Partnerships, Lemons and Efficient Trade

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Abstract

We analyze the possibility of efficient trade with informationally interdependent valuations and with a dispersed ownership. A crucial role is played by the sign of the derivatives that measure how valuation functions depend on others' signals. If valuations are increasing functions of other agents' signals, it is more difficult to achieve efficient trade with interdependent values than with private values (where the respective derivatives are zero.) In contrast, if valuations are decreasing functions of other agents' signals, it is easier to achieve efficient trade with interdependent values. Our results unify and generalize the insights of Cramton et al. [1987], Myerson and Satterthwaite [1983], and Akerlof [1970].

1 Introduction

We inquire whether efficient trade can take place in environments where the agents' valuations depend on their own private information and on the private information of other agents. Such interdependence is natural in many trading situations, e.g., when a seller has private information about the quality of the good which influences the valuations of both the seller and a potential buyer. Especially in situations where property rights are initially dispersed among several agents (e.g., a partnership) it is natural to assume that each agent has private information that also determines the other agents' valuations. For an illustration, consider the situation where each partner is responsible for a particular project (or client, or operative part of the business, etc...) and where the projects are not

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related to each other. It is clear that an estimate of the value of the entire business can be made only by having information on all projects. In addition to the "standard" case where private information influences all agents' valuations in the same direction (e.g., if this information is about quality) situations where "good" news for one agent turns out to be "bad" news for other agents are conceivable[1]. As an example, consider the following excerpt taken from the Economist, 2001, which describes actions taken by partners owning Formula 1 motor racing:

> "...it would not be in Mr. Ecclestone's long-term interest to forgo a deal which could only enhance the value of his family's remaining 50% stake in SLEC...the other 50% stake in SLEC, owned by EM.TV, a debt-ridden German media company, is up for sale. ... The uncertainty created by the dispute between Mr. Eccelstone and Mr. Mosley might depress the value of EM.TV holding. Could that work to Mr. Ecclestone's advantage ? Quite possibly. The lower the value of EM.TV's stake, the higher the relative value of an option Mr. Ecclestone holds to sell a further 25% of SLEC to EM.TV for around $1 billion - and the better the deal Mr. Ecclestone might be able to extract for surrendering the option."

Our paper has two parts. We first show how results from the auction and mechanism design literature with private values can be adapted in order to analyze the possibility of efficient trade in models with interdependent values. The second part illustrates the advantages of this approach by analyzing in detail several trading situations with interdependent values, and in particular the dissolution of a partnership.

The analysis employs three main steps:

1. If signals are independent[2], we show that a Revenue Equivalence Theorem (in the tradition of Myerson's [1981] pioneering contribution) holds for incentive compatible mechanisms in the interdependent valuation case[3].

2. We next construct a value-maximizing, incentive-compatible mechanism. The standard Clarke-Groves-Vickrey (CGV) approach calls for transfers to agent i that depend on the sum of the utilities of the other agents (in the implemented alternative) But here such transfers will depend on i's report, thus destroying incentives for

[1] This may be the case if agents are informed about mutually exclusive properties of the same asset, which they want to use for different applications.

[2] Correlation among types can be used to extract all private information, thus circumventing many of the problems addressed here. But, in such schemes, transfers to agents may grow arbitrarily large. If there is some bound on these transfers (say, due to limited liability of agents in a partnership), we are back to a setting where the questions raised in this paper play a role.

[3] Already Myerson himself allowed for a simple form of interdependent valuations, the so-called "revision effects".

truthful revelation. Hence, we have to use a refinement of the CGV approach. We adapt for our purposes the mechanism described in Maskin [1992] for a one-sided auction setting with one indivisible unit. Achieving incentive compatible value maximization is easy: the construction hinges on a single-crossing property which ensures that the value-maximizing allocation is monotone in the agents' signals.

3. Finally, using Revenue Equivalence, we note that it suffices to analyze the conditions under which generalized CGV mechanisms (which are incentive compatible and value-maximizing) satisfy individual rationality and budget-balancedness. For private values, a similar approach has been used by Williams [1999] and Krishna and Perry [1998] .

It is important to note that in the recent literature on one-sided auction settings with interdependent values[4], the seller (whose private information does not play a role) is a "residual claimant" and receives all payments from the buyers. Budget-balancedness is therefore costless and it is automatically satisfied. Hence, the type of problem posed in the present paper is completely absent in that literature[5].

Our first application concerns the dissolution of a partnership. Cramton, Gibbons and Klemperer [1987] look at situations where each one of several agents owns a fraction of a good, and where agents have independent private values. Assuming symmetric distributions of agents' valuations, they prove that efficient trade is always possible if the agents' initial shares are equal[6].

We analyze a model that uses the symmetry assumptions made by Cramton et. al., and where the private and common value components are separable. A comparison of the cases with private and interdependent values reveals that a crucial role is played by the sign of the derivatives of the common value components (note that the private values case is exactly characterized by setting these derivatives equal to zero.)

If valuations are increasing functions of other agents' signals, it is more difficult to achieve efficient trade with interdependent values than with private values, since the information revealed ex-post is always "bad news" and the agents must be cautious in order to avoid the respective (i.e. winner's or loser's) curses. Even if initial shares are equal, it is not always possible to dissolve a partnership efficiently. Surprisingly, this result continues to hold for arbitrarily small common value components. Indeed, for any

[4]See Maskin [1992], Jehiel, Moldovanu and Stacchetti [1996], Ausubel [1997], Dasgupta and Maskin [2000], and Perry and Reny [1999].

[5]Ignoring individual rationality or budget balancedness, Jehiel and Moldovanu [1998] show that value-maximization is, per-se, inconsistent with incentive compatibility if valuations are interdependent and if different coordinates of a multi-dimensional signal influence utilities in different alternatives (as in a general model of multi-object auctions).

[6]Schweizer [1998] has generalized this result by showing that, even if agents' types are not drawn from the same distribution, there always exists an initial distribution of property rights such that, ex-post, the partnership can be efficiently dissolved.

symmetric and separable valuations that are increasing in other agents' signals, we can construct a symmetric distribution function such that efficient dissolution is impossible, no matter what the initial distribution of property rights is.

If valuations are decreasing functions of other agents' signals, the additional information revealed ex-post is always a "blessing", and it turns out that it is easier to achieve efficient trade with interdependent valuations. We show that, in this case, there exists an open set of partnerships (around the equal partnership) that can always be dissolved efficiently.

An important special case of the partnership model is the situation where, ex-ante, the property rights belong to one agent. In a bilateral private values framework, Myerson and Satterthwaite [1983] show that efficient trade is possible only in a setting where it is common knowledge that the buyer's lowest valuation exceeds the seller's highest valuation. The introduction of interdependent values allows us to connect the Myerson-Satterthwaite result to Akerlof's famous market for lemons [1970]. Akerlof examines a bilateral trading situation where only the seller has private information, but this information influences both traders' valuations. He gives an example where efficient trade is not possible even if it is common knowledge that the buyer's valuation always exceeds the seller's valuation[7].

With extreme ex-ante ownership, the "worst-off" types of traders are unambiguously defined, and we can relax some assumptions made for the analysis of partnerships. We display a general existence condition for efficient trade that generalizes and unifies both Myerson-Satterthwaite's and Akerlof's classical contributions. We also show how efficient trade can take place (even if its possibility is not common knowledge) if agents' valuations are decreasing in other agents' signals. This positive result complements the negative result obtained by Gresik [1991][8] for the case of valuations that increase in other agents' signals. Gresik derived an existence condition for efficient trade as a by-product of his characterization of second-best bilateral mechanisms for several traders with interdependent values[9].

