Microeconomics II (PhD)
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Exercises:

1. *Gibbard-Satterthwaite*
   Consider the setting of Exercise 4 in Tutorial 9.

   Using the results from the previous tutorial, show that the Gibbard-Satterthwaite theorem holds if there are at least two agents.

   Hint: Use an induction argument and in particular the last lemma that we showed:

   Lemma. *Let f be strategy-proof and onto. If we define $f_{ij} : \mathcal{P}^{N-1} \to X$ by $f_{ij}(P_i, P_{-\{i,j\}}) = f(P_i, P_i, P_{-\{i,j\}}$ then f_{ij} is strategy-proof and onto.*

2. (From lecture.) Consider a marriage market with strict preferences and suppose that μ and μ' are stable matchings.

   Show: $\mu \vee_M \mu'$ and $\mu \wedge_M \mu'$ are stable matchings.

3. Consider the following marriage market with four men and four women.
   Preferences are strict and given by

$$\begin{array}{lllll} m_1: & w_1, & w_2, & w_3, & w_4 \\ m_2: & w_2, & w_1, & w_4, & w_3 \\ m_3: & w_3, & w_4, & w_1, & w_2 \\ m_4: & w_4, & w_3, & w_2, & w_1 \end{array} \qquad \begin{array}{lllll} w_1: & m_4, & m_3, & m_2, & m_1 \\ w_2: & m_3, & m_4, & m_1, & m_2 \\ w_3: & m_2, & m_1, & m_4, & m_3 \\ w_4: & m_2, & m_3. & & \end{array}$$

   In the following, you may use without proof that

$$\mu_1 = \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \\ m_3 & m_1 & m_4 & m_2 \end{array}$$

   is a stable matching.

   (a) Show that

$$\mu_2 = \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \\ m_2 & m_4 & m_1 & m_3 \end{array}$$

   is a stable matching.

   (b) Find the men-optimal stable matching μ_M and the women-optimal stable matching μ_W.

   (c) Find two additional stable matchings that are different from μ_1, μ_2, μ_M and μ_W.

4. (From lecture.) Consider a marriage market with strict preferences.

   Show: The matching resulting from the men-proposing deferred acceptance algorithm is men-optimal.