Microeconomics II (PhD)
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Exercises:

1. Solve Exercise 21.D.5 in MWG.
2. Solve Exercise 21.D.10 in MWG.
3. Read Section 8.4 in Börgers (2014) and solve Exercise 8.6.e.

   Let me know if you do not have a copy of the book.
4. *Gibbard-Satterthwaite* (harder)

   Consider the Gibbard-Satterthwaite setting from the lecture: X denotes the set of alternatives, $|X| \geq 3$, and $\mathcal{R}_i = \mathcal{P}$ for all agents i, where $\mathcal{P}$ denotes the set of all rational preference profiles without indifferences.

   Given a preference P_i of agent i, denote by $r_1(P_i)$ the alternative that is ranked first under preference P_i (and analogously for $r_k(P_i)$).

   (a) Show: Suppose the social choice function f is dominant-strategy incentive compatible. Then f is monotonic.

   (b) Show: Suppose the social choice function f is dominant-strategy incentive compatible and onto. Then f is Pareto efficient.

   Suppose from now on that there are only two agents, 1 and 2, and consider a social choice function f that is dominant-strategy incentive compatible and onto.

   (c) Let $x, y \in X$, $x \neq y$ and consider a preference profile $(\overline{P}_1, \overline{P}_2,)$ such that $r_1(\overline{P}_1) = r_2(\overline{P}_2) = x$, $r_2(\overline{P}_1) = r_1(\overline{P}_2) = y$, and suppose that $f(\overline{P}_1, \overline{P}_2,) = x$. Show that $f(P_1, P_2) = x$ for all (P_1, P_2) such that $r_1(P_1) = x$.

   (d) Repeating the previous argument for all pairs of alternatives yields two sets $X_1 \subseteq X$ and $X_2 \subseteq X$ such that $z \in X_1$ if and only if $r_1(P_1) = z$ implies $f(P_1, P_2) = z$ (and $z \in X_2$ if and only if $r_1(P_2) = z$ implies $f(P_1, P_2) = z$).

   Show that $X_1 = X$. What do you conclude?

   (Hint: Use the previous part! Argue that $X_0 = X \setminus \{X_1 \cup X_2\}$ contains at most one alternative and hence $X_1 \cup X_2$ contains at least two distinct alternatives. Argue then that either X_1 or X_2 must be empty. Using the previous part you can show now that X_0 must also be empty.)