How to Dissolve a Partnership

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Abstract

We survey in a unified framework the recent literature on partnership dissolution in settings where agents have interdependent values. Contrary to the private values case where the main obstacle that hinders the construction of efficient trading mechanisms is the asymmetry in initial endowments, we find that informational asymmetries play here a major role. For settings where the first-best cannot be achieved, we study incentive-efficient mechanisms or other well-performing simple mechanisms whose rules do not depend on the exact specification of the models' parameters.

1 Introduction

In one of Aesop's fables the divide-and-choose method for allocating assets in a partnership is vividly described[1]: A lion, a fox and an ass participated in a joint hunt. Upon request, the ass divided the kill in three portions, and invited the others to choose their shares. Enraged, the lion simply ate the ass, and then asked the fox to make the division. The fox piled all the kill in a great heap, except for a tiny morsel. The lion was delighted, and asked "Who has taught you, my very excellent fellow, the art of division ?" The fox replied, "I learned it from the ass, by witnessing his fate." For our purposes, the fable's lessons are: 1) asymmetries among partners do not make life easier; 2) information about partners' preferences is typically not a-priori available; 3) such information can be revealed through various actions; 4) taking into account such information can make all the difference!

The purpose of this paper is to survey in a unified framework several recent developments in the theoretical literature about partnership dissolution. These developments

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[1]This and other (more successfull) instances where the divide-and-choose method has been used to allocate assets in a partnership are described in a recent book by Brams and Taylor (1999). The examples range from Greek mythology to the modern Law of the Sea and include also the unwritten rules for dividing bread in Auschwitz, as described by Primo Levi.

focus on the role of information by extending earlier frameworks to allow for both private and common value components in the functions determining valuations for the partnerships' assets.

Besides the last section which deals with a partnership owning several indivisible objects, we focus on a partnership owning a single indivisible item. The application of the divide-and-choose method to the allocation of a single indivisible good has the following form, which is commonly known as the "*Texas shoot-out*": One agent, the divider, offers a payment. The other agent, the chooser, selects either to receive the payment (sell) or to take the good and make the payment (buy)[2]. The Texas shoot-out is easy to use, since its rules do not depend on the parties' values. Hence the procedure can be implemented by a court or lawyer who have no knowledge about the partners' preferences. If the partners themselves have complete information about all valuations, the Texas shoot-out allocates the good to the party valuing it more, does not require and outside subsidy, and both partners prefer it to the status-quo where the partnership is not dissolved. Unfortunately, McAfee [1992] showed that these properties do not hold anymore if the parties are privately informed about their values and if they behave strategically[3].

Given the above remarks, the obvious question is how should partnerships be dissolved in the more realistic cases where the partners are privately informed ? An important step towards answering that question has been taken by Cramton, Gibbons and Klemperer [1987]. These authors have studied a partnership model with symmetric, independent private values. Their main result is that efficient dissolution is always possible if the initial shares are not too far from the equal partnership[4]. This is in stark contrast to the impossibility result obtained in the extreme-ownership setting considered by Myerson and Satterwaithe[1983], and shows that, at least with private values, the main obstacle on the way to achieve efficiency is not really the presence of information asymmetries but rather the presence of asymmetries in physical endowments (see also McAfee [1991]).

In general, an efficient dissolution mechanism depends on the function governing the distribution of private information, but Cramton et. al. also offer a mechanism, called the $k-$double auction, whose implementation does not require any such knowledge, but which succeeds to efficiently dissolve any equal partnership[5]. In the $k-$double auction the partners submit sealed bids and the entire partnership is allocated to the partner with the highest bid. The highest bidder's payment to the other partners is a convex combination of the highest bid (b_H) and second highest bid (b_L), i.e. the payment for the entire partnership is given by $kb_L + (1-k)\, b_H$ where $k \in [0,1]$[6].

In a private values setting the partners have a precise estimate about their own value for the partnership. But, in many cases, different partners are responsible for different

[2] In order to apply the procedure to instances where the partners' shares are not equal, it is necessary to allow the actual payments to be some function of the one stated by the divider.

[3] Typically, the partner who makes the initial offer does not submit a price reflecting her true valuation.

[4] Schweizer [1998] has generalized this result by showing that, even if agents' types are not drawn from the same distribution, there always exists an initial distribution of property rights such that, ex-post, the partnership can be efficiently dissolved.

[5] McAfee [1992] extends this result to allow for outside options and risk aversion.

