Microeconomics II (PhD)
Tutorial 3, May 5

Andreas Kleiner
akleiner@uni-bonn.de

Exercises:

1. Look again at Exercise 23.C.10, Part d. In case you want to discuss this part, prepare some questions. If there are no questions we will skip this part.

2. Recap the following exercise from last tutorial:

   Interdependent value auction

   Suppose there is one object for sale and N potential buyers. Each agent privately observes a signal X_i, which is independently distributed on $[0, \overline{X}]$ with density f.

   Buyers have quasi-linear utilities: in case of winning the object, buyer i gets utility $v(x_i, x_{-i}) - p$, where p denotes the payment made, and he gets utility of 0 in case of not winning. Suppose that v is strictly increasing in all signals, symmetric in the last $N-1$ signals, and denote by $\overline{v}(x_i, y)$ the expected valuation of agent i given he received signal x_i and the highest signal among all other signals has value y.

   Show: In a second price auction, each agent bidding according to the bid function $\beta(x_i) = \overline{v}(x_i, x_i)$ is a Bayes-Nash equilibrium.

   Is it a dominant strategy to follow this bid function? Is it an ex-post equilibrium?

3. *Roberts Theorem*

   Let $\alpha_1, ..., \alpha_I \in \mathbb{R}_+ \setminus \{0\}$ and $\lambda_1, ..., \lambda_K \in \mathbb{R}$. A function $k : \Theta \to K$ is called an *affine maximizer* if $k(\theta) \in \arg\max_k \sum_{i=1}^{I} \alpha_i \cdot v_i(k, \theta_i) + \lambda_k$.

   (a) Show: $k : \Theta \to K$ is implementable if k is an affine maximizer.

   (b)

   Definition 1. *A SCF $f : \Theta \to K \times \mathbb{R}^I$ satisfies* positive association of differences *(PAD) if for all $i \in 1, ..., I$, $\theta_{-i} \in \Theta_{-i}$ and $\theta_i, \theta_i' \in \Theta_i$ such that $k(\theta_i', \theta_{-i}) = x$ and $v_i(x, \theta_i) - v_i(y, \theta_i) > v_i(x, \theta_i') - v_i(y, \theta_i')$ for all $y \neq x$, it holds that $k(\theta_i, \theta_{-i}) = x$.*

   Show: Every implementable SCF satisfies PAD.

   (c) Suppose now that $\alpha_1, ..., \alpha_I \in \mathbb{R}_+ \cup \{0\}$. Argue that not every affine maximizer is implementable under this definition.

4. Solve Exercise 23.B.4.a in MWG.

5. Consider a seller which has a single indivisible good for sale. Suppose she auctions this good to N potential buyers. Buyers are ex ante symmetric, and buyer i has a value $\theta_i \in [0, \overline{\theta}]$ for the good, where θ_i is independently distributed according to distribution function F with strictly positive density f.

   (a) Compute the expected revenue to the seller if she conducts a second-price auction without reserve price.

   (b) Compute the expected revenue to the seller if she conduts a first-price auction without reserve price and agents follow their symmetric equilibrium strategies that you characterized in the lecture. Compare to part (a).