Microeconomics II (PhD)
Tutorial 4, May 6

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Exercises:

1. Solve Exercise 23.E.3 in MWG.

2. Solve Exercise 23.E.5 in MWG.

3. *(Variant of an old exam question)*
Consider an auction setting: a seller has a single object for sale, which he does not value. There are 2 bidders, who are privately informed about their valuations θ_i. Valuations are drawn independently, where $\theta_1 \sim U[0,1]$ and $\theta_2 \sim U[0,2]$. Each buyer has a utility function $u_i = p_i \cdot \theta_i + t_i$, where p_i denotes the probability that buyer i gets the object and t_i denotes the transfer he receives.

   Any auction has to be Bayesian incentive compatible and give each bidder type an interim expected utility of at least 0.

   You may use all results from the lecture without proof.

   (a) Compute the allocation rule of the revenue maximizing auction. Illustrate graphically why this allocation rule is inefficient.

   (b) Compute the interim expected utility of bidder 1 as a function of his type.

   (c) Suppose the seller has a commonly known reservation value of $\frac{1}{2}$. His utility is therefore given by $u_S = (1 - p_1 - p_2) \cdot \frac{1}{2} - t_1 - t_2$. Compute the allocation rule of the auction that is optimal for the seller.

   (d) Suppose that types are not drawn independently. Instead, types are distributed on $[0,1]^2$ according to the density $f(\theta_1, \theta_2) = 1$ if $\theta_1 = \theta_2$ and 0 else (i.e., types are perfectly correlated).

   How does an optimal auction look like? Check that incentive and participation constraints are fulfilled. Compute the interim expected utilities of the buyers.

   (e) Suppose now that each bidder has either a valuation of 1 or 2.

$$Prob(\theta_1 = 1, \theta_2 = 1) = Prob(\theta_1 = 2, \theta_2 = 2) = \frac{1}{4} + \varepsilon$$
$$Prob(\theta_1 = 1, \theta_2 = 2) = Prob(\theta_1 = 2, \theta_2 = 1) = \frac{1}{4} - \varepsilon$$

   for some $\varepsilon > 0$. Compute the optimal auction and the corresponding revenue.

4. Consider an auction setting with a seller who sells a single object which she values at 0. Buyers are ex-ante symmetric and the valuations are *iid* distributed according to cdf F. Suppose that virtual valuations are increasing.

   Show: The revenue of the optimal auction if there are n bidders is weakly less than the revenue from a standard second-price auction (without reserve price) with $n+1$ bidders.

   (Hint: Construct an auction with $n+1$ bidders that always allocates the object to a buyer and achieves a weakly higher utility than the optimal auction with n bidders. Following the argument in the lecture, construct the optimal auction in the class of auctions that always allocate the object.)