[6] This payment is equally shared by the loosing partners.

parts of their business, and therefore they get different information affecting the value of the partnership as a whole. As a consequence, no partner knows the "true" value of the firm, and there is no secure strategy that ensures a certain payoff. Moreover, the partners have to be extra-cautious in order to avoid "winner's- or looser's curses" that occur when the information revealed post-dissolution happens to be bad news. These factors compound to make strategic manipulation of information a pertinent issue. This manipulation may lead to inefficient allocations if the gains from trade are not sufficient to cover the informational rents (and if there are no outside subsidies). This is the underlying theme of the present survey.

The paper is organized as follows. In Section 2 we describe the partnership model where partners jointly own an indivisible object and define desirable properties for dissolution mechanisms. We introduce private and common value components and assume that these are separable. In addition, we use the symmetry assumptions made by Cramton et. al.[1987]. We also define revelation mechanisms and several of their properties which, combined, yield an efficient dissolution.

In Section 3, which is based on Fieseler, Kittsteiner and Moldovanu [2000] we derive existence conditions for efficient, i.e., incentive compatible, value-maximizing, budget-balanced and individually-rational, mechanisms in our setting. The analysis applies a Revenue Equivalence Result to modified Clarke-Groves-Vickrey mechanisms (the modification is called by the presence of interdependent values). A crucial role is played by the sign of the derivatives of the common value components (the private values case is characterized by setting these derivatives equal to zero.) If valuations are increasing functions of other agents' signals, it is more difficult to achieve efficient trade with interdependent values than with private values, since the information revealed ex-post is always "bad news" and the agents must be cautious in order to avoid the respective (i.e. winner's or loser's) curses. Even if initial shares are equal, it is not always possible to dissolve a partnership efficiently. This result continues to hold for arbitrarily small common value components. These results show that here, contrary to the private value case where problems were posed by asymmetries in endowments, also information asymmetries hinder efficiency. If valuations are decreasing functions of other agents' signals, the additional information revealed ex-post is always a "blessing", and it is easier to achieve efficient trade with interdependent valuations.

In Section 4, which is based on Kittsteiner [2000], we look at the performance of the $k-$double auctions in the presence of interdependent values. As mentioned above, such mechanisms achieve efficient dissolution in sufficiently symmetric settings with private values[7]. The $k-$double auction has a unique equilibrium in pure strategies. This equilibrium awards the partnership to the agent with the highest valuation. But, an important difference to the private values setting appears here: the partners no longer can ensure themselves a positive payoff by participating in the $k-$double auction since, at the interim stage, they do not know their values and therefore cannot submit these values as

[7] A double auction is ex-post efficient in a sufficiently symmetric setting with independent private values in case of risk neutral partners (see Cramton et al. [1987]) or partners with CARA-utility functions (see McAfee [1992]).

secure bids. As a consequence, the partners may refuse to participate at the auction, which creates inefficiencies. This problem arises whenever no budget-balanced, individually rational and incentive compatible mechanism that dissolves the partnership in an ex-post efficient way exists. To solve the participation problem, Kittsteiner extends the rules of the $k-$double auction by allowing for non-participation (or vetoes against the dissolution): If at least one agent does not participate, the status quo is kept. This procedure ensures an efficient dissolution (if this is theoretically possible) and it does possesses an equilibrium which guarantees gains from trade otherwise. In that equilibrium, agents having signals in the middle of the types' interval choose not to participate at the auction.

In Section 5, which is based on Kittsteiner [2000] and Jehiel and Pauzner [2001], we study incentive-efficient mechanisms for settings where first-best (i.e., efficient) dissolution mechanisms do not exist. It turns out that, contrary to the intuition gained in the private values case, the $\frac{1}{2}-$double auction is not incentive-efficient for a symmetric, equal shares setting. Kittsteiner is able to display a mechanism that performs better, but implementing that mechanism requires a detailed knowledge of valuations and distributions (contrary to the double auction). Both the general form of the incentive-efficient mechanism, and the characterization of an incentive-efficient mechanism in the class of procedures that do not depend on the form of valuations and distributions are still open questions. A setting where more is known has been studied by Jehiel and Pauzner[2001]. These authors analyzed an asymmetric setting where there are two partners, but only one partner has private information which affects the valuations of both agents (similarly to Akerlof's [1970] famous example). In this framework they characterize the incentive-efficient mechanism which generally depends on the form of valuations and distributions. Interestingly, also here it is the case that the inefficiency occurs for signals in the middle of the types' interval.

