Resale Markets and the Assignment of Property Rights

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The consumption of an indivisible good causes identity-dependent externalities to non-consumers. We analyse resale markets where the current owner designs the trading procedure, but cannot commit to future actions. We ask the following questions: (1) Does the identity of the initial owner matter for the determination of the final consumer? (2) Is the outcome always efficient? The major conclusion of our paper is that the irrelevance of the initial structure of property rights arises in resale processes even if there are transaction costs that hinder efficiency. This result complements the Coasian view where the irrelevance of the assignment of property rights is a consequence of efficiency.

1. INTRODUCTION

In two seminal papers Coase (1959, 1960) argued that in a zero-transaction costs world the allocation of property rights has no effect on efficiency, and that the laissez-faire always yields optimal outcomes irrespective of the assignment of property rights. He writes: “While the delimitation of rights is an essential prelude to transactions...the ultimate result (which maximizes the value of production) is independent of the legal decision” (Coase (1959)). It is important to note that Coase sees the independence property as a consequence of the efficiency property. His line of argument is as follows: The ultimate outcome must be Pareto-efficient. Otherwise anybody could propose a Pareto-improving outcome, which would be accepted. Moreover, if there are no income effects (agents’ utilities are transferable), all Pareto-efficient outcomes maximize welfare, and are thus welfare-equivalent. The independence result follows.

We follow Coase by considering a situation with a few agents whose utilities are transferable. An indivisible good can be consumed by one of the agents. In addition to the benefit he enjoys from consumption, the consumer of the good imposes identity-dependent externalities on the other agents. As suggested in Jehiel, Moldovanu and Stacchetti (1996), this situation is common and covers diverse contexts such as the sale of patents, the sale of pollution rights, the sale of spectrum rights, changes in ownership in imperfectly competitive markets (e.g. privatizations, mergers, takeovers). In general, the externalities stand as a reduced form for the effects of the interaction between agents after the close of a particular transaction.

Jehiel and Moldovanu (1995) and Jehiel, Moldovanu and Stacchetti (1996) consider the sale of goods involving externalities in a framework without resales.
An important question that we wish to address here is whether resale markets that permit the sequential internalization of external effects can replace contracts including commitments to future actions as a vehicle towards achieving efficiency. In a context with externalities, we show below that contracts leading to efficient allocations may require a certain degree of commitment (e.g. to sell to a particular agent, or not to sell at all). Such commitments are not necessarily *credible*, and we wish to relax the possibility of exogenous commitment. While allowing for resale markets, we make the extreme assumption that commitments to future actions are not feasible.\(^1\)

The role of commitments in strategic interactions is well recognized at least since Schelling (1960). Interestingly, Coase considered the problem faced by a durable good monopolist that cannot commit not to sell (or to sell only for a certain price) in the future. The inability to commit often prevents a trader from extracting much surplus from a transaction. This phenomenon, called the “Coase conjecture”, has been the subject of an extensive literature.\(^2\)

Another kind of transaction cost that we wish to consider is that induced by the constraint of bilaterality. Most of the observed transaction forms are bilateral, *i.e.* involve a transfer of goods and money between two parties. When externalities are present, achieving an efficient allocation may require, however, a multilateral contract. Such multilateral agreements are natural and often observed in frameworks with externalities. For example, Guinness and Grand Metropolitan, two merging British drinks and food firms, agreed to pay £250 million to LVMH, a French drinks and luxury goods company with a stake in Guinness (see *The Economist*, October 18th, 1997). LVMH’s chairman, Bernard Arnault, strongly opposed the merger, and delayed the deal for 5 months. He claimed that LVMH's share in the merged company would generate less revenue than before. Arnault finally accepted the deal, but only after getting the hefty side-payment.

A question of interest is whether a series of bilateral transactions achieved through repeated resales (which allow a sequential internalization of externalities) can form a substitute for a multilateral contract.\(^3\)

Within our setup, the main contribution of this paper is to answer the following questions:

1. *Does the identity of the original property right owner matter for the determination of the final consumer?*
2. *Is the final outcome always efficient?*

In an appropriate setting, the Coasian view is that a negative answer to the first question is a logical consequence of an affirmative answer to the second question. It is instructive to recall that Coase first expressed the views which are collectively known as “The Coase Theorem” in an article that advocated the use of auctions for allocating spectrum rights (Coase (1959)). This idea has only recently been implemented with, apparently, great success (see for example, McMillan (1994)). Previous to auctioning, spectrum has been allocated by lottery. At a first glance, lotteries seem incompatible with efficiency.\(^4\) One may argue, however, that efficiency will nevertheless be achieved provided that resale

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\(^1\) Without commitment and without a resale market, it is easily shown that the outcome may be inefficient. See Example 4.14 below.

\(^2\) See Coase (1972), and Gul *et al.* (1986) for a more recent treatment.

\(^3\) Gale (1986) addresses a related question in a general equilibrium framework without externalities. He analyses whether a sequence of bilateral tradings can replace a Walrasian auctioneer.

\(^4\) Of course, lotteries in which spectrum is allocated for free deprive the Government of a substantial source of revenue. It is estimated that during the 1980s cellular licenses worth $46 billion were given away.
markets are well functioning (\textit{e.g.} resales are not constrained, delays do not occur,\textsuperscript{5} \textit{etc} \ldots). Such an argument has been formulated by opponents to the sophisticated, centralized auction. It is an application of the idea that the initial allocation of property rights is ultimately immaterial in \textit{laissez-faire}. For example, McMillan (1994) reports a "not atypical case" where an obscure partnership was awarded by lottery the license to run cellular telephones on Cape Cod. Not surprisingly, the license was promptly sold to Southwestern Bell for $41 million.

The paper is organized as follows. In Section 2 we describe the economic situation and the dynamic resale market. The resale market is modelled as a $T$-stage undiscounted game where the last stage $T$ stands for the deadline. At the beginning of each stage $t$, $t < T$, the current owner of the good may \textit{either} use the trading procedure, \textit{or} he may keep the object till the next stage. At stage $T$ the good may be consumed, and that consumption creates externalities on the other agents.

Possible interpretations of the deadline $T$ include: (1) The good to be traded is a bond with maturity $T$; (2) The good to be traded is a control right starting at time $T$. Note that imposing a deadline is a simple way to create a changing environment. The deadline expresses a change in regime after some known time.

In Section 3 we define the trading procedures that may be used at each stage of the resale market, and the time structure that connects the various stages.

The stage trading procedures involve a change in the identity of the owner and side-payments from the acquirer and possibly the non-acquirers to the seller. In line with the mechanism design approach, we assume that the current owner can choose the trading procedure. That is she can make a proposal to a subset of agents of her choice. Given a class of feasible trading procedures, the owner will choose the one that maximizes her revenue (while taking into account the possibility of resales in the future).

Concerning the time structure, we follow the \textit{dynamic bargaining} literature: we assume that if one of the approached agents refuses to participate in the mechanism organized by the owner, there is a delay (see Rubinstein’s bargaining model), and the owner must optimize again. The ability to delay captures an essential feature of the LVMH–Guiness–Grand Metropolitan story reported above, and it is a crucial feature of all dynamic bargaining models following Rubinstein’s contribution.

As a benchmark, we consider the class of mechanisms where the current owner can commit in advance to the allocation he is going to implement at future stages. In this framework we show that the outcome of the resale market is always efficient.

A major drawback within the above class of mechanisms is the fact that the seller’s ability to extract revenue hinges on commitments to threats that are not credible. These threats are embodied by the allocations that the seller commits to implement at future stages. In Section 4 we restrict the class of feasible mechanisms to mechanisms that employ only credible threats. Each approached agent has the possibility to delay the transaction by not participating, but the set of approached agents need not consist of the whole society. Given the time friction, the inability to commit represents a transaction cost: without the possibility of resale, this transaction cost may induce inefficiencies, and the final outcome may depend on the identity of the initial owner. In Subsection 4.1 we look at resale markets where, at each stage, only bilateral mechanisms are feasible, \textit{i.e.} the owner chooses the revenue-maximizing procedure among all bilateral mechanisms without commitment. The main result is that, for a long enough horizon, the outcome is always

\textsuperscript{5} The fear of delays was an important reason for the decision to switch to an allocation system based on a well-designed auction mechanism.
independent of the identity of the initial owner, although the outcome need not be efficient. Bilaterality is a serious constraint in frameworks with externalities, and we next relax it by allowing for multilateral trades. In Subsection 4.2 we briefly consider resale markets where the current owner cannot choose the subset of agents to which he makes a proposal, i.e. he must address the whole society in order to trade. Even though such a unanimity requirement is not realistic in most applications, we find it of interest to observe that the outcome is then always efficient. We illustrate how the constraint of unanimity reduces the seller's ability to extract surplus and how, without this constraint, sellers may wish to exclude various agents from proposed multilateral trades.

