# Voting Agendas and Preferences on Trees: Theory and Practice

Andreas Kleiner and Benny Moldovanu*

April 21, 2020

**Abstract**

We study how parliaments and other committees vote to select one out of several alternatives in situations where not all available options can be ordered along a "left-right" axis. Practically all democratic parliaments routinely use sequential binary voting procedures to select one of several alternatives. Which agendas are used in practice, and how should they be designed? We assume that preferences are single-peaked on an arbitrary tree and we study *convex* agendas where, at each stage in the sequential, binary voting process, the tree of remaining alternatives is divided into two subtrees that are subjected to a binary Yes-No vote. In this wide class of situations, we show that dynamic, strategic voting is congruent with sincere, unsophisticated voting, even if agents are privately informed and no matter what their beliefs about other voters are. We conclude the paper by illustrating the empirical implications of our results for two case studies from Germany and the UK.

## 1 Introduction

We study how parliaments and other committees vote to select one out of several alternatives in complex situations where not all available options can be ordered along a "left-right" axis.

*This paper was previously circulated under the title "Abortions, Brexit and Trees". We wish to thank Mirjam Beyer, Tim Bock and Katharina Schneider for research assistance. Moldovanu acknowledges financial support from the German Science Foundation via the Hausdorff Center, Econtribute Cluster of Excellence and CRC TR-224. Kleiner: Department of Economics, Arizona State University, andreas.kleiner@asu.edu; Moldovanu: Department of Economics, University of Bonn, mold@uni-bonn.de.

For example, in a well-known abortion legislation case from the German Bundestag, which we describe below, the main axis of conflict pitted the rights of women versus the rights of unborn life, but many of the eight proposed bills contained additional provisions about deadlines that need to be respected for legal abortions, possible punishments for both women and doctors that perform illegal abortions, the need for counseling, psycho-social indications, etc. Certain pairs of alternatives were easily ordered, while other pairs were not comparable along the main axis. The German Bundestag used a particular, apparently well-designed agenda, and we offer here a theory that allows us to understand the rationale behind the voting procedure and the ensuing consequences for voting behavior and the final outcome.

In another recent and dramatic case from the UK Parliament, the main conflict axis involved a "hard" vs. "soft" (or no) Brexit. However, due to the complexity of the question and the many potential post-Brexit arrangements, some of the proposed bills were not easily comparable along this main conflict line. The employed voting agenda was rather unusual: Premier May's possible strategic calculations did not materialize, and she was subsequently forced to resign.

As in the two cases mentioned above, practically all democratic parliaments routinely use *sequential binary voting procedures* to select one of several alternatives. At each stage in a sequence of votes and starting with the full set of alternatives, the set of remaining alternatives is divided into two strict subsets.[1] Then, a binary Yes-No vote is taken on the two subsets. The subset that gains a majority of the votes advances to the next stage, while the other subset is discarded. This process is repeated until a single alternative remains, and is formally elected. There is considerable variation concerning the precise structure of the divisions into two subsets that are put to vote at each stage. Well-known, stylized representatives include the following:

1) The amendment procedure (AP) is common in the Anglo-Saxon world. It works with a basic bill (proposed by the Government, say), amendments to that bills, amendments to amendments, etc. At each stage, two alternatives (the original bill and an amended version, say) are pitted against each other, and the winner advances to the next stage that has a similar structure.

2) The Successive Procedure (SP) is common in continental Europe and usually works with independent, fully-formed bills. At each stage, a single bill is voted upon (so to say, against the rest of the alternatives), and voting stops as soon as one alternative obtains a majority.

As we shall see below, the *agenda* – defining which subsets of alternatives are considered at each voting stage – plays a crucial role in determining

[1] These need not be disjoint.

individual voting behavior and the identity of the elected alternative.

How should agendas be designed? In previous work, we identified a special class of carefully constructed agendas ensuring that sincere voting at each stage constitutes a very robust, dynamic equilibrium in any sequential binary voting procedure, as long as privately informed voters have single-peaked preferences on alternatives ordered on a *line*, e.g. when the underlying issue is one dimensional (see Kleiner and Moldovanu, [2017]). We also illustrated that such designed agendas are deliberately used in some (but not all) parliaments, and gave examples of documented strategic behavior (so called "manipulations") in cases where the agenda was formed by different criteria.

