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Exercises:

1. Consider a marriage market with strict preferences.

   Show: The set of individuals that remain single is the same for all stable matchings.

2. Consider a marriage market with strict preferences.

   Show: In the men-proposing deferred acceptance algorithm, truthtelling is a dominant strategy for men.

3. There are $i = 1, ..., 8$ sellers each of one horse and $j = 1, ..., 10$ potential buyers each of one horse in a horse market. All agents' utility gains in the horse market can be identified with their monetary gains, and the horses are homogeneous goods. The reserve price c_i of the sellers and the maximal willingness-to-pay h_j of the buyers are known to be given as in the following tables:

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
10	11	15	17	20	21.5	25	26

   and

h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
30	28	26	24	22	21	20	18	17	15

   (a) Identify the gains from trade each coalition can achieve.

   (b) Show that the existence of two buyer-seller pairs who trade at different prices contradicts the requirements of the core.

   (c) Determine the core.

   (d) Show that each allocation in the core can be supported by a Walrasian price. (Note that this is the converse of the standard result whereby each Walrasian allocation is in the core!)

4. (a) Consider an assignment game, let x, y be stable payoff vectors and let u be defined by

   $$u_m = \max(x_m, y_m) \qquad \forall m$$
   $$u_w = \min(x_w, y_w) \qquad \forall w.$$

   Show: u is also a stable payoff vector.

   (b) Show: The set of stable payoff vectors has a minimal and a maximal element.