Microeconomics II (PhD)
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Exercises:

1. Suppose there is one agent, three potential types $(\theta^1, \theta^2, \theta^3)$ and three alternatives (a, b, c). The valuation the agent has for an alternative given his type is given by the following matrix:

	θ^1	θ^2	θ^3
a	0	-1	x
b	1	0	-1
c	-1	1	0

Consider the function k' such that $k'(\theta^1) = a$, $k'(\theta^2) = b$, and $k'(\theta^3) = c$.

(a)
Definition 1. *A decision rule k is* weakly monotone *if for all θ^i, θ^j,*

$$v(k(\theta^i), \theta^i) - v(k(\theta^j), \theta^i) \geq v(k(\theta^i), \theta^j) - v(k(\theta^j), \theta^j).$$

Suppose $x = 1$. Is k' weakly monotone? Is it implementable in the sense that there is a payment rule t such that (k', t) is incentive compatible? How does this relate to the result you saw in the lecture?

(b)
Definition 2. *A decision rule k is* cyclically monotone *if for every sequence of types of length $l \in \mathbb{N}$, $(\theta^1, \theta^2, ..., \theta^l)$, with $\theta^l = \theta^1$, we have*

$$\sum_{\kappa=1}^{l-1} v(k(\theta^\kappa), \theta^{\kappa+1}) - v(k(\theta^\kappa), \theta^\kappa) \leq 0.$$

Show that every implementable decision rule k is cyclically monotone.

(c) For which values of x is k' cyclically monotone?

2. There is one seller with two objects, and one buyer. The seller does not value the objects; the buyer values object k by θ^k $(k = 1, 2)$ and getting both objects by $\theta^1 + \theta^2$.

(a) Suppose that valuations are independently distributed and, for $k = 1, 2$,

$$\theta^k = \begin{cases} 10 & \text{with probability } \frac{1}{2} \\ 22 & \text{with probability } \frac{1}{2}. \end{cases}$$

What are the optimal prices and the corresponding revenue if the seller sells the objects separately? What is the optimal price and the corresponding revenue if the seller only sells the bundle?

(b) Suppose that valuations are independently distributed and, for $k = 1, 2$,

$$\theta^k = \begin{cases} 10 & \text{with probability } \frac{1}{2} \\ 50 & \text{with probability } \frac{1}{2}. \end{cases}$$

What are the optimal prices and the corresponding revenue if the seller sells the objects separately? What is the optimal price and the corresponding revenue if the seller only sells the bundle?

(c) Suppose the seller sets a price for each object and a price for the bundle of both objects. Determine the optimal prices if valuations are identically, independently, and uniformly distributed on $[0, 1]$.

(d) Suppose the seller sets a price for each object and a price for the bundle of both objects. Determine the optimal prices if valuations are independently distributed and, for $k = 1, 2$,

$$\theta^k = \begin{cases} 1 & \text{with probability } \frac{1}{6} \\ 2 & \text{with probability } \frac{1}{2} \\ 4 & \text{with probability } \frac{1}{3} \end{cases}$$

(e) In the previous part, suppose the seller offers the following menu: A lottery which yields with probability $\frac{1}{2}$ object 1 and nothing otherwise, a lottery which yields with probability $\frac{1}{2}$ object 2 and nothing otherwise, and getting the bundle of both objects for sure. Determine the optimal prices. Compare the revenue to the revenue for the prices that you derived in the previous part.

3. *Roberts Theorem*

Let $\alpha_1, ..., \alpha_I \in \mathbb{R}_+ \setminus \{0\}$ and $\lambda_1, ..., \lambda_K \in \mathbb{R}$. A function $k : \Theta \to K$ is called an *affine maximizer* if $k(\theta) \in \arg\max_k \sum_{i=1}^{I} \alpha_i \cdot v_i(k, \theta_i) + \lambda_k$.

(a) Show: $k : \Theta \to K$ is implementable if k is an affine maximizer.

(b)

Definition 3. *A SCF $f : \Theta \to K \times \mathbb{R}^I$ satisfies* positive association of differences *(PAD) if for all $i \in 1, ..., I$, $\theta_{-i} \in \Theta_{-i}$ and $\theta_i, \theta_i' \in \Theta_i$ such that $k(\theta_i', \theta_{-i}) = x$ and $v_i(x, \theta_i) - v_i(y, \theta_i) > v_i(x, \theta_i') - v_i(y, \theta_i')$ for all $y \neq x$, it holds that $k(\theta_i, \theta_{-i}) = x$.*

Show: Every weakly monotone SCF satisfies PAD.

(c) Suppose now that $\alpha_1, ..., \alpha_I \in \mathbb{R}_+ \cup \{0\}$. Argue that not every affine maximizer is implementable under this definition.