This paper is organized as follows: In Section 2 we describe the model. In section 3 we construct value-maximizing, incentive compatible mechanism for interdependent valuations, and we use a revenue equivalence result in order to derive conditions under which

[7] In that environment it suffices to analyze simple fixed-price mechanisms. Because private information in our paper is two-sided, we cannot restrict attention to price mechanisms, and the analysis is more complex. A detailed analysis of the Akerlof one-sided example using mechanism design techniques can be found in Myerson [1985] and Samuelson [1984], which constructs second-best mechanisms.

[8] Other papers focused on impossibility results: Spier [1994] and Schweizer [1989] study models of pretrial negotiation where the outcome of a trial depends on both parties' signals. They observe that not going to trial (which is efficient) cannot occur with probability one. Bester and Wärneyard [1998] study a model of conflict resolution where agents are uncertain about each other's fighting potential, and observe that conflict must arise with positive probability even if peaceful settlement is always efficient.

[9] Since the result is obtained via the solution of a variational problem, Gresik's approach depends on certain assumptions about virtual valuations. Such assumptions are not needed in our treatment

such mechanisms are budget balanced and individually rational. In section 4 we generalize the Cramton et al. [1987] environment to the case with interdependent valuations. In section 5 we briefly look at the case of bilateral trade. Concluding comments are gathered in Section 6. All proofs are relegated to an Appendix.

2 The model

There are n risk-neutral agents and one good. Each agent i owns a fraction α_i of the good, where $0 \leq \alpha_i \leq 1$ and $\sum_{i=1}^{n} \alpha_i = 1$. We denote by θ_i the type of agent i , by θ the vector $\theta = (\theta_1, ..., \theta_n)$, and by θ_{-i} the vector $\theta_{-i} = (\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n)$. Types are independently distributed. Type θ_i is drawn according to a commonly known density function f_i with support $\left[\underline{\theta}_i, \overline{\theta}_i\right]$. The density f_i is continuous and positive (a.e.), with distribution F_i .

The valuation of agent i for the entire good is given by the function $v_i\left(\theta_i, \theta_{-i}\right)$, where the arguments are always ordered by the agents' indices: $v_i\left(\theta_i, \theta_{-i}\right) = v_i\left(\theta_1, ..., \theta_n\right)$. The function $v_i\left(\theta_i, \theta_{-i}\right)$ is strictly increasing in θ_i, and continuously differentiable. We further assume the following single crossing property (SCP):

$$v_{i,i} > v_{j,i} \qquad \forall\ i, j \neq i. \tag{SCP}$$

where $v_{i,x}\left(\theta_1, ...\theta_n\right)$ denotes the $x'th$ partial derivative of $v_i\left(\theta_1, ..., \theta_n\right)$. This assumption[10] guarantees that the functions $v_i\left(\cdot, \theta_{-i}\right)$ and $v_j\left(\cdot, \theta_{-i}\right)$ are equal for at most one θ_i.

Agents have utility functions of the form $q_i v_i + m_i$ where q_i and m_i represent the share of the good and the money owned by i, respectively.

By the revelation principle, it suffices to analyze direct revelation mechanisms (DRM). In a DRM agents report their types, relinquish their shares α_i of the good, and then receive a payment $t_i(\theta)$ and a share $k_i(\theta)$ of the entire good. A DRM is therefore a game form $\Gamma = \left(\left[\underline{\theta}_1, \overline{\theta}_1\right], ..., \left[\underline{\theta}_n, \overline{\theta}_n\right], k, t\right)$, where $k(\theta) = \left(k_1(\theta), ..., k_n(\theta)\right)$ is a vector with components $k_i : \times_{j=1}^{n}\left[\underline{\theta}_j, \overline{\theta}_j\right] \mapsto [0, 1]$ such that $\sum_{i=1}^{n} k_i(\theta) = 1\ \forall\ \theta$, and $t(\theta) = \left(t_1(\theta), ..., t_n(\theta)\right)$ is a vector with components $t_i : \times_{j=1}^{n}\left[\underline{\theta}_j, \overline{\theta}_j\right] \mapsto \mathbb{R}$. We call the k and t the allocation rule and the payments, respectively. To simplify notation, we refer to the pair (k, t) as a DRM if it is clear which strategy sets $\left[\underline{\theta}_i, \overline{\theta}_i\right]$ are meant.

A mechanism (k, t) *implements* the allocation rule k if truth-telling is a Bayes-Nash equilibrium in the game induced by Γ and by the agents' utility functions. Such a mechanism is called *incentive compatible* (IC). A mechanism is (ex post) *efficient* (EF) if it implements an allocation rule where the agent with the highest valuation of the good

[10] Maskin [1992] shows that, without this assumption, the value-maximizing allocation may fail to be monotone in types, and hence it may be impossible to implement it. If that allocation just happens to be monotone (as in Akerlof's original example where SCP is not satisfied, but where the value-maximizing allocation is constant) our main results also go through.

always gets the entire good[11]. A mechanism is called (ex-ante) *budget balanced* (BB) if a designer doesn't expect to pay subsidies to the agents, e.g. $E_\theta\left[\sum_{i=1}^{n} t_i(\theta)\right] \leq 0$. We call a mechanism (interim) *individual rational* (IR) if every agent i who knows his type θ_i wants to participate in the mechanism, given that all players report their types truthfully, e.g. if $U_i(\theta_i) \geq 0$ for all $\theta_i,\ i = 1, .., n$, where $U_i(\theta_i)$ is the utility type θ_i expects to achieve by participating in the mechanism.

We denote characteristic functions as follows:

$$\mathbf{1}(\text{statement}) := \begin{cases} 1, & \text{if statement is true} \\ 0, & \text{if statement is false} \end{cases} \quad \text{or } \mathbf{1}_A(x) := \begin{cases} 1, & \text{if } \quad x \in A \\ 0, & \text{if } \quad x \notin A. \end{cases}$$

3 Efficiency and incentive compatibility

Our analysis uses three main ideas

- A revenue equivalence result implies that any two EF and IC mechanisms yield, up to a constant, the same interim expected transfers.
- We display generalized Groves mechanisms that satisfy EF and IC for the case of interdependent values.
- By revenue equivalence, it is enough to check under which conditions a generalized Groves mechanism satisfies BB and IR to obtain general conditions for the existence of EF, IC BB and IR mechanisms[12].

Krishna and Perry [1998] and Williams [1999] have used the same combination for the analysis of efficient trade in buyer-seller settings with private values.

3.1 The revenue equivalence theorem

The Revenue Equivalence Theorem constitutes the basis of most results in the mechanism-design literature with quasi-linear utility functions, risk-neutral agents and independent types. It states that expected payments are (up to a constant) the same in all IC mechanisms that implement the same allocation. Its proof can be easily extended to environments with interdependent valuations[13].

[11] A more appropriate name for this property is *value-maximization,* since efficiency combines in fact **all** properties listed here. But we keep the common jargon.

[12] Note that mechanisms that satisfy IC and EF without belonging to the CGV class do indeed exist. For instance, Cramton et al. [1987] show that a double auction (which is not a CGV mechanism) dissolves a partnership effciently when agents have private values.

[13] Various such extensions can be found in Myerson (1981), Jehiel et.al. (1996), Jehiel and Moldovanu (1999), and Krishna and Maenner (1999). None of these results covers the present setting.