In the present paper we extend our previous analysis to the much larger class of preferences that are single-peaked on an arbitrary *tree*. Trees represent ideological relations among alternatives that go well beyond the one-dimensional "left-right" framework underlying single-peakedness on a line but still avoid impossibility results that would result in fully-fledged multidimensional problems.[2] This class of preferences was introduced in an elegant paper by Demange [1988]. Demange showed that although the induced majority dominance relation on alternatives is not necessarily transitive, every profile of single-peaked preferences on a tree admits a Condorcet winner.[3] This generalizes the classical insight, due to Black [1948], who showed that the peak of the median voter is a Condorcet winner for single-peaked preferences on a line.[4]

In this paper we introduce *convex* agendas on trees: at each stage in the sequential, binary voting process, the tree of remaining alternatives that has not yet been discarded is divided into two *subtrees* that are subjected to a binary Yes-No vote. Since subtrees are *connected* sets of alternatives, this roughly says that each of the two subsets of alternatives in each Yes-No vote is ideologically coherent (according to the logic induced by the original, underlying tree). Thus, it cannot be the case that "extreme left" and "extreme right" alternatives are grouped together in one subset, and a "moderate" compromise among those extremes appears only in the other subset.

Assume now that preferences of incompletely informed agents are single-peaked with respect to an arbitrary tree, and that an arbitrary sequential, binary voting procedure with an arbitrary convex agenda is used. Our main theoretical result then shows that *sincere*, myopic voting at each stage in the

[2]We note that complex multidimensional voting problems are often divided into several simpler ones. See also Poole [2005].

[3]In other words, cycles may occur, but they never occur at the top of the majority dominance relation.

[4]In that case the majority relation is indeed acyclical.

voting sequence is an ex-post perfect equilibrium (and hence does not depend on the agents' beliefs about each other), and that the Condorcet winner is elected in this equilibrium. The ex-post nature of our dynamic equilibrium concept also embodies a notion of no-regret: even if agents were told ex-post what the actual preferences of others were, they would not want to revise their past voting behavior.[5]

Thus, in a wide class of situations, equilibrium strategies in the dynamic voting game are congruent with sincere voting – that is, by definition, myopic or "unsophisticated" voting. This holds even if agents are privately informed about their preferences, and no matter what their beliefs about other voters are and what the information revealed during the voting process is. Thus, our paper can also be seen as contributing to the *robust design* of dynamic voting procedures.

We conclude the paper by illustrating the empirical implications of our results for the abovementioned case studies from Germany and the UK. Both instances involved binary, sequential voting by more than 600 heterogeneous voters who needed to dynamically select one out of a relatively large number of alternatives (up to eight). Party discipline (or the "whip") – whereby members of parliament have to vote according to a uniform party line – was either institutionally not imposed (Germany), or was simply not respected by many decisive voters (UK). As a consequence, in both cases the final outcome was highly uncertain. Therefore, both voting instances involved highly complex strategic situations, whose precise analysis seems, at least a priori, beyond the reach of standard theory. We are nevertheless able to apply theoretical results because, as mentioned above, we conduct a robust equilibrium analysis.

In each case study, we first construct appropriate trees on which preferences were presumably single-peaked. It is worth mentioning here that if there exists a tree that renders a profile of preferences single-peaked, then, under a very mild richness condition, such a tree is unique (Trick [1989]). This feature significantly restricts our freedom as analysts to presume an underlying structure of preferences that may not exist in reality.

We next derive the few voting patterns (i.e., the individual sequences of Yes and No votes) that could be observed if our theory was correct (given the employed agenda and the assumed structure of preferences). Finally, we compare the predicted patterns with those observed in reality. This special form of *revealed-preference analysis* allows us to estimate the voters' preferences and to decide whether the outcome was the Condorcet winner. Yet

[5]This should not be confused with the stronger notion of an equilibrium in dominant strategies. Such equilibria need not exist in our framework.

again, the empirical identification task is rendered feasible here by our robust equilibrium concept: without it, we would have a joint inference problem of both preferences and beliefs.