In Subsection 4.3 we study the resale market where, at each stage, the current owner may use any multilateral trading mechanism which is based only on credible threats. Here the owner need only satisfy the participation constraints of an (optimally) chosen subset of agents in order to trade with them. Non-approached agents do not have the option to delay agreement. The main result is that, for long enough horizons, the outcome is always independent of the identity of the initial owner. Although efficiency is always obtained for cases with no more than three agents, this need not be the case if there are at least four agents. We observe that the resulting outcome in the multilateral case need not coincide with its counterpart in the bilateral case. Therefore, a sequence of bilateral trades cannot always replace a multilateral agreement.

To sum up, the major conclusion of our paper is that the irrelevance of the initial structure of property rights arises in resale processes even if there are transaction costs that hinder efficiency. This result complements the Coasian argument that views the irrelevance of the assignment of property rights solely as a consequence of efficiency in a zero transaction costs world.

Several concluding comments are gathered in Section 5.

2. THE DYNAMIC RESALE MARKET

The market we consider consists of a set \( N \) of \( n \) agents, \( A_1, A_2, \ldots, A_n \), and an indivisible good. The good has known values to each agent, and the values are contingent on the identity of the consumer. If \( A_i \) is the consumer, he derives payoff \( \pi_i > 0 \), and agent \( A_j \) (\( j \neq i \)) derives payoff \( -\alpha_{ij} \). In other words, we consider identity-dependent externalities between the agents.

The good can be consumed at a given period \( T \), and it can be sold and resold before this period. The resale market is modelled as a \( T \)-stage game where stage \( T \) stands for the deadline. At the beginning of each stage \( t \), \( 1 \leq t < T \), the current owner of the good may either trade, or he may wait for one period. A trade at period \( t \) determines the owner at period \( t + 1 \), and a vector of monetary transfers among agents. In the next section we detail the considered trading procedures. At the final stage \( T \), there is no further sale, and the current owner can only consume the good with the effects on payoffs as described above. We assume that there is no discounting during the resale phase. Hence, besides possible side-payments during trading, the final payoffs to the agents depend only on the identity of the consumer of the good. The owner of the good at stage 1 is called the initial.

6. It is not very realistic because (1) it gives too much power to agents who may not be concerned with the trade and (2) the authority in charge of implementing the unanimity requirement may not be aware of who is really concerned with the trade.

7. We adopt this convention of sign because in competitive interactions externalities are, in general, negative so that \( \alpha_{ij} > 0 \). However, our analysis and results are valid whether externalities are positive or negative, and we make no assumption on the sign of these.
owner. It should be clear that, for a given stage trading procedure, we obtain for each initial owner a different resale game.

We focus our attention on Subgame Perfect Nash Equilibria (SPNE) of the various games. Thus, at each stage, agents take into account the consequences of their actions for the future. In particular, they have to consider the channel of future resales.

Definition 2.1. The outcome of the resale process is said to be efficient if, in equilibrium, the consumer is necessarily agent \( A_t \) such that \( i \in \text{ArgMax}_i (\pi_i - \sum_j \alpha_{ij}) \). The outcome of the resale process is said to be independent of the identity of the initial owner if, no matter who the initial owner was (i.e. in all possible resale games resulting from a given stage trade procedure), in equilibrium the good is consumed by the same agent.

If the outcome is always efficient, then it is necessarily independent of the identity of the initial owner. (This is the Coasian argument discussed in the Introduction.)

We assume that valuations and externalities are generic in the following sense: Consider any two sets of coefficients \( \{e_i\}_{i \in \{1, \ldots, n\}} \) and \( \{e_{ij}\}_{i, j \in \{1, \ldots, n\}, i \neq j} \) such that not all coefficients are null, and such that \( \forall i, e_i \in \{-1, 0, 1\} \), and \( \forall i, j, i \neq j, e_{ij} \in \{-1, 0, 1\} \). Then it must hold that

\[
\sum_i e_i \pi_i + \sum_i \sum_j e_{ij} \alpha_{ij} \neq 0.
\] (2.1)

The genericity assumption ensures that, for each set of parameters and for each horizon \( T \), either there exists a unique SPNE, or all SPNE are payoff-equivalent. It also ensures that all efficient outcomes are associated with the consumption of a unique agent. The above assumption requires that a finite set of equations is not satisfied. The set of excluded parameters has Lebesgue measure zero. Finally, note that the genericity assumption is sufficient for all our results below, but not necessary. All our results have analogues for non-generic settings, but in those settings one must keep track of certain symmetries.

3. TIME FRICTIONS AND COMMITMENT

The static mechanism design paradigm is usually modelled as a three-step game (see for example, Fudenberg and Tirole (1991)): In step 1, the principal proposes a mechanism (i.e. a game form in which agents send costless signals, and in which, based on the signals, an allocation is implemented). In step 2, the agents accept or refuse to participate in the mechanism. In step 3, the agents who agreed to participate play according to the rules specified in the chosen mechanism, and an allocation is implemented accordingly.

In a complete information framework the principal (i.e. the current owner of the good) can implement any feasible allocation subject to participation constraints, and step 3 in the mechanism design approach is trivial. In a "direct revelation approach" the only signals agents need to send concern their participation decisions.

The dynamic bargaining literature (see Osborn and Rubinstein (1990)) considers repeated interactions between two bargaining parties. At each step, one party (the "proposer") offers a way to split the surplus. The other party (the "responder") may either accept or reject the offer. Accepted offers are implemented, and the game proceeds to the

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8. The initial owner may have been determined by history, or he may have been selected at random, or he may have been selected through a mechanism not described here.

9. For example, all our illustrations below use non-generic parameters in order to simplify calculations. Nevertheless, our results hold unchanged also for those non-generic frameworks.
next stage after a rejected offer. From the point of view of mechanism design, the proposer plays the role of the principal, and the mechanisms at each given period specify a surplus division that applies only for that period. If the responder rejects an offer—this can be interpreted as the responder's refusal to participate in the mechanism—there is a time friction (some time elapses), and the game proceeds to the next stage.

We take from the bargaining literature the time friction feature. We take from the mechanism design literature the feature that the current owner is free to choose a mechanism that maximizes her expected payoff.

Our primary interest is the class of mechanisms in which only actions to be taken in the current stage are involved:

**Definition 3.1.** A mechanism without commitment for \( A_i \) at stage \( t \) is given by \( m^t_i = (S^t_i, f^t_i, g^t_i) \) such that:

- \( S^t_i \subseteq N \setminus \{A_i\} \);
- \( f^t_i \) is an agent in \( S^t_i \cup \{A_i\} \);
- \( g^t_i \) is a vector of payments in \( \mathbb{R}^{|S^t_i|} \), where \( |S^t_i| \) is the cardinality of \( S^t_i \).

We denote by \( M^{nc}_i \) the set of all mechanisms without commitment for agent \( A_i \).

The interpretation is as follows: The current owner \( A_i \) chooses a set of agents \( S^t_i \) to whom he wishes to propose a mechanism. The agents in \( S^t_i \) simultaneously decide whether they want to participate or not.\(^{10}\) If all agents in \( S^t_i \) participate, then in stage \( t \) an allocation is implemented such that the good is transferred to \( f^t_i \in S^t_i \cup \{A_i\} \) (it stays with the current owner if \( f^t_i = A_i \)), and the members of \( S^t_i \) make payments to the \( A_i \) according to the vector \( g^t_i \) (a negative payment means that the owner pays a compensation). We denote by \( g^{t}_{\setminus i} \) the payment from agent \( A_j \in S^t_i \) to the owner \( A_i \) according to \( g^t_i \).

If at least one agent in \( S^t_i \) refuses to participate, the game proceeds to stage \( t+1 \) without the good changing hands and without side payments. In other words, there is a time friction in case at least one addressed agent refuses to participate.

The above definition implicitly assumes that the good cannot be dumped upon an agent who does not participate, and that no payments can be obtained from agents who do not participate.\(^{11}\)

We want first to establish results for the case where agents can employ mechanisms that allow for commitments to actions in future stages (which would be the natural assumption in the static mechanism design literature).