In Section 6 we briefly discuss the additional problems appearing in settings where the partnership consists of several heterogenous and indivisible objects. In this setting we show that already incentive compatibility and value-maximization are incompatible with each other (even when we neglect the constraints imposed by budget-balancedness and individual-rationality). The treatment here is based on the impossibility result derived by Jehiel and Moldovanu [1998] for settings with interdependent valuations and multi-dimensional signals. These authors showed that multi-object value-maximizing auctions do not generally exist. Moreover, finding incentive-efficient mechanisms is an extremely complex task since the maximization involves a strong integrability constraint.

Section 7 gathers several concluding comments.

2 The model

There are n risk-neutral agents and one physically indivisible object. Each agent i owns a fraction α_i of the good, where $0 \leq \alpha_i \leq 1$ and $\sum_{i=1}^{n} \alpha_i = 1$. We denote by θ_i the type of agent i , by θ the vector $\theta = (\theta_1, ..., \theta_n)$, and by θ_{-i} the vector $\theta_{-i} = (\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n)$. Types are independently distributed. Type θ_i is drawn according to a commonly known density function f with support $\left[\underline{\theta}, \overline{\theta}\right]$. The density f

is continuous and positive (a.e.), with distribution F .

The valuation of agent i for the entire object is given by the function $v_i(\theta_1, ..., \theta_n) = g(\theta_i) + \sum_{j \neq i} h(\theta_j)$, where g, h are continuously differentiable, g is strictly increasing, and $g' > h'$. Note that :

$$v_i(\theta_1, ..., \theta_n) > v_j(\theta_1, ..., \theta_n) \Leftrightarrow \theta_i > \theta_j, \tag{1}$$

$$v_i(\theta_1, ..., \theta_n) = v_j(\theta_1, ..., \theta_n) \Leftrightarrow \theta_i = \theta_j. \tag{2}$$

Agents have utility functions of the form $q_i v_i + m_i$ where q_i and m_i represent the share of the good and the money owned by i, respectively.

In a direct revelation mechanism (DRM) agents report their types, relinquish their shares α_i of the good, and then receive a payment $t_i(\theta)$ and a share $s_i(\theta)$ of the object. A DRM is therefore a game form $\Gamma = \left(\left[\underline{\theta}, \overline{\theta}\right]^n, s, t\right)$, where $s(\theta) = (s_1(\theta), ..., s_n(\theta))$ is a vector with components $s_i : \left[\underline{\theta}, \overline{\theta}\right]^n \mapsto [0,1]$ such that $\sum_{i=1}^n s_i(\theta) = 1 \ \forall \ \theta$, and $t(\theta) = (t_1(\theta), ..., t_n(\theta))$ is a vector with components $t_i : \left[\underline{\theta}, \overline{\theta}\right]^n \mapsto \mathbb{R}$. We call the s and t the allocation rule and the payments, respectively. To simplify notation, we refer to the pair (s, t) as a DRM if it is clear which strategy sets are meant.

A mechanism (s, t) *implements* the allocation rule s if truth-telling is a Bayes-Nash equilibrium in the game induced by Γ and by the agents' utility functions. Such a mechanism is called *incentive compatible* (IC). A mechanism is (ex post) *value-maximizing* (VM) if it implements an allocation rule where the agent with the highest valuation always gets the entire object. A mechanism is called (ex-ante) *budget balanced* (BB) if a designer doesn't expect to pay subsidies to the agents, e.g. $E_\theta\left[\sum_{i=1}^n t_i(\theta)\right] \leq 0$. We call a mechanism (interim) *individual rational* (IR) if every agent i who knows his type θ_i wants to participate in the mechanism, given that all players report their types truthfully, e.g. if $U_i(\theta_i) \geq 0$ for all θ_i, $i = 1, .., n$, where $U_i(\theta_i)$ is the utility type θ_i expects to achieve by participating in the mechanism.

A mechanism is called *efficient* (EF) if it satisfies IC, VM, BB and IR.

3 Existence Conditions for Efficient Mechanisms

In order to derive conditions for the existence of efficient mechanisms in the partnership dissolution problem, Fieseler et.al. [2000] use three main steps:

1. If signals are independent[8] a Revenue Equivalence Theorem (RET) (see Myerson [1981]) holds for incentive compatible mechanisms in the interdependent valuation case. The RET states that expected payments are (up to a constant) the same

[8] Correlation among types can be used to extract all private information, thus circumventing many of the problems addressed here. But, in such schemes, transfers to agents may grow arbitrarily large. If there is some bound on these transfers (say, due to limited liability of agents in a partnership), we are back to a setting where the questions raised in this paper play a role.

in all IC mechanisms that implement the same allocation. Its proof can be easily extended to environments with interdependent valuations[9].

