Exercises:

1. Look again at Exercise 23.C.9 and prove the remaining direction.

2. Suppose there are two agents and the question whether a bridge should be built. The net valuation of
   agent $i$ for having a bridge is $\theta_i$, which is independently and uniformly distributed on $[-3,3]$. Utilities
   are quasi-linear: agent $i$ gets utility $\theta_i + t_i$ if the bridge is built and $t_i$ otherwise, where $t_i$ denotes the
   transfer he receives.

   (a) Assume agents can either vote in favor or against the bridge and there are no transfers. The bridge
   will be built only if both agents vote for it. What is an equilibrium in dominant strategies? What
   is the expected aggregate welfare if agents follow these strategies?

   (b) Suppose that agents’ valuations were observed by a utilitarian social planner. Which decision rule
   should he implement and what is the resulting expected aggregate utility (that is, the sum of the
   agents expected utilities)?

   (c) Assume that transfers are feasible. What is the expected aggregate utility if the Pivotal mechanism
   is implemented?

3. Solve Exercise 23.C.10 in MWG.
   Assume throughout the exercise that (23.C.8) is a necessary condition for $(k^*, t_1, ..., t_I)$ to be truthfully
   implementable in dominant strategies. In part c insert “implementable” before “ex post efficient social
   choice function” and suppose that $V_i(\theta_{-i})$ is $I$ times continuously differentiable for each $i$.

4. Interdependent value auction
   Suppose there is one object for sale and $N$ potential buyers. Each agent privately observes a signal $X_i$, which
   is independently distributed on $[0, X]$ with density $f$.

   Buyers have quasi-linear utilities, i.e. in case of winning the object buyer $i$ has utility $v(x_i, x_{-i}) - p$,
   where $p$ denotes the payment made and utility of 0 in case of not winning. Suppose that $v$ is increasing
   in all signals, symmetric in the last $N-1$ signals, and denote by $\bar{v}(x_i, y)$ the expected valuation of agent
   $i$ given he received signal $x_i$ and the highest signal among all other signals has value $y$.

   Show: In a second price auction, each agent bidding according to the bid function $\beta(x_i) = \bar{v}(x_i, x_i)$ is a
   Bayes-Nash equilibrium.

   Is it a dominant strategy to follow this bid function? Is it an ex-post equilibrium?