We first need some notation: The interim utility of agent i with type θ_i which participates and announces the type $\widehat{\theta}_i$ (while all the other agents report truthfully) is given by

$$\begin{aligned} U_i\left(\theta_i,\widehat{\theta}_i\right) &= E_{\theta_{-i}}\left[v_i\left(\theta_i,\theta_{-i}\right)\ k_i\left(\widehat{\theta}_i,\theta_{-i}\right)-\alpha_i v_i\left(\theta_i,\theta_{-i}\right)+t_i\left(\widehat{\theta}_i,\theta_{-i}\right)\right] \\ &= : V_i\left(\theta_i,\widehat{\theta}_i\right)+E_{\theta_{-i}}\left[t_i\left(\widehat{\theta}_i,\theta_{-i}\right)\right] \\ &= : V_i\left(\theta_i,\widehat{\theta}_i\right)+T_i\left(\widehat{\theta}_i\right). \end{aligned}$$

To simplify notation we write:

$$U_i\left(\theta_i\right) := U_i\left(\theta_i,\theta_i\right).$$

Theorem 1 *Assume that $v_i\left(\theta_i,\theta_{-i}\right)$ is continuously differentiable in each component and that for all $\widehat{\theta}_i\in[\underline{\theta}_i,\overline{\theta}_i]$ $\lim_{\theta_i\to\widehat{\theta}_i}k_i\left(\theta_i,\theta_{-i}\right)=k_i\left(\widehat{\theta}_i,\theta_{-i}\right)$ for almost every θ_{-i}. Then, for every IC mechanism (k,t), the interim expected utility of agent i in a truth-telling equilibrium can be written as:*

$$\begin{aligned} U_i\left(\theta_i\right) &= U_i(\underline{\theta}_i)+\int_{\underline{\theta}_i}^{\theta_i}V_{i,1}\left(x,x\right)\ dx \\ &= U_i(\underline{\theta}_i)+\int_{\underline{\theta}_i}^{\theta_i}E_{\theta_{-i}}\left[v_{i,1}\left(x,\theta_{-i}\right)\left(k_i\left(x,\theta_{-i}\right)-\alpha_i\right)\right]dx \end{aligned}$$

Corollary 1 *Let (k,t) be an IC mechanism. If there is no BB and IR mechanism of the form $(k,t+q)$ where $q:=(q_1,...,q_n)$ is an arbitrary vector of constants, then there are no IR and BB mechanisms that implement k.*

We now examine under which conditions we can find an BB and IR mechanism in the class of mechanisms of the form $(k,t+q)$ where (k,t) is an IC mechanism. Let $\widetilde{\theta}_i$ be the "worst off" type of agent i in the mechanism (k,t). This is defined by

$$U_i(\widetilde{\theta}_i)\le U_i\left(\theta_i\right) \qquad \forall\theta_i.$$

Theorem 2 *Let (k,t) be an IC mechanism and $T_i,U_i,\widetilde{\theta}_i$ be the associated interim payments, interim utilities and "worst off" types, respectively. There exists an IC, BB and IR mechanism that implements k if and only if*

$$\sum_{i=1}^{n}E_{\theta_i}\left[T_i\left(\theta_i\right)\right]\le\sum_{i=1}^{n}U_i(\widetilde{\theta}_i).$$

The worst type's utility $U_i(\widetilde{\theta}_i)$ can also be viewed as a maximal entry fee that can be collected from agent i in the mechanism (k,t) such that every type of agent i still participates. If these entry fees cover the expected payments needed to ensure IC then (and only then) there exists an IR and BB mechanism that implements k. Such a mechanism is then given by $(k, t+q)$ with $q=(q_1,...,q_n)=(-U_1(\widetilde{\theta}_1),...,-U_n(\widetilde{\theta}_n))$.

In the sequel we focus on EF mechanisms. For a given trading situation it suffices to analyze the allocation rule k^* given by:

$$k_i^*(\theta) := \begin{cases} 1, & \text{if} \quad i = m(\theta) \\ 0, & \text{if} \quad i \neq m(\theta) \end{cases},$$

where $m(\theta) := \max\{j | \; j \in \arg\max_i \; v_i(\theta)\}$.

Any two efficient allocation rules differ only in the tie breaking rule and coincide a.e. We can apply Theorem 1 for the efficient allocation rule k^* since, for all $\widehat{\theta}_i$, we have $\lim_{\theta_i \to \widehat{\theta}_i} k_i^*(\theta_i, \theta_{-i}) = k_i^*\left(\widehat{\theta}_i, \theta_{-i}\right)$ for almost all θ_{-i}.

3.2 The generalized Groves mechanism

We now display a mechanism which applies to the interdependent values case the idea behind Groves mechanisms. Variations on this idea have been used to construct value-maximizing auctions by Ausubel [1997], Dasgupta and Maskin [2000] , Jehiel and Moldovanu [1998] and Perry and Reny [1999].

Theorem 3 *Let k^* be an efficient allocation rule, and let the payments t^* be given by*

$$t_i^*(\theta) := \begin{cases} 0, & \text{if} \quad k_i^*(\theta) = 1 \\ v_i(\theta_i^*(\theta_{-i}), \theta_{-i}), & \text{if} \quad k_i^*(\theta) \neq 1 \end{cases},$$

where $\theta_i^*(\theta_{-i})$ is defined by

$$v_i(\theta_i^*(\theta_{-i}), \theta_{-i}) = \max_{j \neq i} v_j(\theta_i^*(\theta_{-i}), \theta_{-i})$$

if the equation has a solution, and by

$$\theta_i^*(\theta_{-i}) := \begin{cases} \overline{\theta}_i, & \text{if} \quad v_i\left(\overline{\theta}_i, \theta_{-i}\right) < \max_{j \neq i} v_j\left(\overline{\theta}_i, \theta_{-i}\right) \\ \underline{\theta}_i, & \text{if} \quad v_i\left(\underline{\theta}_i, \theta_{-i}\right) > \max_{j \neq i} v_j\left(\underline{\theta}_i, \theta_{-i}\right) \end{cases}$$

if it does not[14]*. Then (k^*, t^*) is incentive compatible*[15]*.*

[14] If $\theta_i^*(\theta_{-i})$ does not exist, it can be arbitrarily chosen out of $[\underline{\theta}_i, \overline{\theta}_i]$. The definition given here simplifies calculations in the next section.

[15] Note that truthtelling is not an equilibrium in dominant strategies, but it is an *ex-post* equilibrium, i.e., it is an equilibrium no matter what the distributions of agents' types are.

3.3 The existence condition

We now have all the needed tools. Theorem 2 shows that an IC,EF,IR and BB mechanism exists if and only if

$$\sum_{i=1}^{n} E_{\theta_i}\left[T_i\left(\theta_i\right)\right] \leq \sum_{i=1}^{n} U_i\left(\widetilde{\theta}_i\right)$$

for an arbitrary IC and EF mechanism. For the mechanism constructed in Theorem 3 we have

$$T_i\left(\theta_i\right) = E_{\theta_{-i}}\left[v_i\left(\theta_i^*\left(\theta_{-i}\right), \theta_{-i}\right) \ \mathbf{1}\left(v_i\left(\theta_i, \theta_{-i}\right) < \max_{j\neq i} \ v_j\left(\theta_i, \theta_{-i}\right)\right)\right].$$

Therefore we can find an IC, EF, IR and BB mechanism if and only if[16]

$$\sum_{i=1}^{n} E_{\theta}\left[v_i\left(\theta_i^*\left(\theta_{-i}\right), \theta_{-i}\right) \mathbf{1} \ \left(v_i\left(\theta_i, \theta_{-i}\right) < \max_{j\neq i} \ v_j\left(\theta_i, \theta_{-i}\right)\right)\right] \leq \sum_{i=1}^{n} U_i\left(\widetilde{\theta}_i\right). \quad (1)$$

4 Dissolving a Partnership

We now apply the above findings to the dissolution of a partnership. We make the following assumptions:

A1 Types are drawn independently from the same distribution function, e.g. $F_i = F$, $\underline{\theta}_i = \underline{\theta}$, $\overline{\theta}_i = \overline{\theta} \quad \forall i$.