**Definition 3.2.** A mechanism with commitment for \( A_i \) at stage \( t \) is given by \( m^t_i = (S^t_i, f^t_i, g^t_i)_{t \geq t_i} \), where:

- \( S^t_i \subseteq N \setminus \{A_i\} \) is the set of addressed agents;
- For \( t' \), \( t \leq t' < T \), \( f^t_i \) is a function that associates to each subset \( R \subseteq S^t_i \) an agent in \( R \cup \{A_i\} \);\(^{12}\)

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10. We restrict attention to equilibria which are robust to trembles. (We could equivalently consider the SPNE of the corresponding sequential game where each agent is asked, in turn, whether he wishes to participate.) This restriction allows us to rule out equilibria where everybody objects because if another agent objects all announcements are equivalent.

11. In principle, the owner may compensate non-participating agents, but, since she is interested in revenue maximization, this never occurs.

12. Additional consistency requirements on the functions \( f^t_i \) are needed. For the sake of brevity, we omit these requirements here.
- For \( t', t \leq t' < T \), \( g^t_i \) is a function that associates to each subset \( R \subseteq S'_i \) a vector of payments \( g^t_i(R) \in \mathbb{R}^{|R|} \), where \(|R|\) is the cardinality of the subset \( R \);
- For \( R \neq S'_i \), \( f^t_i(R) = A_i \), and \( g^t_i(R) = 0^{|R|} \).

We denote by \( M_i \) the set of all possible mechanisms for agent \( A_i \).

A set \( R \subseteq S'_i \) represents the subset of approached agents who accepted to participate in the proposed mechanism; \( f^t_i(R) \) is the agent to whom the object is transferred at stage \( t' \) when the subset \( R \) participates in the mechanism; similarly \( g^t_i(R) \) is the corresponding transfer at stage \( t \). If at least one approached agent refuses to participate, \( i.e. \ R \neq S'_i \), there is a delay, and therefore there is no trade at stage \( t \). Compared to the definition of mechanisms without commitment, the difference lies in the fact that actions at future stages \( t' > t \) may be included in a mechanism with commitment.\(^{13}\)

Proposition 3.4 below shows that efficiency is always achieved if agents can employ mechanisms with commitments. A simple illustration is given in the example below.

**Example 3.3.** There are three agents, \( A_i, i = 1, 2, 3 \). Payoffs are given by: \( \pi_1 = \pi_2 = \pi_3 = 5 \), \( \alpha_{21} = \alpha_{31} = 100 \), \( \alpha_{23} = 10 \), \( \alpha_{32} = 10 \), \( \alpha_{12} = \alpha_{13} = 0 \).

The efficient consumer is \( A_1 \). Assume the initial owner \( A_1 \) is free to choose any mechanism with commitment \( m_i \in M_i \). Consider stage \( t = T - 2 \). If \( A_1 \) is the owner at that stage, then a revenue maximizing procedure is as follows: \( A_1 \) addresses the other two agents and proposes the following mechanism: If both \( A_2 \) and \( A_3 \) participate then \( A_1 \) keeps the object till \( T \) (hence he consumes at stage \( T \)), and asks for payments of 10 from both \( A_2 \) and \( A_3 \). If \( A_2 \) refuses to participate, then \( A_1 \) transfers the good to \( A_2 \) for free at \( t = T - 1 \). If \( A_3 \) refuses to participate then \( A_1 \) transfers the good to \( A_2 \) for free at \( t = T - 1 \). If both \( A_2 \) and \( A_3 \) refuse to participate, then \( A_1 \) keeps the object till stage \( T \) and no payments are made.\(^{14}\)

Given this mechanism, both \( A_2 \) and \( A_3 \) choose to participate.\(^{15}\)

Assume now that \( A_2 \) owns the object at stage \( T - 2 \). A revenue maximizing procedure is as follows: The good is transferred to \( A_1 \) for a payment of 105, and \( A_2 \) is required to pay 10. If either \( A_1 \) or \( A_3 \) refuses to participate, then \( A_2 \) keeps the good, and no payments are made. Given this mechanism, both \( A_1 \) and \( A_3 \) choose to participate. The mechanism for \( A_1 \) is analogous. It follows that the outcome of the resale game for any \( T \geq 3 \) is always efficient, since \( A_1 \) always consumes the good at stage \( T \).

**Proposition 3.4.** Let \( T \geq 3 \), and assume that the initial owner \( A_i \) can choose a mechanism with commitment \( m_i \in M_i \). Then the final outcome is always efficient, irrespective of the identity of the initial owner.

**Proof.** The proof is an adaptation to the present model with a time friction of Proposition 1 in Jehiel, Moldovanu and Stacchetti (1996). From that proposition, a revenue maximizing mechanism \( \bar{m}_i = (\bar{S}_i^t, \bar{f}^t_i, \bar{g}^t_i)_{t \geq 1} \) has the following properties: (1)

\[\begin{align*}
13. & \text{Generally, mechanisms with commitments could also include actions in future stages to be taken by agents other than the current owner, \textit{e.g.} } A_i \text{ sells to } A_j \text{ with the provision that } A_j \text{ will not resell to another. Our result for mechanisms with commitment does not change in this more general set-up.} \\
14. & \text{Formally, } S_{1}^{T-2} = (A_2, A_3), \ f_{1}^{T-3}(S_{1}^{T-2}) = f_{1}^{T-1}(S_{1}^{T-2}) = A_1, \ f_{1}^{T-1}(A_2) = A_2, \ f_{1}^{T-1}(A_3) = A_3, \ f_{1}^{T-1}() = A_1, \ g_{12}^{T-3}(S_{1}^{T-2}) = g_{12}^{T-3}(S_{1}^{T-2}) = 10, \ g_{12}^{T-1}(S_{1}^{T-2}) = g_{12}^{T-1}(S_{1}^{T-2}) = 0, \ g_{12}^{T-1}(R) = 0, \ g_{12}^{T-1}(R) = 0 \text{ for } R \neq S_{1}^{T-2}. \\
15. & \text{Although } A_2, A_3 \text{ are indifferent between participating or not, equilibrium behaviour requires that both participate. Otherwise, } A_i \text{ could infinitesimally reduce the required payments. A general feature is that such indifferences disappear if we introduce a smallest money unit } o. \text{ Participation becomes then a dominant strategy in the revenue maximizing mechanism.}
\end{align*}\]
\[ S^i_t = N \setminus \{ A_i \} \]. (2) All members of \( N \setminus \{ A_i \} \) choose to participate. (3) \( f_t^T(S^i_t) \) is the efficient consumer.

Note that classical mechanisms such as first-price auctions, second-price auctions, etc… are mechanisms with commitment. In order to implement the outcome of a standard auction, the rules of the direct mechanism are such that in case an agent chooses not to participate, the owner commits to sell to another. Also, if the auction has a reserve price, then the owner commits not to sell in some cases even if all agents participate. In the presence of externalities classical auction mechanisms are, in general, neither efficient nor revenue-maximizing.

4. TRADE WITHOUT COMMITMENT

Example 3.3 illustrates a serious drawback of trading mechanisms with commitment: For example, \( A_i \)'s ability to extract payments when he is the stage \( T - 2 \) owner crucially depends on \( A_i \)'s commitment to sell the good for free at stage \( T - 1 \) if one of the other agents refuses to participate. Clearly, given the large negative externality imposed on \( A_i \) in case of a sale, this threat is not credible. In this section we focus on mechanisms that do not suffer from this drawback, and we thus restrict attention to mechanisms without commitment as defined above.

As we shall see below, in the framework without commitment, a crucial part in the seller's revenue maximization problem is the choice of the set of agents to whom a mechanism is proposed, i.e., the choice of \( S^i_t \). We first look at two cases where this choice is either simple or trivial: In Subsection 4.1 we study bilateral mechanisms, where the owner at stage \( t < T \) is constrained to address a single other agent; In Subsection 4.2 we study unanimous mechanisms, where the owner at stage \( t < T \) needs the consent of all other agents for a trade. Finally, in Subsection 4.3 we study the unconstrained revenue maximizing mechanism without commitment.

4.1. Bilateral trading

We start the analysis with bilateral trading procedures because these are the simplest and most common forms of sales. Moreover, the comparison with multilateral mechanisms without commitment provides some insight about the effect of the additional constraint of bilaterality.

Definition 4.1. A bilateral mechanism without commitment for \( A_i \) is a mechanism without commitment \( m^i_t = (S^i_t, f'_i, g'_i) \) such that \( |S^i_t| = 1 \). We denote by \( M^i_{bil} \) the set of all bilateral mechanisms without commitment for agent \( A_i \).

