley outcome').

This argument selects a *unique* separating e'qm!

1.b Cho & Kreps (1987)



The InC destroys all e'qa but *one* sep'g e'qm!

1.b Cho & Kreps (1987)

This is great! but

- notice that the 'least-cost' e'qm is independent of $\mu_i, \ i \in \{L, H\}$, the proportion of skills
- suppose now the prior μ_L gets very small, say, $\mu_L = \delta \rightarrow 0$
- in such a case it seems excessive to pay a high education cost of e_H^* in order to distinguish the high type from the low probability low ability type
- just to get a very small increase over the pooling wage.