A2 The valuation functions $v_i\left(\theta_1, ..., \theta_n\right)$ have the following form: $v_i\left(\theta_1, ..., \theta_n\right) = g\left(\theta_i\right) + \sum_{j\neq i} h\left(\theta_j\right)$, where g, h are continuously differentiable, g is strictly increasing, and $g' > h'$.

To simplify notation, we write $h\left(\theta_{-i}\right) := \sum_{j\neq i} h\left(\theta_j\right)$.

These conditions constitute a natural and simple generalization of the symmetry assumption in Cramton et al. [1987]. Condition A2 is also needed for computational reasons: it allows an explicit characterization of the "worst off" participating types, which otherwise become complex functions of the model's parameters[17] (including valuation functions).

[16] The existence condition for an IC, EF, IR and *ex-post* budget balanced mechanism is the same: by applying the ideas of Arrow [**?**], d'Aspremont and Gérard-Varet [1979] we can find an expected externality mechanism that is ex-post budget-balanced and results in the same interim utilities and payments as the generalized Groves mechanism. In the expected externality mechanism, however, truthtelling is not an ex-post equilibrium.

[17] This assumption does not restrict a main message of this section which states that general possibility results like in private values environments cannot be achieved if valuation functions are increasing in other agents' types.

By the single crossing property and A2, we obtain:

$$v_i(\theta_1,...,\theta_n) > v_j(\theta_1,...,\theta_n) \Leftrightarrow \theta_i > \theta_j, \quad \text{(S1)}$$
$$v_i(\theta_1,...,\theta_n) = v_j(\theta_1,...,\theta_n) \Leftrightarrow \theta_i = \theta_j. \quad \text{(S2)}$$

An EF and IC mechanism is given by (k^*, t^*) of Theorem 3. Because of S1, S2 and A2 we have $\theta_i^*(\theta_{-i}) = \max_{j \neq i} \theta_j$.

Theorem 4 *1) Let* $\widetilde{\theta}_i := F^{-1}(\alpha_i^{\frac{1}{n-1}})$. *1) An EF, IC, BB and IR mechanism exists if and only if:*

$$\sum_{i=1}^{n}\left(\int_{\widetilde{\theta}_i}^{\overline{\theta}} g(\theta)dF^{n-1}(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} g(\theta)F(\theta)\,dF^{n-1}(\theta)\right) + \int_{\underline{\theta}}^{\overline{\theta}} h'(\theta)\left(F^n(\theta) - F(\theta)\right)d\theta \geq 0. \quad (2)$$

2) The set of $(\alpha_1,...,\alpha_n)$ *for which EF, IC, BB and IR mechanisms exist is either empty or a symmetric, convex set around* $\left(\frac{1}{n},...,\frac{1}{n}\right)$.

Condition 2 reduces to that given in Cramton et al. [1987] if $g(\theta_i) = \theta_i$ and $h(\theta_i) \equiv 0$. For that case, they also show that the condition is always fulfilled if $\alpha_1 = \cdots = \alpha_n = \frac{1}{n}$. Observe that the additional term containing the common value component is negative if $h' > 0$ and positive if $h' < 0$. Cramton's et. al. [1987] result implies that, in the latter case, a partnership can be efficiently dissolved if the initial property rights are distributed equally.

Example 1 *To see that both cases in Theorem 4-2 can occur, consider a setting with two agents such that:*

$$\begin{aligned} v_i(\theta_1,\theta_2) &= a\ \theta_i + b\ \theta_{-i}, \qquad a > b > 0, \\ f(\theta_i) &= 1_{[0,1]}(\theta_i)\,;\ \alpha_1 = \alpha_2 = \frac{1}{2}. \end{aligned}$$

Condition 2 reduces to:

$$2a\left(\int_{\frac{1}{2}}^{1} \theta_i\ d\theta_i - \int_0^1 \theta_i^2\ \ d\theta_i\right) + b\int_0^1 \left(\theta_i^2 - \theta_i\right)\ d\theta_i = \frac{1}{12}a - \frac{1}{6}b \geq 0$$

The set of shares (α_1,α_2) *for which an EF, IC, IR and BB mechanism exists is empty if and only if* $0 < a < 2b$. ■

It is interesting to compare the existence condition for interdependent values with that for private values. For this purpose, consider a trading situation with two agents given by $\left(F, \alpha_1, g, h, \underline{\theta}, \overline{\theta}\right)$ and two different valuation functions:

1. An interdependent valuation function $v_i^{IV}(\theta_i, \theta_{-i}) := g(\theta_i) + h(\theta_{-i})$.
2. A private valuation function $v_i^{PV}(\theta_i) := g(\theta_i) + E_{\theta_{-i}}[h(\theta_{-i})] =: g(\theta_i) + H$.

Let (k^*, t^*) be a generalized Groves mechanism (which is a standard Groves mechanism for v_i^{PV}). Observe that, for all types of agent i, the interim valuation $E_{\theta_{-i}}[v_i(\theta_i, \theta_{-i})]$ and interim expected utility are identical in both models. Therefore the "worst off" types are also identical, and so are their equilibrium utilities $U_i(\widetilde{\theta}_i)$. Hence, in both models we can collect the same entry fee while insuring participation of all types. But, according to Theorem 3, the needed (expected) transfers in the interdependent values case are given by

$$E_\theta\left[g\left(\max_i \ \theta_i\right)\right] + E_\theta\left[h\left(\max_i \ \theta_i\right)\right],$$

whereas in the private values case they are given by

$$E_\theta\left[g\left(\max_i \ \theta_i\right)\right] + H = E_\theta\left[g\left(\max_i \ \theta_i\right)\right] + E_{\theta_i}\left[h\left(\ \theta_i\right)\right].$$

If $h' > 0$, the generalized Groves mechanism is more expensive than the standard Groves mechanism[18], and efficiency is harder to achieve with interdependent values. In particular if an efficient mechanism exists for v^{IV}, then an efficient mechanism exists also for v^{PV}. If $h' < 0$, exactly the opposite occurs: efficiency is easier to achieve with interdependent values.

Our next result shows that efficient trade is possible for any valuation functions where $h' \leq 0$ and for any distribution function F, if each individual share is not too small. For example, if there are two partners, efficient dissolution is always possible if the smaller share is at least 25%.

Theorem 5 *Let $\alpha_1 \leq \cdots \leq \alpha_n$, and assume that, for all $i = 1, ..., n-1$, we have $\sum_{j=1}^{i} \alpha_j \geq \left(\frac{i}{n}\right)^n$. Then, for any valuation function $v_i(\theta_i, \theta_{-i}) = g(\theta_i) + h(\theta_{-i})$ with $h'(\theta_{-i}) \leq 0$ and for any distribution function F, the partnership can be dissolved efficiently.*

It is a-priori plausible that the above insight continues to hold if the derivative of the common value component is positive, but sufficiently small. We next show, however, that this is not the case: even if that derivative is arbitrarily small but positive, there exist distribution functions such that an equal partnership cannot be dissolved efficiently.