We now describe the form taken by the revenue-maximizing mechanism in this framework. Let \( X'_j(t + 1) \) be the payoff to agent \( A_j \) when agent \( A_i \) owns the good at stage \( t + 1 \), and assume that agent \( A_i \) owns the good at stage \( t \). If agent \( A_i \) decides to keep the object, then he can extract no money from agent \( A_{ij}, j \neq i \), since otherwise \( A_j \) would refuse the proposal. Hence if agent \( A_i \) keeps the object at stage \( t \) he gets \( X'_i(t + 1) \). If agent \( A_i \) decides to sell the object to agent \( A_{ij}, j \neq i \), then agent \( A_j \) must be willing to participate in the

16. Recall that \( S^i_t \) always contains all potential agents in the revenue maximizing mechanism with full commitment.
mechanism organized by $A_t$. Moreover, $A_t$ is ready to pay at most $X_j'(t+1) - X_i'(t+1)$, since at a higher price agent $A_j$ could profitably decide that he does not participate in $A_t$'s mechanism. Thus, if $A_t$ decides to sell to $A_j$, he will choose $m^l_t \in M_j^{bil}$ such that $S^l_t = \{A_j\}$, $f^l_t = \{A_j\}$, and $g^l_{ij} = X_j'(t+1) - X_i'(t+1)$. Agent $A_t$ thus obtains $X_j'(t+1) + X_j'(t+1) - X_i'(t+1)$ by selling to $A_j$. Revenue maximization means that $A_t$ will sell to $A_k$ such that $k = \text{Argmax}_j X_i'(t+1) + X_j'(t+1) - X_k'(t+1)$ if $X_k'(t+1) + X_j'(t+1) - X_i'(t+1)$ exceeds $X_i'(t+1)$, and will keep the object otherwise.

Note that the above described mechanism is just a take-it-or-leave-it bargaining procedure where the current owner is free to choose the agent to whom he wishes to sell. If there were no externalities (i.e. $\alpha_{ij} = 0$, $\forall i, j, i \neq j$) any owner would choose to make a take-it-or-leave-it offer to the efficient consumer $i = \text{argmax}_i \pi_i$ at a price of $\pi_i$, and $A_t$ would always consume. Without externalities, this procedure (which is bilateral and without commitment) is the overall revenue maximizing procedure (i.e. in the class of general feasible mechanisms). In our framework, the determination of the buyer to whom the offer is addressed, and of the asked price is slightly more complex.

**Lemma 4.2.** Assume that the parameters values are generic and that at all stages $t, t < T$, an owner $A_t$ can choose any mechanism $m^l_t \in M_j^{bil}$. Let $\Gamma_i^t$ be the subgame starting at stage $t$ with $A_i$ being the current owner. Then, in all SPNE of $\Gamma_i^t$ the good is consumed by the same consumer, denoted by $FC(A_i, t, T)$. Moreover, for any agent $A_j$, all SPNE of $\Gamma_i^t$ yield the same payoff to $A_j$, denoted by $X_j'(t)$. It holds that

\[
\begin{align*}
& (i) \quad \forall t, \forall i, j, j \neq i, \exists k, \quad \text{s.t.} \quad X_j'(t) = -\alpha_{kj}, \\
& (ii) \quad \forall t, \exists \epsilon_{jk} \in \{-1, 0, 1\}, \quad \text{s.t.} \quad X_j'(t) = \pi_j + \sum_j \sum_k \epsilon_{jk} \alpha_{jk},
\end{align*}
\]

where $A_{i_1} = FC(A_i, t, T)$.

**Proof.** See Appendix. ||

**Proposition 4.3.** Assume that at all stages $t, t < T$, an owner $A_t$ can choose any mechanism $m^l_t \in M_j^{bil}$. Assume that the parameter set takes generic values. Then, if $T$ is large enough, the identity of the final consumer (and thus the equilibrium total welfare) is independent of the identity of the initial owner. That is,

$$\exists T^* \quad \text{such that} \quad \forall T, T' > T^*, \forall i, j, \quad FC(A_i, 1, T) = FC(A_j, 1, T').$$

**Proof.** See Appendix. ||

The strategy of the proof is as follows. We first observe that, in equilibrium, a finite, fixed number of stages is sufficient to exhaust all gains from trade. Moreover, at a stage where no profitable transaction exists between agents $A_i$ and $A_j$, the sum of equilibrium payoffs for $A_i$ and $A_j$ when $A_i$ is the owner must be the same as the sum when $A_j$ is the owner. (Otherwise, either $A_i$ or $A_j$ would be willing to transact.) From Lemma 4.3, we know that both these sums are combinations of valuations and externalities with coefficients that belong to the set $\{-1, 0, 1\}$. This allows us to conclude that, for generic parameters, the two sums of payoffs must be associated with the consumption of the same agent. Hence, the final consumer must be the same, irrespective of whether $A_i$ or $A_j$ was the initial owner.
To get some idea of how the resale market as well as the independence result work, consider the following example.

**Example 4.4.** There are three agents, $A_i, i = 1, 2, 3$. Payoffs are given by: $\pi_1 = \pi_2 = \pi_3 = 5$, $\alpha_{12} = 2$, $\alpha_{23} = 3$, $\alpha_{31} = 3$, and all other externality terms are null. The total number of stages is $T = 4$.

The equilibrium process of sales is described by the following flow diagram. At stage $t = T = 4$, if agent $A_i$ owns the good, he consumes it (since $\pi_i > 0$).

At stage $t = 3$, if agent $A_1$ owns the good, he sells it to agent $A_2$ at price $\pi_2 + \alpha_{12} = 7$. He then obtains: $\pi_2 + \alpha_{12} - \alpha_{21} = 7$, which is more than what he could get by: (1) not selling ($\pi_1 = 5$), or (2) by selling to agent $A_3$ ($\pi_3 + \alpha_{13} - \alpha_{31} = 2$). The vector of equilibrium payoffs to agents $A_1, A_2, A_3$ is $(7, -2, -3)$. Similarly, if agent $A_2$ owns the good he sells it to agent $A_3$ at price 8, and the payoffs are $(-3, 8, -3)$. If agent $A_3$ is the owner, he sells to $A_1$ at price 8, and the payoffs are $(-3, -2, 8)$.

At stage $t = 2$, when agent $A_1$ is the owner he sells to agent $A_3$ at price 11. The payoff to $A_1$ is then $11 - 3 = 8$ (since $-3$ is $A_1$'s payoff when $A_3$ is the owner at $t = 3$). This is more than what $A_1$ could have got by selling to agent $A_2$, i.e. $8 + 2 - 3 = 7$, or by waiting, i.e. 7. The resulting payoff vector is $(8, -2, 3)$. Similarly, $A_2$ sells to $A_3$ at price 11, and the resulting payoffs are $(-3, 9, -3)$. Finally, $A_3$ chooses to wait and the resulting payoffs are $(-3, -2, 8)$.

At stage $t = 1$, whoever owns the good finds it optimal to keep it.

Observe that, irrespective of who owns the good at stage $t = 1$, the final consumer is invariably $A_1$. The same result applies for all $T \geq 4$, since equilibrium behaviour requires
then that all initial owners wait till stage \( T - 3 \). Moreover, when \( A_1 \) is the initial owner, the good goes first to \( A_3 \) and then goes back to \( A_1 \). Hence the SPNE displays cycles in terms of ownership. Such cycles may seem, at first glance, paradoxical, and one may wonder why agent \( A_1 \)rationally sells to \( A_3 \) if he knows that he will eventually buy the good back. He does so because agent \( A_3 \) is so afraid that agent \( A_1 \) sells to agent \( A_2 \) at a later stage (\( \alpha_{23} \) is large) that \( A_3 \) is ready to pay a lot today. As a result, \( A_1 \) finds is strictly more advantageous to sell to \( A_3 \) rather than keep the good till the deadline.

If no resales were permitted, the outcome would be that of stage \( t = 3 \). That is, if agent \( A_1 \) is the initial owner, agent \( A_2 \) gets the good. If agent \( A_2 \) is the initial owner, agent \( A_3 \) gets the good. If \( A_3 \) is the initial owner, agent \( A_1 \) gets the good. Thus, if resales were not permitted the identity of the initial owner would matter for the determination of the final owner, and welfare would be affected accordingly.

Finally, observe that agent \( A_1 \) is the welfare-maximizing consumer. Hence, in Example 4.4 bilateral trade with resales leads to an efficient outcome. In particular, it is superior to the institution where resales are not permitted, in which case the final outcome is efficient only if \( A_3 \) is the initial owner.

Example 4.4 illustrates the independence property of Proposition 4.3. It also suggests that the final outcome may be efficient. The efficiency result always holds when there are only two agents, but, as shown in Example 4.5, may fail with three or more agents.

**Example 4.5.** There are three agents, \( A_i, i = 1, 2, 3 \). Payoffs are given by: \( \pi_1 = \pi_2 = \pi_3 = 10, \alpha_{12} = 5, \alpha_{21} = 6, \alpha_{31} = 1 \), and all other externality terms are null. The total number of stages is \( T = 4 \).