The paper is organized as follows: In the next subsection we review the related literature. In Section 2, we recall several fundamental definitions and results about graphs that are trees. In Section 3, we introduce the social choice model, the sequential binary voting procedures under incomplete information, and their agendas. In Section 4, we prove our main theoretical result that connects sincere and strategic voting for convex agendas. In Section 5, we present two case studies, one each from the German and UK parliaments. In Section 6, we discuss the results and the underlying assumptions.

## Related literature

The study of strategic, sequential binary voting was pioneered by Farquharson [1969]. The literature has often assumed that agents are completely informed about the preferences of others (see, for example, the classic papers by Miller [1977], McKelvey and Niemi [1978] and Moulin [1979]). Under complete information sophisticated voters can use backward induction: at each stage they foresee which alternative will be finally elected, essentially reducing each decision to a vote among two alternatives. Under simple majority, a Condorcet winner is selected by sophisticated voters whenever it exists, independently of the particular structure of the binary voting tree, and independently of its agenda. Thus, that body of theory cannot fully account for the use of carefully designed, special agendas in those cases. If a Condorcet winner does not exist, then a member of the top Condorcet cycle is elected, and the agenda influences which particular element of the cycle prevails.[6] The influence of agenda manipulations on the outcome of binary, sequential voting under complete information has been studied by Ordeshook and Schwartz [1987] and, more recently, by Barbera and Gerber [2017]. An observational equivalence between strategic voting and sincere voting that occurs in our paper for any voting procedure with any fixed, convex agenda and for incompletely informed voters was established by Austen-Smith [1987] for completely informed voters who use the amendment procedure in a setting where the agenda is set endogenously. Rasch [2000] has surveyed the employed voting and agenda-setting procedures in democratic parliaments all over the world, while Poole and Rosenthal [2000] offer a history of roll-call voting in the US Congress.

[6] The influence of agenda manipulations on the outcome of binary, sequential voting under complete information has been emphasized by Ordeshook and Schwartz [1987].

Several researchers have conducted empirical studies of observed voting behavior in parliaments. Leininger [1993] and Pappi [1992] analyze the 1991 decision about the post-reunification location of the German capital, and attempt to reconstruct the legislators' preferences from the sequence of observed votes. Based of the inferred preferences, they also conduct simulations with other, hypothetical voting procedures and compare the results. Pappenberger and Wahl [1995] look at the regulation of abortion in 1992, which we also analyze here, while Von Oertzen [2003] discusses several other cases from the Bundestag.

In a revealing study, Ladha [1994] analyzes a large number of cases from the US Congress and focuses on cases where the agenda followed a natural left-right order on a line: for those cases he observes patterns of behavior that have a natural monotonicity property that is also associated with sincere voting patterns.[7]

Roughly speaking, the abovementioned empirical papers – and many other similar ones – try to infer preferences from observed behavior. A crucial difference from our paper is that they are all based on the premise that members of parliaments vote sincerely, i.e., there is no presumption of optimal individual behavior and no equilibrium analysis.

Attempts to investigate when and why sincere voting might occur are often based on external explanations: behavior is assumed to be governed by motives and institutions outside the scope of the basic social choice model itself and the voting procedure, such as *home-style* a la Fenno [1978]. Moreover, the prevalent guiding social choice intuitions in those empirical studies are mostly gathered from the classical literature on binary, sequential voting under complete information. As we noted above, that literature cannot offer a fine enough analysis of observed agendas and strategic behavior for cases such as those analyzed in the present paper where complete information does not seem to be a good approximation and where, therefore, the final outcome is highly uncertain

An early analysis of strategic, sequential, binary voting under incomplete information is offered by Ordeshook and Palfrey [1988]. These authors constructed relatively complex Bayes-Nash equilibria for an *amendment* voting procedure in a situation with three alternatives and with three possible preference profiles that potentially lead to a Condorcet paradox. Such a Bayesian analysis crucially depends on the assumed agents' beliefs about other voters. In particular, a similar theoretical analysis of our real-life case studies that involved up to 8 distinct alternatives and more than 600 voters with hetero-

[7]The implications for strategic vs. sincere voting are also discussed by Groseclose and Miljo [2010].

geneous preferences does not seem to be feasible. Even if it were feasible, the analyst then needs to infer from the observed voting data both the voters' beliefs and their preferences, and identification is much more complex.