[18] Bergemann and Välimäki [2000] focus on the differences between transfers in the CGV mechanisms in the private and interdependent values cases in order to compare the resulting incentives for information acquisition.

Theorem 6 *For any valuation function* $v_i(\theta_i, \theta_{-i}) = g(\theta_i) + h(\theta_{-i})$ *with* $h'(\theta_{-i}) > 0$ *there exists a distribution function* F *such that the equal partnership cannot be efficiently dissolved. By Theorem 4-2, for this* F *there is no ex-ante distribution of shares that leads to efficient trade.*

A distribution F with the above property puts mass on types close to the extremities of the types' interval. For these types, the payment difference between a standard Groves mechanism and the generalized Groves mechanism is relatively large. Since the later is much more costly, inefficiency occurs.

5 Bilateral trade

We now briefly look at the case of two agents, one who a-priori owns the whole good (the seller) and another one who wants to buy the good (the buyer). We denote the agents by S and B for seller and buyer, respectively, so that $i \in \{S, B\}$, $\alpha_S = 1$ and $\alpha_B = 0$.

For this special case the "worst off" types do not depend on the functional form of the valuation functions[19], and we can allow for general valuations (as introduced in section 2).

The following Theorem[20] exhibits a condition under which efficient trade is possible.

Theorem 7 *An EF, IC, BB and IR mechanism exists if and only if*

$$\int_{\underline{\theta}_S}^{\overline{\theta}_S} \int_{\underline{\theta}_B}^{\overline{\theta}_B} \left(v_S\left(\theta_S^*\left(\theta_B\right), \theta_B\right) - v_B\left(\theta_S, \theta_B^*\left(\theta_S\right)\right)\right) \times \mathbf{1}\left(k_B^*\left(\theta\right) = 1\right) f_B\left(\theta_B\right) d\theta_B f_S\left(\theta_S\right) d\theta_S \leq 0.$$

Assume that agents' valuations are increasing in other agents' types, i.e. $v_{S,B} \geq 0, v_{B,S} \geq 0$. *An EF, IC, BB and IR mechanism exists if and only if:*

$$E_{\theta_S}\left[v_B\left(\theta_S, \underline{\theta}_B\right)\right] \geq E_{\theta_B}\left[v_S\left(\overline{\theta}_S, \theta_B\right)\right] \quad \text{or} \quad v_B\left(\underline{\theta}_S, \overline{\theta}_B\right) \leq v_S\left(\underline{\theta}_S, \overline{\theta}_B\right)$$

In other words, if valuations are increasing in the other agent's type, efficient trade is only possible if a price p exists such that we can always have trade at this price[21], i.e. if.

$$E_{\theta_S}\left[v_B\left(\theta_S, \underline{\theta}_B\right)\right] \geq p \geq E_{\theta_B}\left[v_S\left(\overline{\theta}_S, \theta_B\right)\right].$$

[19] The "worst off" seller is always a seller of type $\overline{\theta}_S$ and the "worst off" buyer is always a buyer of type $\underline{\theta}_B$.

[20] The proof follows the intuition used in the previous section and is ommitted here. It can be found in the discussion paper version, available from the authors.

[21] In contrast to the Akerlof model, however, we cannot a-priori restrict attention to simple mechanisms that set prices.

Note also that a necessary condition for $E_{\theta_S}\left[v_B\left(\theta_S, \underline{\theta}_B\right)\right] \geq E_{\theta_B}\left[v_S\left(\overline{\theta}_S, \theta_B\right)\right]$ to hold is that $v_B\left(\theta_S, \theta_B\right) > v_S\left(\theta_S, \theta_B\right)$ for all θ_S, θ_B.

On the other hand, if the negative dependence of agents' valuations on the other agent's type is strong enough, then efficient trade is possible even if the distributions of types have overlapping support (i.e., even if the possibility of efficient trade is not common knowledge)[22]. This phenomenon is illustrated below.

Example 2 *Assume that valuations are:*

$$v_B(\theta_S, \theta_B) = a\theta_B + b\theta_S; \; v_S(\theta_S, \theta_B) = a\theta_S + b\theta_B$$

with $a > b$, $a > 0$, *and assume that* $f_B(\theta_B) = \mathbf{1}_{[0,1]}(\theta_B)$, $f_S(\theta_S) = \mathbf{1}_{[0,1]}(\theta_S)$. *Theorem 3 shows that the following mechanism is IC and EF:*

$$\begin{aligned} k_B(\theta) &= \begin{cases} 1 & if \quad \theta_B \geq \theta_S \\ 0 & if \quad \theta_B < \theta_S \end{cases} \\ t_S(\theta) &= (a+b)\,\theta_B \quad k_B(\theta)\,; \; t_B(\theta) = -(a+b)\,\theta_S \quad k_B(\theta) \end{aligned}$$

Because worst-off types never trade, the mechanism designer cannot collect entry fees. He has to pay (in expectation):

$$(a+b)\int_0^1\int_0^1 (\theta_B - \theta_S) \quad k_B(\theta_B, \theta_S)\, d\theta_S d\theta_B = \frac{1}{6}(a+b)\,.$$

For $b > 0$ *it is more costly to achieve efficiency than in the private values case (where the mechanism designer has to pay* $\frac{1}{6}a$*). For* $b = -a$ *, everybody tells the truth without receiving any payments at all, so that BB and IR are also fulfilled. For* $b < -a$ *the designer can even extract money from the traders!* ■

6 Conclusion

In a recent survey of Mechanism Theory, Mat Jackson writes:

> "There is still much that is not known about the existence or properties of incentive compatible mechanisms that are efficient[23] (much less the balanced and individual rational), when there are general forms of uncertainty and interdependencies in the preferences of individuals." (Jackson, 2000)

Our study represents the first attempt at a systematic study of the above problems. Focusing on the possibility of efficient trade, we have highlighted the similarities and differences between the private value case and the case with interdependent valuations.

[22] Gresik (1991) generally concludes that efficient trade is impossible. But some parts of his analysis hold in fact only for settings where valuations increase in the other agent's signal.

[23] value maximizing (n.a.)

Our analysis generalizes and unifies several well-known results that were obtained in special cases. We showed how the comparison of the private and interdependent cases crucially depends on whether valuations are increasing or decreasing in other agents' signals.

For the Myerson-Satterthwaite and Akerlof "extreme-ownership" settings second-best mechanisms have been exhibited in the literature (see Myerson and Satterthwaite [1983], Samuelson [1984], Gresik [1991]). The construction of a second-best mechanisms for the partnership model with interdependent values is still an open question. First steps have been undertaken by Jehiel and Pauzner [1999] and Kittsteiner [2000]. Jehiel and Pauzner study second-best mechanisms in a setting with a single informed partner. They show that the second-best allocation method coincides with the ex-post efficient allocation only outside an interior interval of types where no trade takes place. Kittsteiner calculates the bidding equilibrium in a specific mechanism - the double auction - in the partnership model of section 4, and shows that an interior interval of types will not participate in the auction. He also shows that, in some situations, there are mechanisms which are welfare superior to the double auction[24].