The sale process is given by the following flow diagram:

The final consumer is invariably \( A_1 \), which leads to a suboptimal allocation, since the optimum is achieved when \( A_3 \) is the final consumer.

We now briefly explain the role of the various constraints that lead to inefficiency in Example 4.5. Assume that agent \( A_3 \) (the welfare maximizing agent) owns the good at stage 3. The optimal, full commitment, contract for \( A_3 \) (which, by Proposition 3.3, necessarily leads to an efficient outcome) has the following form: Agent \( A_3 \) commits not to sell the good in exchange for a payment of 5 from \( A_2 \) and a payment of 6 from \( A_1 \). If \( A_1 \) (resp. \( A_2 \)) refuses to pay, agent \( A_3 \) commits to sell to agent \( A_2 \) (resp. \( A_1 \)). Our noncommitment paradigm does not allow for such contracts. (If \( A_3 \) does not sell the good at stage 3, then he is the owner of the good at stage 4, the deadline, and therefore agent \( A_3 \) is unable to make his threats to the other two agents credible.) Moreover, the bilateral trading constraint does not permit multilateral agreements that involve all three agents, even if those agreements are based on credible threats for the future. In fact, we show below (see Subsection 4.3) that, when multilateral agreements without commitment are feasible, the introduction of resale markets is sufficient to restore efficiency in situations with no more than 3 agents.

So far we have shown that the identity of the final consumer is the same irrespective of the initial owner, and that the final consumer need not be the welfare-maximizing agent. It is of interest to identify the final consumer as a function of valuations and externalities. The general identification is difficult, but, to get some insight, we now briefly consider two special cases.

When all externality terms are equal, it is readily verified that the outcome of the resale game is efficient (it is the same insight as that when there are no externalities). We
now prove a stronger version of this result (Proposition 4.6), and exhibit another class of parameter sets for which the final consumer can be explicitly identified (Proposition 4.7).

**Proposition 4.6.** Assume that externalities depend only on the identity of the sufferer, i.e. $\alpha_{ij} = \alpha_{kj} = \alpha_j$ for all $i,k \neq j$. Let $T \geq 2$, and assume that at all stages $t$, $t < T$, an owner $A_i$ can choose any mechanism $m_i \in M_i^{bil}$. Then the outcome of the resale game is always efficient.17

**Proof.** The efficient consumer $A_i$ satisfies $i \in \arg\max_j (\pi_j - \sum_{k \neq j} \alpha_k)$. Consider period $T - 1$, and assume that the owner at that period is $A_i$. The maximum price that any agent $A_j$, $j \neq i$ is willing to pay is given by $\pi_j + \alpha_j$. Owner $A_i$ can obtain a maximum payoff of

$$
\max (\pi_i, \max_{j \neq i} (\pi_j + \alpha_j - \alpha_i)) = \max_j (\pi_j + \alpha_j) - \alpha_i
$$

$$
= \max_j (\pi_j + \alpha_j - \sum_k \alpha_k) - \alpha_i + \sum_k \alpha_k
$$

$$
= \max_j (\pi_j - \sum_{k \neq j} \alpha_k) - \alpha_i + \sum_k \alpha_k,
$$

by selling to the efficient consumer. Hence, no matter what happens before stage $T - 1$, the efficient consumer will consume at stage $T$. ||

17. This is not a generic setting, but, as we show in the proof, the independence results holds as well. To be consistent with the genericity assumption made throughout the paper, we could consider generic parameters that lie in a sufficiently small neighbourhood of the parameters considered here. The proof trivially extends to that case as well. The same comment applies to Propositions 4.7 and 4.12(iii) below, and will not be repeated there.
Proposition 4.7. Assume that externalities depend only on the identity of the consumer i.e. $\alpha_{ij} = \alpha_{ik} = \alpha'$ for all $j, k \neq i$. Let $T \geq 2$, and assume that at all stages $t, t < T$, an owner $A_i$ can choose any mechanism $m_i^t \in M_i^t$. Then, the final consumer is always $A_{i'}$ where $i' = \text{argmax}_i (\pi_i - \alpha')$.

Proof. Consider stage $T - 1$, and let $A_i$ be the owner at that stage. The maximum price that any agent $A_j, j \neq i$ is willing to pay is given by $\pi_j + \alpha'$. Hence $A_i$ can obtain a maximum payoff of $\max (\pi_i, \max_{j \neq i} (\pi_j + \alpha' - \alpha')) = \max_i (\pi_j - \alpha') + \alpha'$ by selling to $A_{i'}$.

When there are $n$ agents the above result shows that the outcome is efficient only if $\pi_i - n\alpha' = \text{maximal}$ among all $\pi_i - n\alpha'$. This need not be the case in general, but if all $\alpha'$ are equal, i.e. $\alpha' = \alpha$, the final outcome is always efficient (since Proposition 4.6 applies).

4.2. Unanimous trading

A simple form of multilateral trading without commitment is that where the owner of the good must address all agents in order to trade. Hence, the owner cannot maximize over the set of agents to whom he wishes to make a proposal.

Definition 4.8. A unanimous mechanism without commitment for $A_i$ is a mechanism without commitment $m_i^t = (S_i^t, f_i^t(\cdot), g_i^t(\cdot))$ such that $S_i = N \setminus \{A_i\}$. We denote by $M_i^u$ the set of unanimous mechanisms without commitment.

Proposition 4.9. Let $T \geq 2$, and assume that at all stages $t, t < T$, an owner $A_i$ can choose any mechanism $m_i^t \in M_i^u$. Then the outcome of the resale process is always efficient (and hence independent of the identity of the initial owner).

Proof. Let $A_i$ be the owner at stage $T - 1$. Waiting one more stage yields $\pi_i$. Assume that $A_i$ wants to sell to $A_j$. This trade needs the approval of all agents. Agent $A_j$ is prepared to pay $\pi_j + \alpha_{ij}$ for the good, and causes an externality of $-\alpha_{ij}$ on $A_i$. Any other agent $A_k$, is prepared to pay $\alpha_{ik} - \alpha_{jk}$ for the good to be transferred to $A_j$ (note that the “payment” may be negative, in which case $A_k$ must be compensated for the transfer). Owner $A_i$ obtains the highest possible payoff max $\max_j (\pi_j + \alpha_{ij} - \alpha_{jk} + \sum_{k \neq i} \alpha_{ik} - \alpha_{jk}) = \max_j (\pi_j - \sum_{k \neq j} \alpha_{jk}) + \sum_{k \neq i} \alpha_{ik}$ by selling to the efficient consumer. Hence, no matter what happens before stage $T - 1$, the efficient consumer will consume at stage $T$.

Although institutions with the above feature do exist, they seem hardly compatible with the common sense of “property rights”. Quite often such institutions are inefficient because there is a risk that trades are delayed by agents who are not actually harmed by the transaction.

4.3. Revenue maximizing trading without commitment

The analysis above shows that the requirement of unanimity imposes severe constraints on the seller, since she is sometimes obliged to compensate agents for her actions. We now wish to study the channel of sales and resales where each stage $t$ owner can choose any mechanism without commitment. It will now be in the interest of the seller to sometimes exclude agents from a transaction.
Let us describe the form of the revenue-maximizing mechanisms. Let $X_j'(t+1)$ be the payoff to agent $A_j$, when agent $A_i$ owns the good at stage $t+1$, and assume that agent $A_i$ owns the good at stage $t$.

If agent $A_i$ decides to keep the object, then he can extract no money from agent $A_j$, $j \neq i$, since otherwise $A_j$ would refuse the proposal. Hence if agent $A_i$ keeps the object at stage $t$ he gets $X_i'(t+1)$. If agent $A_i$ decides to sell the object to agent $A_j$, $j \neq i$, then agent $A_i$ must be willing to participate in the mechanism organized by $A_i$. Moreover, as in the take-it-or-leave-it institution, $A_j$ is ready to pay at most $X_j'(t+1) - X_i'(t+1)$. Whether $A_i$ will choose to exclude agent $A_k$ or not from his mechanism depends on whether agent $A_k$ prefers agent $A_i$, or agent $A_j$ having the good at stage $t+1$. If agent $A_k$ prefers agent $A_i$ having the good, then agent $A_i$ will exclude $A_k$ from the mechanism, since in case $A_k$ were to be included, obtaining $A_k$'s participation would require that $A_i$ pays something to $A_k$.

Symmetrically, if agent $A_k$ prefers agent $A_j$ having the good, then $A_i$ will include $A_k$ in his mechanism and ask for a side payment equal to the difference between what $A_k$ gets in the two situations, i.e. $X_j'(t+1) - X_i'(t+1)$.