More recently, several papers have studied binary, sequential voting under incomplete information. Gershkov, Moldovanu and Shi [2017] analyzed voting by qualified majority in the successive procedure in settings where agents have "private values", and where preferences are assumed to be single-peaked on a line.[8] Kleiner and Moldovanu [2017] generalized their finding: under single-peaked, private-value preferences on a line, sincere voting constitutes an ex post perfect equilibrium in any sequential, binary voting procedure if the agenda is *convex*. Kleiner and Moldovanu [2020] apply this theory to explain both the emergence and rarity of *killer amendments* and illustrate this theory with a case study involving the Nazi party. Gershkov et al. [2019] also consider sequential voting with single-peaked preferences on a line but assume that preferences are interdependent, an assumption that includes both private and common values. In their model, not all alternatives are fixed ex-ante: these authors study the emergence and location of compromise alternatives (e.g., the location of a compromise deal in the Brexit case and the emergence of the composite flag of the Weimar Republic).

Finally, it is interesting to note that the type of analysis performed here resembles the one that would be necessary, say, to infer valuations for auctioned objects (these are analogous to our preferences) from bids (analogous to individual voting profile) submitted at various prices in a dynamic auction procedure (analogous to our voting procedure and its agenda). If the dynamic auction procedure has a robust, ex-post perfect equilibrium,[9] then the beliefs of bidders about the valuations of others does not play a role, and thus need not be jointly inferred. Otherwise, the observed bids mix valuations with beliefs in a much more complex way.

# 2 Graphs and Trees

We first briefly recall here several basic graph-theoretic definitions and results that will be useful for our analysis below. While the concepts appear to be abstract, their utility will become apparent below.

**Definition 1** *1. A* graph $G$ *on a set of nodes* $\mathcal{A}$ *with typical elements* $A$*,*

[8]Their focus was on finding a welfare-maximizing procedure. This was achieved by varying the thresholds needed for the adoption of each alternative.

[9]See, for example, Ausubel's [2004] generalization of the English auction for multiple goods.

*$B$, $C$, ... is a set of unordered pairs of distinct elements of $\mathcal{A}$, called* edges.

2. *A* path *$P$ of $G$ is a sequence of distinct nodes $A_1, ..., A_m$ such that $(A_i, A_{i+1})$ is an edge for $i = 1, 2, ..., m-1$.*

3. *A graph is* connected *if, for any pair of nodes $A_i, A_j$, there is a path with initial node $A_i$ and terminal node $A_j$.*

4. *A* cycle *(or circuit) is a path in which the initial node coincides with the terminal node.*

5. *The* circuit rank *of a graph is the minimum number of edges that must be removed from the graph to break all its cycles.*

6. *The* degree *of a node $A_i$, denoted by $d(A_i)$, is the number of edges having $A_i$ as element.*

**Definition 2** *A* tree *$\Psi$ is a connected graph that contains no cycles. A node $A$ is a* leaf *of tree $\Psi$ if it has degree 1; that is, there is exactly one edge of $\Psi$ containing this node.*

In our application below, nodes will correspond to the social alternatives among which voters have to choose, and the edges in a graph will correspond to ideological proximity relations among alternatives.

**Theorem 1 (Berge, 1962)** *Any one of the following equivalent properties characterizes trees:*

1. *$\Psi$ contains no cycles and has $k-1$ edges (where $k$ is the number of nodes).*

2. *$\Psi$ is connected and has $k-1$ edges.*

3. *$\Psi$ contains no cycles, and if a new edge is added, one, and only one, cycle is formed.*

4. *$\Psi$ is connected but ceases to be so if any edge is deleted.*

5. *Any two nodes $A$ and $B$ in $\Psi$ are linked by a unique path, denoted below by $P_{AB}$.*

The key to the proof of the above equivalences is the following elegant result:

**Theorem 2 (Berge, 1962)** *Consider a graph $G$ with $k$ nodes, $m$ edges and $p$ connected components. The* circuit rank *of $G$ is given by $\nu(G) = m-k+p$. Thus, the graph $G$ contains no cycle (a unique cycle) if and only if $\nu(G) = 0$ ($\nu(G) = 1$).*

Let us prove, for example, the result in Theorem 1-1: If $\Psi$ is a tree, then it is connected, hence $p = 1$. It contains no cycles, hence $\nu(\Psi) = 0$. Thus, we obtain $0 = m - k + 1$, which implies that $m = k - 1$. The other points follow from Theorem 2 by similar arguments.