A Appendix

Proof of Theorem 1: The proof follows that for private values (see e.g. Myerson [1981]). There is one additional argument needed to justify the differentiation under the integral. Incentive compatibility implies:

$$U_i(\theta_i,\theta_i) \geq U_i\left(\theta_i,\widehat{\theta}_i\right) \text{ and } U_i\left(\widehat{\theta}_i,\widehat{\theta}_i\right) \geq U_i\left(\widehat{\theta}_i,\theta_i\right) \quad \forall\ \theta_i,\widehat{\theta}_i.$$

We therefore obtain the following inequalities:

$$V_i\left(\widehat{\theta}_i,\widehat{\theta}_i\right) - V_i\left(\theta_i,\widehat{\theta}_i\right) \geq U_i\left(\widehat{\theta}_i,\widehat{\theta}_i\right) - U_i(\theta_i,\theta_i) \geq V_i\left(\widehat{\theta}_i,\theta_i\right) - V_i(\theta_i,\theta_i).$$

Dividing by $\widehat{\theta}_i - \theta_i$ gives

$$E_{\theta_{-i}}\left[\frac{v_i\left(\widehat{\theta}_i,\theta_{-i}\right) - v_i(\theta_i,\theta_{-i})}{\widehat{\theta}_i - \theta_i}\left(k_i\left(\widehat{\theta}_i,\theta_{-i}\right) - \alpha_i\right)\right] \geq \frac{U_i\left(\widehat{\theta}_i,\widehat{\theta}_i\right) - U_i(\theta_i,\theta_i)}{\widehat{\theta}_i - \theta_i}$$
$$\geq E_{\theta_{-i}}\left[\frac{v_i\left(\widehat{\theta}_i,\theta_{-i}\right) - v_i(\theta_i,\theta_{-i})}{\widehat{\theta}_i - \theta_i}\left(k_i(\theta_i,\theta_{-i}) - \alpha_i\right)\right].$$

[24] Note that a double auction is a second-best mechanism in private values environments.

Because v_i is continuously differentiable and because $\lim_{\widehat{\theta}_i \to \theta_i} k_i(\theta_i, \theta_{-i}) = k_i\left(\widehat{\theta}_i, \theta_{-i}\right)$ a.e. we can take the limit $\widehat{\theta}_i \to \theta_i$ and apply the Dominated Convergence Theorem to obtain

$$E_{\theta_{-i}}\left[v_{i,1}(\theta_i, \theta_{-i})(k_i(\theta_i, \theta_{-i}) - \alpha_i)\right] \geq \frac{dU(\theta_i)}{d\theta_i} \geq E_{\theta_{-i}}\left[v_{i,1}(\theta_i, \theta_{-i})(k_i(\theta_i, \theta_{-i}) - \alpha_i)\right]$$

and therefore that $U(\theta_i)$ is differentiable with $\frac{dU(\theta_i)}{d\theta_i} = E_{\theta_{-i}}\left[v_{i,1}(\theta_i, \theta_{-i})(k_i(\theta_i, \theta_{-i}) - \alpha_i)\right]$

■

Proof of Corollary1: Fix an IC mechanism that implements k and has the payment functions $t_i(\theta_1, ..., \theta_n)$, $i = 1, ..., n$. Observe that a mechanism $(k, t + r)$ with $r = (r_1, ..., r_n)$, where r_i is an arbitrary constant also implements k. Consider an arbitrary mechanism that implements k and has payment functions $s_i(\theta)$ and interim payment functions $S_i(\theta_i) := E_{\theta_{-i}}[s_i(\theta_i, \theta_{-i})]$. Denote by $U_i^s(\theta_i)$ and by $U_i^{t+r}(\theta_i)$ the interim equilibrium utilities of agents participating in (k, s) and $(k, t + r)$, respectively. Because the interim utilities of the participating agents are (up to a constant) the same for all IC mechanisms that implement k, we can find constants q_i such that $U_i^s(\theta_i) = U_i^{t+q}(\theta_i)$ .This means that for every IC mechanism (k, s) we can find a mechanism $(k, t + q)$ that is equivalent to (k, s) in terms of interim utilities. This leads to the following important observation: If the mechanism (k, s) is BB and IR, then the mechanism $(k, t + q)$ is also BB and IR. To check this note that $U_i^{t+q}(\theta_i) = U_i^s(\theta_i) \geq 0$ and that $\sum_{i=1}^n E_{\theta_i}[T_i(\theta_i) + q_i] = \sum_{i=1}^n E_{\theta_i}\left[U_i^{t+q}(\theta_i) - V_i(\theta_i, \theta_i)\right] = \sum_{i=1}^n E_{\theta_i}[S_i(\theta_i)] \leq 0.$ ■

Proof of Theorem 2: Given a mechanism (k, t) that implements k, let $\widetilde{\theta}_i$ be the "worst off" type of agent i. Let $q = (q_1, ..., q_n)$ be a vector of constants. Because of $U_i^{t+q}(\theta_i) = U_i^t(\theta_i) + q_i$ the "worst off" type of player i in the mechanism $(k, t + q)$ is also given by $\widetilde{\theta}_i$. We are looking for constants q_i such that the mechanism $(k, t + q)$ is BB and IR, i.e., $\sum_{i=1}^n (E_{\theta_i}[T_i(\theta_i)] + q_i) \leq 0$ and $U_i^{t+q}\left(\widetilde{\theta}_i\right) = U_i^t\left(\widetilde{\theta}_i\right) + q_i \geq 0 \quad \forall i$. These conditions can hold if and only if $\sum_{i=1}^n E_{\theta_i}[T_i(\theta_i)] \leq \sum_{i=1}^n U_i^t\left(\widetilde{\theta}_i\right)$. ■

Proof of Theorem 3: Consider agent i and assume that all agents other than i report their types θ_{-i} truthfully. Assume first, that $\forall\ \theta_i$ we have $v_i(\theta_i, \theta_{-i}) > \max_{j \neq i} v_j(\theta_i, \theta_{-i})$ or $v_i(\theta_i, \theta_{-i}) < \max_{j \neq i} v_j(\theta_i, \theta_{-i})$. Then i's report does not change the allocation k^*. Because payments do not depend on θ_i, it is optimal for i to report truthfully.

Assume now that $v_i(\theta_i^*, \theta_{-i}) = \max_{j \neq i} v_j(\theta_i^*, \theta_{-i})$ for $\theta_i^* \in \left[\underline{\theta}_i, \overline{\theta}_i\right]$, and that the true type of agent i is θ_i. We distinguish several cases:

1. $\theta_i > \theta_i^*$: Any report $\widehat{\theta}_i > \theta_i^*$ does not change the allocation (agent i still gets the good) because we have

$$v_i(\theta_i^*, \theta_{-i}) = \max_{j \neq i} v_j(\theta_i^*, \theta_{-i}) \text{ and } v_{i,i}(\theta_i, \theta_{-i}) > v_{j,i}(\theta_i, \theta_{-i}) \quad \forall\ j \neq i$$

$$\Rightarrow v_i\left(\widehat{\theta}_i, \theta_{-i}\right) > \max_{j\neq i}\ v_j\left(\widehat{\theta}_i, \theta_{-i}\right).$$

Payments are not affected by reporting $\widehat{\theta}_i > \theta_i$ either. If i reports $\widehat{\theta}_i < \theta_i^*$ he won't get the good any more but receives the payment $v_i(\theta_i^*, \theta_{-i}) < v_i(\theta_i, \theta_{-i})$ and therefore it is optimal to report θ_i instead of $\widehat{\theta}_i$. If $\widehat{\theta}_i = \theta_i^*$ agent i gets either $v_i(\theta_i^*, \theta_{-i})$ or $v_i(\theta_i, \theta_{-i})$. So he cannot improve his payoff by lying.