To summarize, if the owner $A_i$ chooses to sell to $A_j$ he will choose a mechanism $m_i'$ such that $S'_i = \{A_j\} \cup \{A_k\}$ such that $X'_k(t+1) - X'_i(t+1) \geq 0$, $f'_i = A_j$, and $g'_k = X'_k(t+1) - X'_i(t+1)$ for all $A_k \in S'_i$ (including $A_j$). Thus, $A_i$ overall obtains $X'_i(t+1) + X'_j(t+1) - X'_i(t+1) \geq 0 + \sum_{k \neq i,j} \max(0, X'_k(t+1) - X'_i(t+1))$ by selling to $A_j$. Note that the main addition to the bilateral revenue maximizing procedure is that the seller is able to extract revenue also from non-acquirers.

In order to determine whether $A_i$ will sell the good, and if yes to whom, it is convenient to define

$$W_i'(t+1) = \sum_k X'_k(t+1),$$

as the welfare associated with the ownership of agent $A_i$ at stage $t+1$, and

$$\Delta'_i(t+1) = \max(X'_i(t+1) - X'_k(t+1), 0) - (X'_i(t+1) - X'_k(t+1))$$

$$= \max(X'_i(t+1) - X'_k(t+1), 0).$$

(4.2)

It is readily verified that $A_i$ prefers to wait rather than sell to $A_j$ if

$$X'_i(t+1) \geq X'_i(t+1) + X'_j(t+1) - X'_i(t+1) + \sum_{k \neq i,j} \max(0, X'_k(t+1) - X'_i(t+1)),$$

or, after simple manipulations, if

$$W_i'(t+1) + \sum_{k \neq i,j} \Delta'_k(t+1) \leq W'_i(t+1).$$

(4.3)

Similarly $A_i$ prefers to sell to $A_j$ rather than to $A_j'$ if

$$W_{i}'(t+1) + \sum_{k \neq i,j'} \Delta'_k(t+1) < W_{i}'(t+1) + \sum_{k \neq i,j} \Delta'_k(t+1).$$

(4.4)

Note that $\Delta'_k(t+1)$ is always non-negative. When $A_i$ sells to $A_j$, he excludes $A_k$ if $\Delta'_k(t+1) > 0$, and includes $A_k$ in the mechanism if $\Delta'_k(t+1) = 0$. The conditions above allow us to find out the mechanism chosen by the owner $A_i$ at stage $t$, given expected equilibrium payoffs $X'_j(t+1)$.

**Lemma 4.10.** Assume that the parameters values are generic, and that at all stages $t, t < T$, an owner $A_i$ can choose any mechanism $m'_i \in M'^{ro}$. Let $\Gamma'_i$ be the subgame starting at stage $t$ with $A_i$ being the current owner. Then, in all SPNE of $\Gamma'_i$ the good is consumed by the same agent, denoted by $FC(A_i, t, T)$. Moreover, for any agent $A_j$, all SPNE of $\Gamma'_i$ yield
the same payoff to $A_j$, denoted by $X_j^i(t)$. It holds that

(i) $\forall t, \forall i, j \neq i, \exists k, \text{ s.t. } X_j^i(t) = -\alpha_{kj},$

(ii) $\forall t, \exists \epsilon_{jk} \in \{-1, 0, 1\} \text{ s.t. } X_j^i(t) = \pi_{i*} + \sum_j \sum_k \epsilon_{jk} \alpha_{jk},$

where $A_i* = FC(A_i, t, T)$.

Proof. See Appendix. ||

**Proposition 4.11.** Assume that at all stages $t, t < T$, an owner $A_i$ can choose any mechanism $m_i' \in M_i^\text{me}$. Assume that the parameter set takes generic values. Then, if $T$ is large enough, the identity of the final consumer and thus the equilibrium total welfare is independent of the identity of the initial owner. That is,

$\exists T^* \text{ such that } \forall T, T' > T^*, \forall i, j, FC(A_i, 1, T) = FC(A_j, 1, T').$

Proof. See Appendix. ||

The intuition for the proof is very similar to that for the proof of Proposition 4.3, and we omit it here. We now wish to analyse whether the final consumer (who is well defined because of the independence property) maximizes total welfare, i.e. whether the process is efficient.

**Proposition 4.12.** Assume that at all stages $t, t < T$, an owner $A_i$ can choose a mechanism $m_i' \in M_i^\text{me}$. Assume that the parameter set takes generic values, and that $T$ is large enough.

(i) If $n \leq 3$, the outcome is always efficient.

(ii) If $n > 3$, the outcome is not necessarily efficient.

(iii) Assume that externalities depend only on the identity of the consumer i.e. $\alpha_{ij} = \alpha_{ik} = \alpha'_{ij}$ for all $j, k \neq i$. Then, the outcome is always efficient, irrespective of the number of agents.

Proof. The proof of (i) is relegated to the Appendix. The proof of (ii) is by means of Example 4.14 below. For the proof of (iii), let $t* = \text{ArgMax}_t \alpha'$. It is enough to show that when agent $A_{t*}$ is the stage $t, t < T$, owner, the final consumer is the welfare-maximizing agent. Together with the independence property, this yields the wished for result. Consider stage $T - 1$. If $A_{t*}$ sells to $A_i$ he invites every other agent $A_k$ to participate because $A_k$ fears $A_{t*}$ more than $A_i$. In other words, $\forall k, j, \Delta_j^{t*, T}(T) = 0$. Conditions 4.3 and 4.4 allow us to conclude for stage $T - 1$. The argument for earlier stages is obtained by induction. ||

To illustrate Proposition 4.12(i) we revisit Example 4.5 which yielded inefficiency in the bilateral framework.

**Example 4.13.** There are three agents, $A_i, i = 1, 2, 3$. Payoffs are given by: $\pi_1 = \pi_2 = \pi_3 = 10, \alpha_{12} = 5, \alpha_{21} = 6, \alpha_{31} = 1$, and all other externality terms are null.

Note first that without resale possibilities the outcome is not necessarily efficient, even if agents can use any mechanism without commitment: When $A_3$ (the efficient consumer) is the initial owner he sells to $A_2$, and this agent consumes.
When resales are possible, the sequence of equilibrium sales is described in the following flow diagram.

It invariably leads to $A_3$ (the welfare maximizing agent) consuming the good. The comparison of the flows in Figures 2 and 3 shows that at stage 3, $A_2$ sells to $A_1$ in the bilateral trading institution and to $A_3$ in the general noncommitment trading institution. The reason is that, in the general noncommitment trading institution, by selling to $A_3$, $A_2$ is also able to extract some payment from $A_1$ (because $A_1$ fears $A_2$ more than $A_3$), and that sale is more profitable.

The following example illustrates Proposition 4.12(ii).

**Example 4.14.** Let $n = 4$, $T \geq 2$, $\pi_1 = 6.5$, $\pi_2 = 10$, $\pi_3 = 9$, $\pi_4 = 7$, $\alpha_{12} = 0$, $\alpha_{13} = 0$, $\alpha_{14} = 0$, $\alpha_{21} = 1$, $\alpha_{23} = 1$, $\alpha_{24} = 2$, $\alpha_{31} = 2$, $\alpha_{32} = 0$, $\alpha_{34} = 1$, $\alpha_{41} = 0$, $\alpha_{42} = 0$, $\alpha_{43} = 1$.

We show that no matter who is the owner at stage $T-1$, the good is never sold to $A_1$ (nor does $A_1$ choose to wait at that stage). Since consumption by agent $A_1$ yields a higher welfare than the consumption by the other three agents, the final outcome cannot be efficient.

At stage $T-1$ agent $A_1$ sells to $A_2$ (while excluding the other two agents). Agent $A_2$ sells to $A_3$ (while inviting agent $A_4$ and excluding agent $A_1$). Agent $A_3$ sells to $A_2$ (while inviting agent $A_4$ and excluding agent $A_1$). Agent $A_4$ sells to $A_2$ (while excluding the other two agents).

Propositions 4.11 and 4.12(ii) show together that the independence property may hold in resale markets even if the outcome is not efficient. Proposition 4.12(iii) shows that...
the externality terms must be sufficiently heterogenous to get an inefficient outcome. Points (i) and (iii) of Proposition 4.12 show together that resale markets can sometimes restore efficiency, even if commitments are not possible. Recalling the inefficiencies obtained in the same contexts with bilateral mechanisms (see Example 4.5 and Proposition 4.7), it is important to note the crucial role played here by multilateral. Finally, point (ii) shows that resale markets are not sufficient to restore efficiency, even if multilateral agreements are feasible. More precisely, the remaining source of inefficiency can be attributed to two elements: (1) the lack of commitment to future actions which forces the threats used by the agents to be credible, and (2) the possibility of exclusion (recall that unanimous mechanisms without commitment yield efficient outcomes).