Finally, to quantify the enumeration problem of finding a suitable ideological structure represented by a tree – but also to emphasize the richness of tree structures – we recall the following result:

**Theorem 3 (Berge, 1962)**

1. *The number of trees with $k$ nodes having respective degrees $d_1, ..., d_k$ is*

$$N(k, d_1, ..., d_k) = \binom{k-2}{d_1 - 1, d_2 - 1, ..., d_k - 1}$$

2. *(Cayley's Formula, 1889) The number of trees with $k$ nodes is $k^{k-2}$.*[10]

# 3 The Social Choice Model

We now apply the graph-theoretical structures to a social choice model. Suppose that there are $2n + 1$ voters who need to select one alternative out of a finite set $\mathcal{A}$ with $k \geq 2$ elements. The set of alternatives corresponds to the set of nodes of a graph, and this graph is assumed here to be a tree $\Psi$. Intuitively, two alternatives are directly connected by an edge if they are ideologically close, and are indirectly connected by a longer path if they are ideological more distant.

Each voter $i$ is characterized by a preference relation $\succ_i$ on $\mathcal{A}$, and preferences are private: an agent only knows her own preference, but not others' preferences.

Single-peakedness on trees requires that, on each isolated path (which can be seen as a line), the agent has a preferred alternative (the peak), and alternatives become worse from her point of view as one moves farther away (in terms of number of edges) from that peak. Formally, we have the following:

[10]Cayley's formula follows from the first statement by summing up all the relevant multinomials over $d_1 \geq 1, ..., d_k \geq 1$ such that $\sum_k d_i = 2(k-1)$.

**Definition 3** *1. An individual preference relation $\succ_i$ is an irreflexive, asymmetric, complete and transitive order on $\mathcal{A}$.*

2. *The preference $\succ_i$ is* single-peaked *on the path $P_{AC}$ of $\Psi$ if, for any node $B$ that lies on this path, it is not the case that both $A \succ_i B$ and $C \succ_i B$ hold.*

3. *The preference $\succ_i$ is* single-peaked on the tree $\Psi$ *if it is single-peaked on every path $P$ of $\Psi$.*[11]

When a tree $\Psi$ consists of a single path, we are in the classic case where alternatives can be ordered on a line, from "left" to "right". Single-peakedness on a tree with many distinct paths is thus a significant generalization of classic single-peakedness on a line, and many more preference profiles are potentially compatible with it. Nevertheless, a tree structure still restricts preferences in a way that allows for meaningful social choice, e.g., avoids standard impossibility results.

**Definition 4** *Given a preference profile $\{\succ_i\}_{i=1}^{2n+1}$, a Condorcet winner is an alternative $CW \in \mathcal{A}$ such that $|\{i : CW \succ_i A\}| > |\{i : A \succ_i CW\}|$ for any $A \neq CW$.*

The existence of a Condorcet winner for any profile of single-peaked preferences on a given tree was established by Demange [1988], who thus significantly generalized the classic result for lines due to Black [1948]. We note that the existence of a Condorcet winner is naturally preserved in subsets of alternatives that preserve the tree structure:

**Lemma 1** *Consider a tree $\Psi$ and a subtree $\Psi' \subset \Psi$. If a preference relation $\succ_i$ is singled-peaked with respect to $\Psi$, then its natural restriction is single-peaked on $\Psi'$. In particular, there is a Condorcet winner among the alternatives in $\Psi'$.*

**Proof.** Take any two nodes (alternatives) in $\Psi'$, $A$ and $C$. Since $\Psi'$ is a tree and hence connected, there exists a path $P$ in $\Psi'$ that goes from $A$ to $C$. Since $P$ is also a path in $\Psi$, the result follows by single-peakedness with respect to $\Psi$. The last part follows from Demange's result. ■

Finally, note that to define a game with incomplete information and to conduct a strategic analysis, it is usually necessary to also specify beliefs

[11] This is equivalent to the following: If $A$ is the peak of $\succ_i$ and $B$ belongs to a path between $A$ and $C$, then $B \succ_i C$.

that agents hold about the other agents' preferences. Since our analysis will be *robust* – namely, independent of those beliefs and independent of other information that becomes available during the voting sequence – we need not specify beliefs here.

