# Microeconomics II (PhD)
# Tutorial 11, July 13

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*Part A*
**Exercises:**

1. Consider a marriage market with strict preferences.

   Show: The set of individuals that remain single is the same for all stable matchings.

2. Consider a marriage market with strict preferences. Consider a direct mechanism, where each agent submits a preference ranking. The designer then runs the men-proposing deferred acceptance algorithm on the reported preferences and implements the resulting matching.

   Show that truthtelling is a dominant strategy for men.

   *Hint: Fix an agent $i$, reports by all other agents, and suppose that $i$ has a best response that strictly improves over truthtelling. Denoting his match by $j$, show first that $i$ gets a weakly preferred partner if he uses a cutoff strategy that ranks all agents he prefers to $j$ truthful and all other agents as unacceptable.*

3. There are $i = 1, ..., 8$ sellers each of one horse and $j = 1, ..., 10$ potential buyers each of one horse in a horse market. All agents' utility gains in the horse market can be identified with their monetary gains, and the horses are homogeneous goods. The reserve price $c_i$ of the sellers and the maximal willingness-to-pay $h_j$ of the buyers are known to be given as in the following tables:

| $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ | $c_8$ |
|---|---|---|---|---|---|---|---|
| 10 | 11 | 15 | 17 | 20 | 21.5 | 25 | 26 |

   and

| $h_1$ | $h_2$ | $h_3$ | $h_4$ | $h_5$ | $h_6$ | $h_7$ | $h_8$ | $h_9$ | $h_{10}$ |
|---|---|---|---|---|---|---|---|---|---|
| 30 | 28 | 26 | 24 | 22 | 21 | 20 | 18 | 17 | 15 |

   (a) Identify the gains from trade each coalition can achieve.

   (b) Show that the existence of two buyer-seller pairs who trade at different prices contradicts the requirements of the core.

   (c) Determine the core.

   (d) Show that each allocation in the core can be supported by a Walrasian price. (Note that this is the converse of the standard result whereby each Walrasian allocation is in the core!)

4. There is a set of $n$ men, $M = \{m_1, ..., m_n\}$, and a set of $p$ women, $W = \{w_1, ..., w_p\}$. If man $m_i$ and woman $w_j$ are paired, they create a monetary value of $v(m_i, w_j)$, single individuals do not create value. Utility is given by the monetary value an agent realizes.

   (a) Suppose that utility is transferable, $n = p = 2$ and match values are given as follows:

| | $w_1$ | $w_2$ |
|---|---|---|
| $m_1$ | 10 | 18 |
| $m_2$ | 1 | 10 |

   Compute the core of the game and draw the set of payoffs for men that are part of core allocations.

   (b) Now one additional man arrives, corresponding match values are given as follows:

| | $w_1$ | $w_2$ |
|---|---|---|
| $m_1$ | 10 | 18 |
| $m_2$ | 1 | 10 |
| $m_3$ | 3 | 5 |

   Determine the payoff vector in the core that men prefer the least. Compare to the payoff vector men prefer the least in part (a).

   (c) Consider the general setting with $n$ men and $p$ women and arbitrary match values. Suppose that utility is not transferable across agents and each men that is matched receives a share $s \in (0, 1)$ of the match value, each woman receives a share $1 - s$. Assume that match values are such that individuals have strict preferences. Show that there is a unique stable matching.