5. CONCLUSION

In a framework with externalities, and where commitments to future actions are not available, we have shown that the outcome of resale markets need not be efficient, even though it is independent of the initial structure of property rights. This result has several policy implications: (1) Well-functioning resale markets may not be enough to achieve efficiency. (2) There may be no way to achieve efficiency, even if a sophisticated rule for the assignment of initial property rights is used. (3) Legal frameworks for multilateral trading negotiations may be desirable (and do not necessarily constitute a form of collusion).

It is interesting to recall that one of the main reasons for the creation of the FCC was that, prior to 1927, the courts held that the Secretary of Commerce (who granted licenses to use spectrum) was not authorized to deny licenses on the ground that they would cause interference, nor limit the licensees' power, frequency, or hours of operation. As a consequence, airwaves became filled with interfering signals, severely reducing the ability to make use of spectrum. The present FCC has the power to regulate the use of a licence after it has been awarded, but the Commission’s Deputy Chief Economist (see Rosston and Steinberg (1997)) thinks that the Commission should:

1. “avoid mandating that spectrum be used to provide specific services”;
2. “create mechanisms for voluntary changes in spectrum use, including where appropriate, procedures for new licensees and incumbents to negotiate compensated relocation of incumbents” and “also consider expanding spectrum users’ flexibility to negotiate among themselves interference limitations that may differ from those specified in the rules”;
3. “continue to define the extent to which each spectrum user may expect freedom from interference... Under some circumstances, this definition may consist of a determination that particular users enjoy little or no freedom from interference”.

It seems to us that the above quotations point to a trading framework that is somewhat similar to the one studied in this paper: there are resale possibilities, multilateral agreements are encouraged, and the Commission does not force users to commit to certain future actions (such as not selling to someone who creates interferences).

We can relate our model to two strands of the literature. The noncommitment paradigm also appears in the dynamic mechanism design literature which identifies the ratchet effect in contexts where the principal is uninformed about the agent’s type, and the principal’s as well as the agent’s type do not change over time (see Freixas et al. (1985), Laffont and Tirole (1988)). In contrast, our focus here has been on the dynamic change in ownership in a complete information setting.
Most of the bargaining literature (see Osborne and Rubinstein (1990)) considers models where only bilateral trades are feasible, and where resale markets are not available.\footnote{A notable exception is Gul (1989). However Gul does not derive inefficiency results.} Groes and Tranaes (1995) study a model of bargaining with resale possibilities between a seller and two buyers with different valuations where trades are constrained to be bilateral. In the case of a durable good, they show how resales may correct the rather artificial inefficiency appearing in such a decentralized model of trading. Even with a non-durable good inefficiencies would disappear in their framework if multilateral trades were allowed. In contrast, our analysis shows that externalities may be a robust source of inefficiency when commitments to future actions are not feasible even if there are resale possibilities and multilateral trades are allowed.

We now briefly mention some results obtained when other departures from the zero-transaction costs paradigm are considered. First, when utilities are non-transferable, the independence property is no longer true, even though the final outcome may be Pareto-efficient. Second, as shown by Myerson and Satterthwaite (1983), the outcome of the bargaining between two asymmetrically informed parties is not, in general, \textit{ex post} efficient. Hence asymmetric information induces inefficient bargaining.\footnote{For the dissolution of a partnership, Cramton \textit{et al}. (1987) show that \textit{ex-post} efficiency can be achieved if ownership is sufficiently diffused. Hence, the structure of ownership affects the efficiency of the final outcome.} Third, the incomplete contract literature shows that the structure of ownership has efficiency implications and derives the optimal structures of ownership (see Grossman and Hart (1986), and Hart and Moore (1990)), assuming contracts cannot be signed on unforeseen and unverifiable contingencies.

\section*{APPENDIX}

\textit{Proof of Lemma 4.2}

The proof is by induction on \( T - t \). At stage \( T \), the property is immediate given the payoffs described in Section 2 and the fact in equilibrium the period \( T \) owner finds it optimal to consume the good (given that \( \pi_i > 0 \) for all \( i \)).

Assume that Lemma 4.2 holds at all stages \( t' \), \( t < t' \leq T \), and for every stage \( t' \) owner. We will show that it holds for the \( t \) stage game.

At stage \( t \), the current owner \( A_i \) may either decide to wait for one more stage, which yields a payoff of \( X'(t+1) \) for him and of \( X_j'(t+1) \) for every agent \( A_j, j \neq i \), or he may decide to sell to some agent \( A_j, j \neq i \). In the latter case, agent \( A_i \) would refuse any offer such that his resulting payoff is less than \( X_j'(t+1) \), in case of non-acceptance, \( A_i \) has to keep the object for one more stage, and thus \( A_i \) can guarantee \( X_j'(t+1) \). In equilibrium, agent \( A_i \) will optimally make a proposal such that agent \( A_j \) is kept at his minimum payoff. It follows that if agent \( A_i \) sells to \( A_j, A_i \), obtains \( X_j'(t+1) + X_j'(t+1) - X_j'(t+1) \), agent \( A_j \) obtains \( X_j'(t+1) \) and all other agents \( A_{i'}, i' \neq i, j \) obtain \( X_{i'}'(t+1) \) (since there are no side-payments from \( A_i \) who does not acquire the good).

If agent \( A_i \) strictly prefers one of the above alternatives, then there is a unique SPNE of \( \Gamma' \). Moreover, if agent \( A_i \) optimally decides to wait for one more period, then \( X_j'(t) = X_j'(t+1) \) for all \( j \), and therefore \( X_j'(t), j \neq i \), and \( X_i'(t) \) satisfy the properties (i) and (ii), respectively. If agent \( A_i \) optimally decides to sell to agent \( A_j \), then \( X_j'(t) = X_j'(t+1) \) and \( X_i'(t) = X_i'(t+1) \) for all \( i' \neq i, j \), and thus property (i) is satisfied. Regarding property (ii), note that the welfare term \( W(t) = X_i'(t) + \sum_{k \neq j} X_k'(t) \) must be associated with the consumption of some agent \( A_{i'} \), i.e., \( W(t) = \pi_i - \sum_{k \neq j} \alpha_{r_{ik}} \); given the form of \( X_i'(t) \) just shown, we conclude that \( X(t) \) satisfies property (ii).

Consider now the case where \( A_i \) is indifferent between several alternatives. Assume that \( A_i \) is indifferent between selling to \( A_j \) or to \( A_k \), where \( j \neq k \neq i \). (The proof for the case where \( A_i \) is indifferent between waiting and selling is completely analogous.) It must be the case that \( X_j'(t+1) + X_j'(t+1) - X_j'(t+1) = X_k'(t+1) + X_k'(t+1) - X_k'(t+1) \). By the genericity assumption, and by the assumed form of the terms \( X_i'(t+1) \), we can conclude that the equilibrium final consumer in \( \Gamma_{i}^{-1} \) is the same as that in \( \Gamma_{k}^{-1} \). But then the welfare terms \( X_j'(t+1) + \sum_{i \neq j} X_i'(t+1) \) and \( X_k'(t+1) + \sum_{i \neq k} X_i'(t+1) \) are the same. The genericity assumption,
the fact that \( X_j'(t+1) + X_i'(t+1) - X_i'(t+1) - X_j'(t+1) = X_{ik}'(t+1) + X_{ik}'(t+1) - X_{ik}'(t+1) \), and the form of \( X_{ik}'(t+1) \), imply that \( X_i'(t+1) = X_{ik}'(t+1) \) for \( i' \neq j, k \), and that \( X_i'(t+1) = X_j'(t+1) \) and \( X_j'(t+1) = X_{ik}'(t+1) \). Hence, whatever optimal choice made by agent \( A_i \), the ensuing equilibrium payoffs are the same for every agent, and they satisfy properties (i) and (ii).

**Proof of Proposition 4.3**

For any \( T > \tau \) we define \( Y_i'(\tau) \) as the equilibrium payoff of \( A_i \) in the subgame starting at stage \( T - \tau \) with current owner \( A_j \) (this is well defined by Lemma 4.2). With the notation of Lemma 4.2, we have \( Y_i'(\tau) = Y_i'(T - \tau) \). By Lemma 4.2(i) we know that \( \forall \tau, Y_i'(\tau) \) is of the form \( \pi_i + \sum_{h} e_{jk} \alpha_{jk} \) where \( h \in \{1, \ldots, n\} \) and \( \forall j, k, e_{jk} \in \{-1, 0, 1\} \). When \( h \in \{1, \ldots, n\} \) and \( e_{jk} \in \{-1, 0, 1\} \) vary, the set of terms of the form \( \pi_i + \sum_{j,k} e_{jk} \alpha_{jk} \) is finite. Hence, for each \( i, \mu_i = \max_{1 \leq s \leq \infty} Y_i'(\tau_i) \) is well defined, and there exists a \( \tau_i \) such that \( \mu_i = Y_i'(\tau_i) \).