### 3.1 Voting Procedures and Their Agendas

As mentioned in the introduction, practically all democratic parliaments use voting procedures that share several important structural properties and belong to the same general class. At each stage of a *sequential binary voting procedure*, the set of remaining alternatives (starting with the full set) is divided into two strict subsets (these need not be disjoint). Each voter approves one of the two subsets. The subset that gains a majority advances to the next stage, while the other subset is discarded. The process is repeated until a single alternative remains and is formally elected. While binary sequential procedures can also be graphically described by means of binary trees – with two branches protruding from each nonfinal node (see, for example, Figures 2 and 4 in the Brexit case studied below) – such voting trees vary with the chosen procedure and should **not** be confounded with the distinct and fixed tree that governs the ideological proximity relations among alternatives.

The following important property connects the agenda of sequential binary voting procedures to the underlying structure of preferences: we shall show that it induces very desirable strategic properties.

**Definition 5**

1. *An* agenda *specifies for each stage of a binary, sequential voting procedure the two subsets of alternatives that are put to vote against each other. Their union forms the set of alternatives that have not yet been rejected.*

2. *An agenda is* convex *with respect to a tree* $\Psi$ *if, at each stage of the voting process, it divides the set of remaining alternatives into two* ***subtrees*** *of* $\Psi$.

The main ingredient in the above definition is the requirement that the division of alternatives at each voting stage is among two distinct (not necessarily disjoint) subsets that are ideologically connected. By the previous lemma, the restricted preferences then continue to be single-peaked on each subtree, and thus, each binary division in a convex agenda is ideologically coherent: if two alternatives $A$ and $B$ belong to one of the subtrees, all alternatives on the path $P_{AB}$ must also belong to the same subtree. In other words, it cannot be the case that voters have to decide between, say, a subset of "centrist" alternatives on the one hand, and a subset containing only "extreme right" and "extreme left" alternatives on the other. In such a case, the unique path connecting the extreme nodes may need to go via one of the centrist nodes, violating the requirement that each subset is a connected subtree.

It is important to note that convex agendas are necessarily *content based* rather than *procedural* (see Kleiner and Moldovanu [2017]). In other words, to construct such an agenda for a given tree, one needs to take into account the actual content of the proposed alternatives in a particular legislative instance, and therefore also the logical/ideological connections between them. In contrast, purely procedural agendas follow predetermined rules that are independent of the content of alternatives in a particular application. For example, the agenda may be defined in terms of some order in which alternatives were submitted to the relevant parliamentary committee (see Rasch [2000] for a survey of the various types of rules used by democratic parliaments).[12]

**Example 1**

1. *Consider the successive voting procedure on a set of alternatives $\mathcal{A}$. An agenda for this procedure is* convex *with respect to a tree $\Psi$ if, at each stage, the alternative that is put to vote is a leaf of the subtree of remaining alternatives. If alternative $C$ is considered at a particular stage, the binary division into two subtrees is $[C,\ \mathcal{A} \setminus \{C\}]$.*

2. *Consider the amendment procedure on $\mathcal{A}$. An agenda for this procedure is* convex *with respect to a tree $\Psi$ if, at each stage, both alternatives that are pitted against each other are leaves of the subtree of the remaining alternatives. If alternatives $B$ and $C$ are considered at a particular stage, the binary division in two subtrees is $[\mathcal{A} \setminus \{C\},\ \mathcal{A} \setminus \{B\}]$.*

Intuitively, both of the above agendas prescribe that more "extreme" alternatives should be put to a vote before more "moderate" ones. These agendas are indeed well-defined and convex based on the following lemma:

**Lemma 2**

1. *Any tree $\Psi$ has at least two leaves.*

2. *Let $A$ be a leaf and denote by $e$ the unique edge of $\Psi$ that contains $A$. Then $\Psi \setminus \{e\}$ is a tree on $\mathcal{A} \setminus \{A\}$.*

***Proof. 1.*** *Let $P = \{A_1, ..., A_m\}$ be the longest path in $\Psi$. Then $A_1$ and $A_m$ must be leaves. Alternatively, note that the sum of degrees in any graph equals twice the number of edges. By Theorem 1 we obtain:*

$$\sum d(A_i) = 2(k-1) = 2k-2$$

[12] As Ladha [1994] documents for the US, in certain cases, procedural agendas may nevertheless be convex.

