Microeconomics II (PhD)
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Exercises:

1. Look again at Exercise 23.C.9 and prove the remaining direction.
2. Suppose there are two agents and the question whether a brigde should be built. The net valuation of agent i for having a bridge is θ_i, which is independently and uniformly distributed on [-3,3]. Utilities are quasi-linear: agent i gets utility $\theta_i + t_i$ if the bridge is built and t_i otherwise, where t_i denotes the transfer he receives.
   (a) Assume agents can either vote in favor or against the bridge and there are no transfers. The bridge will be built only if both agents vote for it. What is an equilibrium in dominant strategies? What is the expected aggregate welfare if agents follow these strategies?
   (b) Suppose that agents' valuations were observed by a utilitarian social planner. Which decision rule should he implement and what is the resulting expected aggregate utility (that is, the sum of the agents expected utilities)?
   (c) Assume that transfers are feasible. What is the expected aggregate utility if the Pivotal mechanism is implemented?
3. Solve Exercise 23.C.10 in MWG.

   Assume throughout the exercise that (23.C.8) is a necessary condition for $(k^*, t_1, ..., t_I)$ to be truthfully implementable in dominant strategies. In part c insert "implementable" before "ex post efficient social choice function" and suppose that $V_i(\theta_{-i})$ is I times continuously differentiable for each i.
4. *Interdependent value auction*

   Suppose there is one object for sale and N potential buyers. Each agent privately observes a signal X_i, which is independently distributed on $[0, \overline{X}]$ with density f.

   Buyers have quasi-linear utilities, i.e. in case of winning the object buyer i has utility $v(x_i, x_{-i}) - p$, where p denotes the payment made and utility of 0 in case of not winning. Suppose that v is increasing in all signals, symmetric in the last $N-1$ signals, and denote by $\overline{v}(x_i, y)$ the expected valuation of agent i given he received signal x_i and the highest signal among all other signals has value y.

   Show: In a second price auction, each agent bidding according to the bid function $\beta(x_i) = \overline{v}(x_i, x_i)$ is a Bayes-Nash equilibrium.

   Is it a dominant strategy to follow this bid function? Is it an ex-post equilibrium?