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*Part B*
**Exercises:**

5. (a) Consider an assignment game, let $x, y$ be stable payoff vectors and let $u$ be defined by

$$u_m = \max(x_m, y_m) \qquad \forall m$$
$$u_w = \min(x_w, y_w) \qquad \forall w.$$

Show: $u$ is also a stable payoff vector.

(b) Show: The set of stable payoff vectors has a minimal and a maximal element.

6. Let $(P, Q, \alpha)$ be an assignment game, where $P$ denotes the set of potential buyers, $Q$ the set of potential sellers and $\alpha_{ij}$ is the valuation of buyer $i$ for the object of seller $j$. Assume that $\alpha_{ij} \geq 0$ for all $i, j$.

(a) Suppose there are three buyers and three sellers. The valuations are given by the following matrix (the entry in the $i$th row and $j$th column is buyer $i$'s valuation for $j$'s object):

$$\alpha = \begin{pmatrix} 9 & 8 & 13 \\ 11 & 9 & 11 \\ 7 & 0 & 7 \end{pmatrix}$$

All sellers have a reservation value of 0. Calculate the buyer-optimal payoff vector in the core. (Hint: Multi-object auction)

(b) Consider a general assignment game and fix some buyer $i \in P$. Show that if there is an optimal assignment $x$ such that buyer $i$ is not assigned, then for any stable payoff vector $(u, v)$ we must have $u_i = 0$.

(c) Provide an example showing that even when $i \in P$ is assigned to some object in all optimal assignments, there may exist stable payoff vectors $(u, v)$ such that $u_i = 0$.