2. The standard Clarke-Groves-Vickrey (CGV) approach calls for transfers to agent i that depend on the sum of the utilities of the other agents (in the implemented alternative). But here such transfers will depend on i's report, thus destroying incentives for truthful revelation. Hence, a refinement of the CGV approach is needed in order to construct IC and VM. For a one-sided auction setting with one indivisible unit, such a mechanism has been described by Maskin [1992]. The construction is relatively easy and it hinges on a single-crossing property which ensures that a value-maximizing allocation is monotone in the agents' signals.

3. Finally, using RET it suffices to analyze the conditions under which generalized CGV mechanisms (which are IC and VM) satisfy IR and BB. The idea is first to identify the "worst off" type for each agent i in a mechanism (s,t). The worst type's utility can be viewed as a maximal entry fee that can be collected from agent i in the mechanism (s,t) such that every type of agent i still participates. If these entry fees cover the expected payments needed to ensure IC then (and only then) there exists an IR and BB mechanism that implements s.

Using the above method, Fieseler et.al. [2000] derive the following results:

Theorem 1 *1) The worst-off types are given by $\widetilde{\theta}_i := F^{-1}(\alpha_i^{\frac{1}{n-1}})$. An efficient mechanism exists if and only if:*

$$\sum_{i=1}^{n}\left(\int_{\widetilde{\theta}_i}^{\overline{\theta}} g(\theta)dF^{n-1}(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} g(\theta)F(\theta)\,dF^{n-1}(\theta)\right) + \int_{\underline{\theta}}^{\overline{\theta}} h'(\theta)\,(F^{n}(\theta) - F(\theta))d\theta \geq 0. \qquad (3)$$

2) The set of initial shares $(\alpha_1, ..., \alpha_n)$ for which efficient mechanisms exist is either empty or a symmetric, convex set around $\left(\frac{1}{n}, ..., \frac{1}{n}\right)$.

Special cases of the above results have been obtained by Cramton et al. [1987] for the private values case where $h \equiv 0$. For that case, they also show that the existence condition is always fulfilled if $\alpha_1 = \cdots = \alpha_n = \frac{1}{n}$. Observe that, in our condition 3, the additional term containing the common value component is negative if $h' > 0$ and positive if $h' < 0$. Cramton's et. al. [1987] result implies that, in the latter case, a partnership can always be efficiently dissolved if the initial property rights are distributed equally.

The next result identifies bounds on the initial shares for which efficient dissolution is possible for any valuation functions where $h' \leq 0$, independently of the distribution function F. For example, if there are two partners, efficient dissolution is always possible if the smaller share is at least 25%.

[9] Various such extensions can be found in Myerson [1981], Jehiel et.al. [1996], Jehiel and Moldovanu [1998], Krishna and Maenner [1999] and Reny [1999].

Theorem 2 *Let* $\alpha_1 \leq \cdots \leq \alpha_n$ *, and assume that, for all* $i = 1, ..., n-1,$ *we have* $\sum_{j=1}^{i} \alpha_j \geq \left(\frac{i}{n}\right)^n$ *. Then, for any valuation function where* $h' \leq 0$ *and for any distribution function* F *, the partnership can be dissolved efficiently.*

It is a-priori plausible that, independently of the distribution function, efficient dissolution is possible if the derivative of the common value component is positive, but sufficiently small. We next show, however, that this is not the case: even if that derivative is arbitrarily small but positive, there exist distribution functions such that an equal partnership cannot be dissolved efficiently. Hence, in this case, are not caused by the asymmetries among partners, but rather by the winner's curse effects stemming from the interdependent valuations.

Theorem 3 *For any valuation functions where* $h' > 0$ *there exists a distribution function* F *such that the equal partnership cannot be efficiently dissolved. By Theorem 1-2, for this* F *there is no ex-ante distribution of shares that leads to efficient trade.*

4 The $k-$Double Auction

In this section, which is based on Kittsteiner [2000], we assume that two agents own equal shares in the partnership. The valuation function v_i is assumed to be strictly increasing in θ_i and increasing in θ_{-i}. Hence $g' > 0$, and $h' \geq 0$.