We assume here that the game is long enough in the sense that \( T \geq \alpha \tau_i \).

The function \( t \rightarrow X_i'(t) \) is non-increasing since, at stage \( t \), an owner \( A_i \) always chooses to wait one more stage and obtain \( X_i'(t+1) \). This monotonicity yields \( X_i'(t) = X_i'(t+1) \) for all \( t < T - \tau_i \). Note that \( X_i'(t) = X_i'(t+1) \) implies that agent \( A_i \) can optimally choose to wait at stage \( t \). Let \( t^* = \min_i(T - \tau_i) \). At any stage \( t < t^* \), any owner \( A_i \) finds it optimal to wait. Hence, \( \forall i, j, X_i'(t) = X_j'(t+1) \). This implies that at stage \( t < t^* \), agent \( A_j \) does not strictly prefer to sell to agent \( A_i \) rather than waiting, i.e.

\[
X_i'(t+1) \geq X_j'(t+1) - X_j'(t+1) + X_i'(t+1). 
\]

The right-hand side of equation (6.1) is non-strictly monotonic, and agent \( A_j \) is set to \( X_j'(t+1) \). Similarly, agent \( A_j \) does not strictly prefer to sell to agent \( A_i \) rather than waiting

\[
X_j'(t+1) \geq X_j'(t+1) - X_i'(t+1) + X_j'(t+1). 
\]

Combining equations (6.1) and (6.2) yields

\[
X_i'(t+1) = X_j'(t+1) - X_j'(t+1) + X_i'(t+1). 
\]

By Lemma 4.2 we have \( X_i'(t+1) = \pi_i + \sum_{h} e_{jk} \alpha_{jk} \), \( X_j'(t+1) = \pi_j + \sum_{h} e_{jk} \alpha_{jk} \), and \( X_i'(t+1) = - \alpha_{ij} \), \( X_j'(t+1) = - \alpha_{jk} \). Generically, equation (6.3) implies that \( t^* = j^* \). Hence, the final consumer is the same whether agent \( A_i \) or agent \( A_j \) is the stage t owner.

**Proof of Lemma 4.10**

The proof is by induction on \( T - t \), and it is very similar to that of Lemma 4.2. It is therefore omitted. Observe that, when at stage \( t \) the owner \( A_i \) sells to \( A_j \) and invites \( A_k \) to participate, the resulting payoff to \( A_k \) is \( X_i'(t) = X_j'(t+1) \). This allows to conclude as in Lemma 4.2.20

**Proof of Proposition 4.11**

For the proof of Proposition 4.11 we first need the following.

**Lemma 6.1.** Let agent \( A_i \) be the stage t owner, and let \( W_i'(t+1) = \sum_{i'} X_{i'}'(t+1) \) be the welfare associated with the stage \( t + 1 \) ownership of agent \( A_i \), as defined in equation (4.1). If, at stage \( t \), agent \( A_i \) optimally chooses to wait, then \( \forall j, W_j'(t+1) \geq W_j'(t+1) \).

**Proof.** At stage \( t \), agent \( A_i \) prefers to wait rather than sell to \( A_j \). Hence, by equation (4.3), \( W_i'(t+1) \geq W_i'(t+1) + \sum_{i' \neq k} \Delta_{ij}(t+1) \geq W_i'(t+1) + \sum_{i' \neq k} \Delta_{ik}(t+1) \) for all \( t \), and for all \( i' \neq k \).
optimally chooses to wait (because there is a finite number of terms $X'(t)$ of the form shown in Lemma 4.10 and $X'(t)$ is monotonically decreasing). Let $\tau^* = \max_{t_i},$ and assume that $T > \tau^*.$ At any stage $t < T - \tau^*,$ any owner finds it optimal to wait. Since $A_i$ prefers to wait rather than sell to $A_j$ and vice-versa, Lemma 6.1 yields: $W^i(t + 1) \geq W^j(t + 1)$ and $W^j(t + 1) \geq W^i(t + 1).$ Together, these equations yield $W^i(t + 1) = W^j(t + 1).$ Hence, the welfare associated with the stage $t + 1$ ownership of the good is constant, and independent of who is the owner at that stage. By the genericity condition, this implies that the final consumer is always the same agent. \[\]

**Proof of Proposition 4.12(i)**

When there are two agents the result is trivially obtained. Assume $n = 3.$ There are three possible subgames at stage $t,$ depending on whether $A_1,$ $A_2,$ or $A_3$ owns the good at that stage. We define the set $F(t)$ as follows: An agent $A_i$ belongs to $F(t)$ if and only if $A_i$ is the final consumer in one of the possible subgames at stage $t.$ Note that $|F(T)| = 3,$ while for large enough $T,$ $|F(1)| = 1.$ (This is the independence property of Proposition 4.11.)

**Claim.** Let $t'$ be the latest stage such that $|F(t')| \leq 2.$ The efficient consumer belongs to $F(t').$

**Proof.** Assume first that $|F(t')| = 1.$ Then one of agents $A_1,$ $A_2,$ or $A_3$ prefers to wait at stage $t'.$ (Otherwise we would have at least two different final outcomes.) From Lemma 6.1 we can conclude that the final outcome is necessarily efficient.

Assume now that $|F(t')| = 2.$ If one of the agents prefers to wait, then Lemma 6.1 allows us to conclude as above. If no agent chooses to wait at stage $t',$ then we can assume without loss of generality (i.e. up to permutations of the three agents) that at stage $t'$, $A_1$ sells to $A_2,$ $A_2$ sells to $A_1,$ and $A_3$ sells to $A_2.$ We have to show that $A_1$ cannot be the efficient consumer. Since $A_3$ and $A_2$ do not sell to $A_1,$ we have

$$W^i(t' + 1) + \Delta_{12}^i(t' + 1) \geq W^j(t' + 1) + \Delta_{12}^j(t' + 1),$$

and

$$W^j(t' + 1) + \Delta_{23}^j(t' + 1) \geq W^l(t' + 1) + \Delta_{23}^l(t' + 1).$$

(6.4)

Since either $\Delta_{12}^i(t' + 1) = 0$ or $\Delta_{12}^j(t' + 1) = 0,$ and for all $i \neq j \neq k,$ $\Delta_{ij}(t' + 1) \geq 0,$ we obtain that $W^i(t' + 1) \leq \max (W^j(t' + 1), W^k(t' + 1))$ \[\]

To complete the proof of Proposition 4.12(i), we need to show that if, for some $t,$ $|F(t + 1)| = 2$ and $|F(t)| = 1,$ then the consumer in $F(t)$ is the more efficient one among the two agents in $F(t + 1).$ Without loss of generality, assume that one element in $F(t + 1)$ corresponds to the outcome in the subgame $G_1^{i+1}$ and $G_3^{i+1},$ while the other element corresponds to the outcome in the subgame $G_2^{i+1}.$ As before, if one of the three agents prefers to wait at stage $t$ we can immediately conclude using Lemma 6.1. We can therefore assume that no agent prefers to wait.

The only way to eliminate one alternative at stage $t,$ without having one agent whose most preferred action is to wait, is thus to assume that at stage $t$ agent $A_1$ sells to $A_3,$ $A_3$ sells to $A_1,$ and $A_2$ sells to either $A_1$ or $A_3.$ We have to show that $A_2$ cannot be the efficient consumer. At stage $t$ agent $A_1$ prefers to sell to $A_3$ rather than to $A_2,$ which gives

$$W^i(t + 1) + \Delta_{13}^i(t + 1) \geq W^2(t + 1) + \Delta_{13}^2(t + 1).$$

Similarly, at stage $t$ agent $A_3$ prefers to sell to $A_1$ rather than $A_3,$ which gives

$$W^i(t + 1) + \Delta_{32}^i(t + 1) \geq W^2(t + 1) + \Delta_{32}^2(t + 1).$$

Since either $\Delta_{13}^i(t + 1) = 0$ or $\Delta_{32}^i(t + 1) = 0,$ and since for all $i \neq j \neq k,$ $\Delta_{ij}(t + 1) \geq 0,$ we conclude that $W^i(t + 1) \leq \max (W^1(t + 1), W^3(t + 1)).$ \[\]

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