2. $\theta_i < \theta_i^*$: As long as i announces $\widehat{\theta}_i < \theta_i^*$ he doesn't change the allocation because we have $v_i\left(\widehat{\theta}_i, \theta_{-i}\right) < \max_{j\neq i}\ v_j\left(\widehat{\theta}_i, \theta_{-i}\right)$. If $\widehat{\theta}_i > \theta_i^*$ he will get the good but values it $v_i(\theta_i, \theta_{-i})$ which is less than the payment he gets by reporting truthfully, $v_i(\theta_i^*, \theta_{-i})$. As above truth-telling yields at least the same as reporting $\widehat{\theta}_i = \theta_i^*$.

3. $\theta_i = \theta_i^*$: In this case agent i will always have the utility $v_i(\theta_i^*, \theta_{-i})$ (independent of his announcement) and therefore optimally reports the truth. ■

Proof of Theorem 4: 1) We have to show that conditions 1 and 2 are equivalent. For this, we first determine the interim utilities of the "worst off" types and the expected payments to the agents. The interim expected utility of agent i with type θ_i is given by:

$$\begin{aligned} U_i(\theta_i) &= E_{\theta_{-i}}\Big[v_i(\theta_i, \theta_{-i})\ \mathbf{1}(\theta_i > \max_{j\neq i}\ \theta_j) + \\ &\quad + v_i\left(\max_{j\neq i}\ \theta_j, \theta_{-i}\right)\ \mathbf{1}(\theta_i < \max_{j\neq i}\ \theta_j) - \alpha_i v_i(\theta_i, \theta_{-i})\Big]. \end{aligned}$$

The "worst off" type $\widetilde{\theta}_i$ satisfies $\widetilde{\theta}_i = \arg\min_{\theta_i} U_i(\theta_i)$. The first order condition of this minimization problem gives

$$\begin{aligned} 0 &= E_{\theta-i}\left[\frac{\partial}{\partial\ \theta_i} v_i(\theta_i, \theta_{-i})\ \left(\mathbf{1}(\theta_i > \max_{j\neq i}\ \theta_j) - \alpha_i\right)\right] \\ &= g'(\theta_i)\ E_{\theta_{-i}}[\mathbf{1}(\theta_i > \max_{j\neq i}\ \theta_j) - \alpha_i] = g'(\theta_i)\ (F^{n-1}(\theta_i) - \alpha_i) \end{aligned}$$

This yields $\widetilde{\theta}_i = F^{-1}(\alpha_i^{\frac{1}{n-1}})$. This is the only minimum because $F^{n-1}(\theta_i) - \alpha_i$ is negative for $\theta_i < \widetilde{\theta}_i$ and positive for $\theta_i > \widetilde{\theta}_i$. The interim utility $U_i\left(\widetilde{\theta}_i\right)$ is given by

$$U_i\left(\widetilde{\theta}_i\right) = E_{\theta_{-i}}\left[v_i(\widetilde{\theta}_i, \theta_{-i})\ \mathbf{1}(\widetilde{\theta}_i > \max_{j\neq i}\ \theta_j) + v_i\left(\max_{j\neq i}\ \theta_j, \theta_{-i}\right)\ \mathbf{1}(\widetilde{\theta}_i < \max_{j\neq i}\ \theta_j) - \alpha_i v_i(\widetilde{\theta}_i, \theta_{-i})\right].$$

Using $\theta_i^*(\theta_{-i}) = \max_{j\neq i}\ \theta_j$ condition (1) writes:

$$\sum_{i=1}^{n} E_{\theta_{-i}}\left[v_i(\widetilde{\theta}_i,\theta_{-i})\ \mathbf{1}(\widetilde{\theta}_i > \max_{j\neq i}\ \theta_j) + v_i\left(\max_{j\neq i}\ \theta_j,\theta_{-i}\right)\ \mathbf{1}(\widetilde{\theta}_i < \max_{j\neq i}\ \theta_j) - \alpha_i v_i(\widetilde{\theta}_i,\theta_{-i})\right]$$
$$-\sum_{i=1}^{n} E_{\theta}\left[v_i\left(\max_{j\neq i}\ \theta_j,\theta_{-i}\right)\mathbf{1}\ \left(v_i\left(\theta_i,\theta_{-i}\right) < \max_{j\neq i}\ v_j\left(\theta_i,\theta_{-i}\right)\right)\right] \geq 0.$$

Using separability and symmetry of the valuation functions, we obtain (for some i)

$$\sum_{j=1}^{n}\left[\int_{\widetilde{\theta}_j}^{\overline{\theta}} g(\theta)\ dF^{n-1}(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\ F(\theta)\ dF^{n-1}(\theta)\right]$$
$$+(n-1)\ E_{\theta_{-i}}\left[\sum_{j\neq i} h(\theta_j)\right] - n\ E_{\theta_{-i}}\left[\sum_{j\neq i} h(\theta_j)\ F\left(\max_{j\neq i}\ \theta_j\right)\right] \geq 0.$$

Integration by parts gives

$$\begin{aligned}
&(n-1)\ E_{\theta_{-i}}\left[\sum_{j\neq i} h(\theta_j)\right] - n\ E_{\theta_{-i}}\left[\sum_{j\neq i} h(\theta_j)\ F\left(\max_{j\neq i}\ \theta_j\right)\right]\\
=\ &(n-1)\ n\int_{\underline{\theta}}^{\overline{\theta}} h(\theta)\ \left[\int_{\theta}^{\overline{\theta}} F^{n-2}(M)\, f(M)\, dM - \frac{1}{n}\right]\ f(\theta)\, d\theta.
\end{aligned}$$

Using $\int_{\theta}^{\overline{\theta}} F^{n-2}(M)\, f(M)\, dM = \frac{1-F^{n-1}(\theta)}{n-1}$ and $n\int_{\underline{\theta}}^{\overline{\theta}} h(\theta)\, F^{n-1}(\theta)\ f(\theta)\ d\theta = h(\overline{\theta}) - \int_{\underline{\theta}}^{\overline{\theta}} h'(\theta)\, F^{n}(\theta)\, d\theta$ we get the wished result.

2) Consider $\psi : [0,1]^n \mapsto \mathbb{R}$ with

$$\begin{aligned}
\psi(\alpha_1,...,\alpha_n) =\ &\sum_{i=1}^{n}\left(\int_{\widetilde{\theta}_i}^{\overline{\theta}} g(\theta)\ dF^{n-1}(\theta)\ d\theta - \int_{\underline{\theta}}^{\overline{\theta}} g(\theta)\ F(\theta)\ dF^{n-1}(\theta)\ d\theta\right) +\\
&+\int_{\underline{\theta}}^{\overline{\theta}} h'(\theta)\ \left(F^{n}(\theta) - F(\theta)\right)\, d\theta.
\end{aligned}$$

The function ψ is symmetric in its arguments and concave (recall that $\alpha_i = F^{n-1}\left(\widetilde{\theta}_i\right)$):

$$\begin{aligned}
\frac{\partial\psi}{\partial\alpha_i} &= -\ g(\widetilde{\theta}_i)\ (n-1)\, F^{n-2}\left(\widetilde{\theta}_i\right) f(\widetilde{\theta}_i)\frac{d\widetilde{\theta}_i}{d\alpha_i} = -g\left(\widetilde{\theta}_i\right)\frac{dF^{n-1}\left(\widetilde{\theta}_i\right)}{d\alpha_i} = -g\left(\widetilde{\theta}_i\right),\\
\frac{\partial^2\psi}{\partial\alpha_i^2} &= -g'\left(\widetilde{\theta}_i\right)\ \frac{d\widetilde{\theta}_i}{d\alpha_i} < 0.
\end{aligned}$$