In the k- *double auction* agent i submits a bid $b_i \in \mathbb{R}$. Denote the agent who submits the higher bid by H and the other agent by L. Given the bids b_L and b_H and the parameter $k \in [0,1]$ the agent with the higher bid[10] gets the entire partnership and pays to the other agent the amount of $\frac{1}{2}\left((1-k)\, b_H + k b_L\right)$. Note that a $k-$double auction can be implemented by, say, a court without any knowledge about valuation and distribution functions. As mentioned above, Cramton et.al. showed that, in the private values case, the $k-$double auction dissolves an equal partnership efficiently. Given the inefficiency results in the previous section, this result cannot hold for the setting with interdependent valuations. Recall that inefficiencies may occur even for equal partnerships and for arbitrarily small common-value components. But, it is still of great interest to characterize the properties of the double auctions, and, in particular, to get some insight in the type of inefficiency they create. Note first that, by definition, the $k-$double auctions are budget balanced, since what one agent pays, the other receives.

Theorem 4 ***1)*** *The k− double auction has a unique equilibrium in pure strategies. In this equilibrium each agents bids according to:*

$$b\left(\theta_i\right) = g\left(\theta_i\right) + h(\theta_i) - \frac{\int_{F^{-1}(k)}^{\theta_i} (g'\left(u\right) + h'(u))\left(F\left(u\right) - k\right)^2 du}{\left(F\left(\theta_i\right) - k\right)^2} \tag{4}$$

[10] In case both agents submit the same bid the partnership is given to agent i with probability $\frac{1}{2}$ and the "winning bidder" (the bidder who gets the entire partnership), pays 0 (or any other fixed amount) to the other agent.

2) The $k-$double auction awards the object to the partner with the highest valuation, i.e., it satisfies VM.

The formula above generalizes the one obtained by Cramton et.al. in the private values case. Since the $k-$double auction satisfies VM, BB and since the equilibrium requirements are equivalent to the IC constraint (that was formulated for revelation mechanisms), we obtain by the results of the previous Section that the $k-$double auction cannot always satisfy IR. Hence, agents may refuse to participate in such a mechanism. The next result shows that the lack of individual rationality occurs precisely (i.e., not more often) when the theoretical impossibility of efficient dissolution holds.

Theorem 5 *A $k-$double auction satisfies IR if and only if an efficient direct revelation mechanism exists. Hence, by Theorem 1 , the $k-$double auction is IR if and only if condition 3 is satisfied for $\widetilde{\theta}_i := F^{-1}(\frac{1}{2})$.*

If (given existing law or other previous contracts) the partners can be forced to participate in a double auction, then the this mechanism will yield allocative efficiency. But, in most cases, agents cannot be forced to participate in a procedure that is likely to yield losses, and therefore we have to adjust the rules of the auction in order to secure participation.

4.1 Individually Rational Double Auctions

In the private values case the k-double auction automatically satisfies IR since each agent can guarantee herself a positive outcome (regardless of the other bid) by bidding exactly her valuation. With interdependent valuations, a partner cannot bid her value, since this depends on unavailable private information. This creates the risk of relinquishing one's share for less than one's valuation, and it implies that agents may refuse to participate in a double auction, thus preventing efficient dissolutions. To ensure that the $k-$double auction satisfies IR, Kittsteiner extends the strategy spaces in such a way that every agent has the right to veto the dissolution. The agent's strategy spaces are given by the set of functions: $\{b \mid b : [\underline{\theta}, \overline{\theta}] \mapsto \mathbb{R} \cup \{\mathrm{N}\}\}$.The outcome of the game is defined as follows: If $b_1 = \mathrm{N}$ or $b_2 = \mathrm{N}$ then the partnership is not dissolved (or, equivalently, each agent gets the partnership with probability $\frac{1}{2}$). In any other case, the partnership is given to the agent with the higher bid. He pays $\frac{1}{2}((1-k)\, b_H + k b_L)$ to the other agent. This game is called a *$k-$double auction with veto*[11].

A $k-$double auction with veto is always IR because every type can, by vetoing, assure that she never makes losses . Furthermore if the $k-$double auction (without veto) is IR then its equilibrium is also an equilibrium of the $k-$double auction with veto. Since the double auction is always BB, and by the results of Section 3, we obtain that, in any equilibrium of the $k-$double auction with veto, the indivisible good may not always be

[11] An equivalent specification is as follows: There are two stages. In the first stage each agent decides whether she participates in the 2nd stage or not. If at least one agent decides not to participate, the partnership is not dissolved. Otherwise, in the 2nd stage a $k-$double auction (without veto) is played.

allocated to the agent that values it most, i.e., VM is not always satisfied. But how do the inefficiencies look like ? It is obvious that the double auction with veto has a very inefficient equilibrium where both agents veto, no mater what their types are. The next result identifies another equilibrium that allocates the good efficiently for open sets of types' realizations.

