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Exercises:

1. Consider a social choice setting with two alternatives (A and B) and two agents. Each agent has 3 possible types and each type is equally likely (and types are independently distributed). An allocation rule specifies for each type profile the probability with which alternative A is chosen.

   (a) Suppose marginals are given by

   $$\Phi_1(x_1^1) = 0.1 \qquad \Phi_2(x_1^2) = 0.2$$
   $$\Phi_1(x_2^1) = 0.4 \qquad \Phi_2(x_2^2) = 0.5$$
   $$\Phi_1(x_3^1) = 0.9 \qquad \Phi_2(x_3^2) = 0.7$$

   Is there an allocation rule inducing these marginals?

   (b) Consider the following allocation rule (where the ij's entry denotes the probability with which alternative A is chosen if agent 1 has type t_i and agent 2 has type t_j):

0	0.5	0.1
0.2	0	1
0.4	1	1

   Construct an equivalent allocation rule that is pointwise nondecreasing.

2. Consider the bilateral trade setting from the lecture (see slide 62) but suppose that the seller is forced to participate in the mechanism (i.e., there is no participation constraint for the seller). An allocation prescribes a probability with which trade takes place and a payment by the buyer to the seller.

   (a) Which mechanism in the class of BIC mechanisms maximizes expected aggregate utility?

   (b) Is there a DIC mechanism which achieves the same expected aggregate utility?

3. Consider a setting with 3 alternatives (A, B and C) and 2 symmetric agents, each with two equally likely and independent types $x^1 < x^2$. The utility of an agent with type x^l is $x^l + c^A$ in alternative A, $ax^l + c^B$ with $0 < a < 1$ in alternative B, and c^C in alternative C (where a and c^k are given parameters).

   Consider the following allocation rule (where $s = \frac{1}{20}$ and q_{ij}^k is the probability of choosing alternative k if agent 1 is of type x^i and agent 2 is of type x^j):

   $$q^A = as \begin{pmatrix} 1 & 1 \\ 1 & 13 \end{pmatrix}, \qquad q^B = s \begin{pmatrix} 9 & 1 \\ 1 & 1 \end{pmatrix}, \qquad q^C = 1 - q^A - q^B$$

   (a) Is this allocation rule implementable in Bayesian equilibrium?

   (b) All symmetric allocation rules that are P-equivalent are given by

   $$\tilde{q}^A = as \begin{pmatrix} 2-\alpha & \alpha \\ \alpha & 14-\alpha \end{pmatrix}, \qquad \tilde{q}^B = s \begin{pmatrix} 10-\beta & \beta \\ \beta & 2-\beta \end{pmatrix}, \qquad \tilde{q}^C = 1 - \tilde{q}^A - \tilde{q}^B$$

   for $0 \le \alpha \le 2$ and $0 \le \beta \le 2$. For which values of α and β are these allocation rules implementable in dominant strategies?

   (c) Construct an U-equivalent allocation rule that is implementable in dominant strategies!