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Exercises:

1. *Interdependent value auction*

   Suppose there is one object for sale and N potential buyers. Each agent privately observes a signal X_i, which is independently and identically distributed on $[0, \overline{X}]$ with cdf F and density f. Denote by $G = F_{(1:N-1)}$ the cdf of the first-order statistic of $N-1$ of these random variables.

   Buyers have quasi-linear utilities: in case of winning the object, buyer i gets utility $v(x_i, x_{-i}) - p$, where p denotes the payment made, and he gets utility of 0 in case of not winning. Suppose that v is positive, strictly increasing in all signals, symmetric in the last $N-1$ signals, and denote by $\overline{v}(x_i, y)$ the expected valuation of agent i given he received signal x_i and the highest signal among all other signals has value y.

   (a) Show: In a second price auction, each agent bidding according to the bid function $\beta(x_i) = \overline{v}(x_i, x_i)$ is a Bayes-Nash equilibrium.

   Is it a dominant strategy to follow this bid function? Is it an ex-post equilibrium?

   (b) Consider an open English auction. A symmetric strategy in an English auction is a collection $\boldsymbol{\beta} = (\beta^N, \beta^{N-1}, ..., \beta^2)$ of $N-1$ functions $\beta^k : [0, \overline{X}] \times \mathbb{R}_+^{N-k} \to \mathbb{R}_+$. The interpretation is that $\beta^k(x, p_{k+1}, ..., p_N)$ is the price at which bidder 1 will drop out of the auction if the number of bidders who are still active is k, his own signal is x, and the prices at which the other $N-k$ bidders dropped out were $p_{k+1} \geq p_{k+2} \geq ... \geq p_N$.

   Describe a symmetric Bayes-Nash equilibrium of the open English auction and show that this strategy profile constitutes indeed an equilibrium.

   Is it an equilibrium in dominant strategies? Is it an ex-post equilibrium?

   (c) Show that the symmetric bidding strategies $\beta(x) = \frac{1}{G(x)} \int_0^x v(y, y) dG(y)$ form a Bayes-Nash equilibrium of the first-price auction.

   (d) Suppose $N = 2$, bidder i's valuation is $v_i(\theta_i, \theta_j) = \eta\theta_i + (1-\eta)\theta_j$. For which η is the outcome of the second-price auction efficient?

2. Consider the Akerlof framework from the lecture.

   Definition 1. *A* competitive equilibrium *in the Akerlof model is a price p^* and a set Θ^* of seller types who trade such that*

$$\Theta^* = \{\theta | R(\theta) \leq p^*\} \qquad \textit{and}$$
$$p^* = \mathbb{E}[\theta | \theta \in \Theta^*].$$

   Suppose the distribution function of θ is

$$F(\theta) = \begin{cases} 0 & \text{for } \theta < 1 \\ \theta - 1 & \text{for } 1 \leq \theta \leq 2 \\ 1 & \text{for } 1 < \theta \end{cases}$$

   and $R(\theta) = 0.9 \cdot \theta$.

   Describe a competitive equilibrium of this model.