Theorem 6 *Assume that the $k-$ double auction without veto is not IR. Then there always exist $c^*, d^* \in [\underline{\theta}, \overline{\theta}]$, $c^* < d^*$, such that the strategy profile where each agent bids according to*

$$b(\theta_i) = \begin{cases} g(\theta_i) + h(\theta_i) - \frac{\int_{c^*}^{\theta_i} (g'(t)+h'(t))(F(t)-F(c))^2 dt}{(F(\theta_i)-F(c))^2} & \text{if} \quad \theta_i \in [\underline{\theta}, c^*) \\ N & \text{if} \quad \theta_i \in [c^*, d^*] \\ g(\theta_i) + h(\theta_i) - \frac{\int_{d^*}^{\theta_i} (g'(t)+h'(t))(F(t)-F(d))^2 dt}{(F(\theta_i)-F(d))^2} & \text{if} \quad \theta_i \in (d^*, \overline{\theta}]. \end{cases} \tag{5}$$

constitutes a symmetric Bayes-Nash equilibrium. The $k-$ double auction with veto satisfies IR and BB. Moreover, it satisfies VM for all $(\theta_1, \theta_2) \in ([\underline{\theta}, \overline{\theta}] \backslash [c^*, d^*])^2$.

Since the standard procedure to derive equilibria (which is based on assuming that strategies are strictly increasing and on the use of a first-order condition) fails here, Kittsteiner derives the above equilibrium through an ingenious construction. The idea is to guess how the equilibrium allocation may look like, and then to use the Revenue Equivalence Theorem in order to construct appropriate bidding strategies yielding exactly that allocation[12].

In the above equilibrium types in the interval $[c^*, d^*]$ veto the auction since their chances of becoming either buyer or seller are close in magnitude. In either case, they cannot expect high gains from participation in an auction where the partnership is dissolved. In contrast, types outside this interval choose to participate in the auction since they rather expect to sell (for types in $[\underline{\theta}, c)$) or to buy (for types in $\theta_i \in (d, \overline{\theta}]$) In either case, a substantial share out of the gains from trade created by a dissolution is expected by the respective agents.

5 Incentive Efficient Mechanisms

In cases where no efficient dissolution procedures exist, some misallocation of the good is implied in the equilibrium of any mechanism satisfying IR and BB. We have seen above exactly how the misallocation is created by the veto possibility in the $k-$ double auctions. An important question is whether some mechanism in that class is ex-ante

[12] A similar approach has been used by Jehiel, Moldovanu and Stacchetti [1999] in their multidimensional auction setting.

incentive-efficient in the sense that it maximizes ex-ante expected gains from trade under the BB and IR constraints.

For the Myerson-Satterthwaite and Akerlof "extreme-ownership" settings incentive efficient mechanisms have been exhibited in the literature (see Myerson and Satterthwaite [1983], Samuelson [1984], Gresik [1991], Wilson [1985]). For example, a $\frac{1}{2}$- double auction is incentive-efficient in a symmetric (i.e., in terms of distributions) buyer-seller setting a la Myerson and Satterthwaite. In a symmetric equal partnership framework a la Cramton et.al., such an auction is even first-best. Kittsteiner [2000] shows, however, that, in the present setting, the $\frac{1}{2}-$double auction is not incentive-efficient and he constructs a mechanism that achieves higher expected gains from trade. That mechanism is also inefficient, but the region of inefficiency is quite different from the "cross" resulting from the $k-$double auctions: if at least one type is very high or very low the partnership is dissolved, whereas the status-quo is preserved only if both types are close to the middle of the types' interval. This reflects the intuition that types close to the middle of the interval contribute very little to the overall gains from trade (while they still have to be paid "information rents"). Therefore they should not trade when we optimize under BB and IR constraints.

It is important to note that the construction of that superior mechanism requires a detailed knowledge of valuation and distribution functions, in stark contrast to the double auctions which can be implemented without such knowledge. It is still an open question whether a $k-$double auction is incentive-efficient in the class of mechanisms whose rules do not depend on v and F. The characterization of a general incentive efficient mechanism (which may depend on v and F) is also still an open question.

