

Exercises:

1. Solve Exercise 21.D.5 in MWG.
2. Solve exercise 21.D.10 in MWG.
 Note that in the fourth line of part (c) it should be “Show that preferences over X ”.
3. Consider the following marriage market with four men and four women.
 Preferences are strict and given by

$m_1 : w_1, w_2, w_3, w_4$	$w_1 : m_4, m_3, m_2, m_1$
$m_2 : w_2, w_1, w_4, w_3$	$w_2 : m_3, m_4, m_1, m_2$
$m_3 : w_3, w_4, w_1, w_2$	$w_3 : m_2, m_1, m_4, m_3$
$m_4 : w_4, w_3, w_2, w_1$	$w_4 : m_2, m_3.$

In the following, you may use without proof that

$$\mu_1 = \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \\ m_3 & m_1 & m_4 & m_2 \end{array}$$

is a stable matching. You may use all results from the lecture without proof.

(a) Show that

$$\mu_2 = \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \\ m_2 & m_4 & m_1 & m_3 \end{array}$$

is a stable matching.

- (b) Find the men-optimal stable matching μ_M and the women-optimal stable matching μ_W .
 - (c) Find two additional stable matchings that are different from μ_1, μ_2, μ_M and μ_W .
4. (From lecture.) Consider a marriage market with strict preferences and suppose that μ and μ' are stable matchings.
 Show: $\mu \vee_M \mu'$ and $\mu \wedge_M \mu'$ are indeed matchings.
 5. (From lecture.) Consider a marriage market with strict preferences.
 Show: The matching resulting from the men-proposing deferred acceptance algorithm is men-optimal.
 6. Consider a marriage market with strict preferences. A mechanism asks the agents to reveal their preferences and applies the men-proposing deferred acceptance algorithm to the reported preferences. Suppose that (i) men report truthfully and (ii) each woman reports a preference list such that all reports form a Nash equilibrium (under complete information).
 Show: The resulting matching is stable.