Because of symmetry, ψ takes its maximum on the simplex $\sum_{i=1}^{n}\alpha_i = 1$ at $\frac{1}{n}, ..., \frac{1}{n}$, and the set of $(\alpha_1, ..., \alpha_n)$ that lead to positive values of ψ is either symmetric and convex, or empty. ■

Proof of Theorem 5: We have $\widetilde{\theta}_i = F^{-1}\left(\alpha_i^{\frac{1}{n-1}}\right)$ and therefore $\widetilde{\theta}_1 \leq \cdots \leq \widetilde{\theta}_n$. Integration by parts in condition 2 yields:

$$
\begin{aligned}
& \sum_{i=1}^{n}\left(g\left(\overline{\theta}\right) - \alpha_i g\left(\widetilde{\theta}_i\right) - \int_{\widetilde{\theta}_i}^{\overline{\theta}} g'\left(\theta\right) F^{n-1}\left(\theta\right) d\theta\right) - (n-1)\, g\left(\overline{\theta}\right) + (n-1)\int_{\underline{\theta}}^{\overline{\theta}} g'\left(\theta\right) F^{n}\left(\theta\right) d\theta \\
& + \int_{\underline{\theta}}^{\overline{\theta}} h'\left(\theta\right)\ \left(F^{n}\left(\theta\right) - F(\theta)\right) d\theta \\
\geq\ & \sum_{i=1}^{n}\int_{\widetilde{\theta}_i}^{\overline{\theta}} g'\left(\theta\right)\ \left(\alpha_i - F^{n-1}\left(\theta\right)\right)\ d\theta + (n-1)\int_{\underline{\theta}}^{\overline{\theta}} g'\left(\theta\right)\ F^{n}\left(\theta\right)\ d\theta \\
=\ & \int_{\widetilde{\theta}_n}^{\overline{\theta}} g'\left(\theta\right)\left(1 - nF^{n-1}\left(\theta\right) + (n-1)\, F^{n}\left(\theta\right)\right) d\theta + \sum_{i=1}^{n-1}\int_{\widetilde{\theta}_i}^{\widetilde{\theta}_n} g'\left(\theta\right)\ \left(\alpha_i - F^{n-1}\left(\theta\right)\right)\ d\theta \\
& + (n-1)\int_{\underline{\theta}}^{\widetilde{\theta}_n} g'\left(\theta\right)\ F^{n}\left(\theta\right)\ d\theta \\
\geq\ & \sum_{i=1}^{n-1}\int_{\widetilde{\theta}_i}^{\widetilde{\theta}_{i+1}} g'\left(\theta\right)\ \left(\sum_{j=1}^{i}\alpha_j - iF^{n-1}\left(\theta\right) + (n-1)\, F^{n}\left(\theta\right)\right)\ d\theta
\end{aligned}
$$

Since for all i, $1 \leq i \leq n-1$, $iF^{n-1}\left(\theta\right) - (n-1)\, F^{n}\left(\theta\right) \leq \left(\frac{i}{n}\right)^{n}$ the last expression is non-negative. ■

Proof of Theorem 6: It is sufficient to show that for any function $v_i = g\left(\theta_i\right) + h\left(\theta_{-i}\right)$ with $h' > 0$ there exists a distribution function F such that efficient trade fails for $\alpha_1 = \cdots = \alpha_n = \frac{1}{n}$. Integration by parts shows that condition 2 is equivalent to:

$$
\begin{aligned}
& \int_{\widetilde{\theta}}^{\overline{\theta}} g'\left(\theta\right)\ \left(1 - nF^{n-1}\left(\theta\right) + (n-1)\, F^{n}\left(\theta\right)\right)\ d\theta + (n-1)\int_{\underline{\theta}}^{\widetilde{\theta}} g'\left(\theta\right)\ F^{n}\left(\theta\right)\ d\theta \\
& + \int_{\underline{\theta}}^{\overline{\theta}} h'\left(\theta\right)\ \left(F^{n}\left(\theta\right) - F(\theta)\right) d\theta \geq 0.
\end{aligned}
$$

Let $a := \max_{\theta\in\left[\underline{\theta},\overline{\theta}\right]} g'(\theta) > 0$ and $b := \min_{\theta\in\left[\underline{\theta},\overline{\theta}\right]} h'(\theta) > 0$. Since $1 - nF^{n-1}\left(\theta\right) +$

$(n-1)F^n(\theta) \geq 0$ it suffices to show that there exists a distribution F such that

$$a\int_{\widetilde{\theta}}^{\overline{\theta}} \left(1 - nF^{n-1}(\theta) + (n-1)F^n(\theta)\right) d\theta + a(n-1)\int_{\underline{\theta}}^{\widetilde{\theta}} F^n(\theta)\ d\theta$$
$$+b\int_{\underline{\theta}}^{\overline{\theta}} \left(F^n(\theta) - F(\theta)\right) d\theta < 0. \qquad (3)$$

We first show that this is the case for the discontinuous distribution F^* given by:

$$F^*(\theta) = \begin{cases} \left(\frac{1}{n}\frac{b}{(n-1)a+b}\right)^{\frac{1}{n-1}} & \text{if} \quad \theta \in [\underline{\theta}, \overline{\theta}) \\ 1 & \text{if} \quad \theta = \overline{\theta} \end{cases}.$$

We set $\widetilde{\theta} = \overline{\theta}$ because $F^*(\theta) < \left(\frac{1}{n}\right)^{\frac{1}{n-1}}$ for all $\theta < 1$. Calculating (3) for F^* yields:

$$a(n-1)\int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{1}{n}\frac{b}{(n-1)a+b}\right)^{\frac{n}{n-1}} d\theta$$
$$+b\int_{\underline{\theta}}^{\overline{\theta}} \left(\left(\frac{1}{n}\frac{b}{(n-1)a+b}\right)^{\frac{n}{n-1}} - \left(\frac{1}{n}\frac{b}{(n-1)a+b}\right)^{\frac{1}{n-1}}\right) d\theta$$
$$= \left(\overline{\theta} - \underline{\theta}\right)\left[(a(n-1)+b)\left(\frac{1}{n}\frac{b}{(n-1)a+b}\right) - b\right]\left(\frac{1}{n}\frac{b}{(n-1)a+b}\right)^{\frac{1}{n-1}} < 0.$$

We now construct a sequence off cumulative distribution functions that are feasible and "arbitrarily close" to F^* . Therefore, such distribution functions will also violate the existence condition. Let $\theta_M := \frac{(\overline{\theta}-\underline{\theta})}{2}$ and $K := \left(\frac{1}{n}\frac{b}{(n-1)a+b}\right)^{\frac{1}{n-1}}$, and consider the following sequence of cumulative distribution functions[25] F_m for odd $m > 1$ [26]:

$$F_m(\theta) = \begin{cases} \left(\frac{\theta-\theta_M}{\overline{\theta}-\theta_M}\right)^m (1-K) + K & \text{if} \quad \theta \geq \theta_M \\ K\left(\frac{\theta-\theta_M}{\theta_M-\underline{\theta}}\right)^m + K & \text{if} \quad \theta < \theta_M. \end{cases}$$

For m large enough F_m satisfies condition (3) , which completes the proof. ■

[25] Observe that they are strictly increasing and differentiable.

[26] This will yield $\widetilde{\theta}_m = F^{-1}\left(\frac{1}{m^{\frac{1}{m-1}}}\right)$.

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