5.1 One-sided incomplete information

Jehiel and Pauzner [2001] study an asymmetric setting where only one out of two partners has information about the partnership, and where this information affects both partners' valuations. This generalizes Akerlof's extreme ownership setting. Suppose then that agent 1 has a type θ_1, drawn according to a commonly known density function f with support $\left[\underline{\theta}, \overline{\theta}\right]$. Valuations are given by $v_1(\theta_1) = \theta_1$ and $v_2(\theta_1) = h(\theta_1)$, where $h' > 0$ and $h' < 1$. Jehiel and Pauzner also assume that the function $\frac{F}{f}$ is increasing, while the function $\frac{1-F}{f}$ is decreasing. In this setting they are able to construct an incentive-efficient mechanism.

Theorem 7 *In the incentive-efficient mechanism, the partnership is efficiently dissolved only for values of θ_1 outside an interval $[c^*, d^*] \subset \left[\underline{\theta}, \overline{\theta}\right]$. For values $\theta_1 \in [c^*, d^*]$ the partners keep their initial shares. In general, the values c^*, d^* depend on valuation and distribution functions.*

In order to implement the above mechanism, the designer must have precise knowledge about h and F. It is not yet known whether a simple game-form (e.g., an auction) that does not require this information, and that implements the same allocation exists. Note that here, similarly to the double auction with veto in the symmetric case, the inefficiency arises in the middle of the types' interval. But, as suggested by Kittsteiner's construction of a superior mechanism, it is unlikely that this feature extends to the case of double-sided incomplete information.

After having constructed the incentive-efficient mechanism for any initial distribution of property rights, Jehiel and Pauzner ask what is the initial ex-ante distribution that generates highest ex-post gains from trade. They find that, for large ranges of the parameters' values, the answer is that one agent should get full property rights. This is in contrast to a symmetric private value framework, where the optimal initial distribution is the equal one (hence property rights should be dispersed).

6 Partnerships with Several Indivisible Objects

In 1991 the divorce of Donald and Ivana Trump made the headlines[13]. Besides business assets to which Ivana was not entitled[14], the contested real estate included: A 46-room estate in Greenwich, Connecticut, a 118-room mansion in Palm Beach, Florida, a Trump Plaza apartment in Manhattan and a 50-room Trump Tower triplex also in Manhattan. This is good example for the dissolution of a partnership which jointly owns several heterogenous and indivisible objects. Donald and Ivana had asymmetric preferences over the objects, with Ivana putting much weight on the Greenwich estate (which was the family home for the couple and their 3 children), while Donald was keen to get the Palm Beach property in order to divide it in 8 large development areas. Moreover, at least Donald had some information that could influence Ivana's valuation: this concerned the probability of the various assets being seized by other creditors in face of Trump's near bankruptcy. Is there an efficient way method to dissolve this kind of partnership ? Brams and Taylor [1999] propose a method whose main feature is that both parties award a total of 100 points to the various items and then each item is awarded to the party who allocates more points to it. This is in fact a sequence of double auctions connected by a common budget constraint. While praising the features of this procedure, Brams and Taylor note that it can perform badly if agents have private information[15]. In a private values framework, McAfee [1992] studied another mechanism where the partners pick objects in alternation, and he displays (relatively strong) conditions under which this procedure performs satisfactorily. We now turn to a simple model of a partnership owning several goods, and sketch a basic impossibility result.

There are n risk-neutral partners who jointly own m indivisible, heterogenous objects. Agent i owns a fraction α_i of the partnership, where $0 \leq \alpha_i \leq 1$ and $\sum_{i=1}^{n} \alpha_i = 1$. We denote by $\theta_i = (\theta_i^1, ..., \theta_i^m)$ the type of agent i . . Types are independently distributed, and type θ_i is drawn according to a commonly known density function f with support $\left[\underline{\theta}, \overline{\theta}\right]$. The density f is continuous and positive (a.e.), with distribution F .

For an agent i and a subset of objects J, denote by θ_i^J the vector $(\theta_i^j)_{j \in J}$. The valuation of agent i for a subset of objects J is given by $v_i^J = g^J\left(\theta_i^J\right) + \sum_{l \neq i} h^J\left(\theta_l^J\right)$. A

[13] For the full story, see the Brams and Taylor [1999].

[14] such as a 282-yacht and a Boeing 727 jet.

[15] Moreover, with interdependent values, the partners also lack a secure strategy, so that participation becomes an issue

mechanism is VM if the proposed allocation of objects maximizes the sum of the agents' utilities. The other definitions of desirable properties are unchanged. This model keeps the separability and symmetry assumptions made in Section 2. The crucial assumption is that the objects are heterogenous, i.e., agents get different signals on different items and there are different functions g^J, h^J for each subset of items. Applying the main result in Jehiel and Moldovanu [1998], we obtain:

Theorem 8 *Value-maximizing, incentive-compatible mechanisms do not exist unless restrictive algebraic conditions relate the functions in the set* $\{g^J, h^J\}_J$.

Example 9 *Consider a setting with two objects, called A and B and two agents. Let the valuation functions for individual objects be* $v_i^A = \theta_i^A + \alpha\theta_{-i}^A$ *and* $v_i^B = \theta_i^B + \beta\theta_{-i}^B$ *for* $i = 1, 2$ *($-i$ denotes the agent other than i). For simplicity, assume that the only feasible alternatives are those where each partner gets at least one object. Then we do not have to specify valuations for the pair. Assume also that* $\alpha, \beta < 1$. *Consider a realization of signals such that the social welfare in the alternative where agent* 1 *gets object A and agent* 2 *gets object B is equal to the social welfare in the alternative where the objects are switched, i.e.* $\theta_1^A + \alpha\theta_2^A + \theta_2^B + \beta\theta_1^B = \theta_2^A + \alpha\theta_1^A + \theta_1^B + \beta\theta_2^B$. *Fix the signal of agent* 2 *and note that the two-dimensional signal of agent* 1 *can vary on a line with slope* $\frac{1-\alpha}{1-\beta}$ *without affecting the above equality. Incentive-compatibility dictates that whenever welfare in the above alternatives is the same, agent* 1 *must also be indifferent between getting object A or B, i.e.,* $\theta_1^A + \alpha\theta_2^A = \theta_1^B + \beta\theta_2^B$. *Otherwise, agent* 1 *would choose an action that yields him the more preferred object. Fixing again the signal of agent* 2 *, note that agent* 1 *is indifferent between the two objects if his two-dimensional signal varies on a line with slope* 1. *To conclude, we can keep agent* 1 *indifferent whenever the society is indifferent (which is necessary for VM to be compatible with IC) only if* $\frac{1-\alpha}{1-\beta} = 1 \Leftrightarrow \alpha = \beta$. *In other words, both objects must be identical.*

The above result is in stark contrast to the insight gained in Section 3 where the existence problems were caused by the conjunction of IC, VM, IR and BB. In that one-object setting, mechanisms a la Clarke-Groves-Vickrey which satisfy VM and IC exist for a large class of preferences where a single-crossing condition holds. Symmetry in valuations does not play a role for that existence result[16]. Here we completely neglected BB and IR while still getting an impossibility result[17]. Even if we further neglect the IR and BB constraints, it is not yet known how incentive-efficient mechanisms look in general, since the maximization problem typically involves a new, complex constraint (called the *integrability constraint)* which is due to the presence of multidimensional signals[18]. The difficulties are compounded if the partners value subsets of objects in a non-additive fashion (e.g., if some objects are substitutes or complements, etc...)

[16] This existence result extends to a framework with several objects only if the signals about each object are one-dimensional and if, for each bundle, its value is obtained by adding the values of the included objects.

[17] In the present formulation, we already imposed a lot of symmetry among agents (which, as illustrated by the Trump example, is not always realistic). The presence of asymmetries in valuations will add more restrictive conditions.

[18] Jehiel, Moldovanu and Stacchetti [1999] contains a more detailed discussion of this issue.

7 Conclusion

In a recent survey of Mechanism Design Theory, Mat Jackson writes:

> "There is still much that is not known about the existence or properties of incentive compatible mechanisms that are efficient[19] (much less the balanced and individual rational), when there are general forms of uncertainty and interdependencies in the preferences of individuals." (Jackson, 2000)

We have surveyed here several very recent papers that address this challenge. The comparison of the private and interdependent cases crucially depends on whether ex-post the agents are cursed or blessed by seeing the outcome. Contrary to the private values case where the main obstacle to the construction of efficient mechanisms was the presence of allocative asymmetries, with have shown that, with interdependent values, also informational asymmetries may hinder efficiency even if all partners have, a-priori, symmetric endowments. The difficulties are much compounded when the partnership has to divide several heterogenous goods. Given the large range of settings where a first-best mechanism does not exist, two important tasks are: 1) the construction of incentive-efficient mechanisms; 2) the identification of well-performing mechanisms whose rules do not depend on features of the problem (e.g., valuation and distribution functions) that are unlikely to be known to a third party (e.g., a court). Given the involved analytical complexity, only modest advances in that direction have been made so far. These are exciting topics for future research.

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