*If there are fewer than two leaves we obtain:*

$$\sum d(A_i) \geq 2(k-1)+1 = 2k-1 > 2k-2$$

*which is a contradiction.*

*2. Consider any two nodes $B, C \in \mathcal{A} \setminus \{A\}$. Then the unique path $P_{BC}$ that connects them in $\Psi$ is also the unique path that connects them in $\Psi \setminus \{e\}$.*

■

# 4 Sincere and Strategic Voting on Trees

We now study strategic voting in sequential, binary voting procedures where the set of alternatives can be endowed with a tree structure, and where the voters are incompletely informed about the preferences of others.

A profile of voting strategies for a sequential, binary voting procedure constitutes an *ex-post perfect equilibrium* if, at each stage in the voting sequence and for each history of voting outcomes, voters play best responses for each realization of preferences. Thus, in an ex-post perfect equilibrium no voter regrets her equilibrium strategy even after learning the preference realizations of all other voters. This is a particularly useful equilibrium notion for our empirical analysis because it does not depend on the (unobserved) beliefs voters entertain.

Below, we will relate strategic voting to the following concept of "unsophisticated" voting:

**Definition 6** *A voting strategy for a sequential, binary voting procedure is* sincere *if, at each stage in the voting sequence, it prescribes voting Yes for the subset of alternatives that contains the most preferred alternative among all remaining ones. If that alternative is contained in both subsets that are put to vote at a certain stage, then a sincere voting strategy proceeds lexicographically (vote yes for the subset that contains the second-best alternative, and so on).*

Our main theoretical result is as follows:

**Theorem 4** *Assume that preferences are single-peaked with respect to a tree $\Psi$ and that a sequential binary procedure with a convex agenda is used. Then sincere voting is an ex-post perfect equilibrium, and the Condorcet winner is elected in this equilibrium.*[13]

[13]Once trivial equilibria (that always exist in voting situations) are discarded, this equilibrium is essentially unique.

**Proof.** Assume first that all voters vote sincerely, and let $CW$ be the Condorcet winner given the agents' preferences. We first show that $CW$ must be elected under such a strategy profile.

Assume, by contradiction, that $CW$ is not elected under sincere voting. Consider then the first stage in the voting process where the majority approved subtree is $\Psi'$ such that $CW \notin \Psi'$. Then, there exist $m \geq n+1$ agents whose preferred alternative among the remaining ones is in $\Psi'$. Denote those most preferred alternatives by $A_1, A_2, ..., A_m$, respectively (these need not be distinct). If $A_1 = A_2 = ... = A_m$ then there are $m \geq n+1$ agents that prefer $A_1$ to $CW$, which is impossible by the definition of $CW$. Assume then without loss of generality that $A_1 \neq A_2$. Because $\Psi'$ is a tree, there exists a unique path, $P_{A_1A_2}$, which is entirely contained in $\Psi'$ and which connects these two nodes. In particular, $CW$ cannot be on this path since $CW \notin \Psi'$. Consider next the uniquely defined paths $P_{CWA_1}$ and $P_{CWA_2}$ in $\Psi$. Then there must exist an alternative, denoted by $B$, such that $B$ belongs to $P_{CWA_1}$, $P_{CWA_2}$ and $P_{A_1A_2}$. Otherwise, the concatenation of $P_{CWA_1}$, $P_{A_1A_2}$ and $P_{CWA_2}$ contains a cycle, contradicting the assumption that $\Psi$ is a tree.

By single-peakedness, we conclude that **all** agents whose most preferred alternative is **either** $A_1$ or $A_2$ prefer alternative $B$ to $CW$. Arguing in the same manner for $A_3, ..., A_m$ shows that there must be an alternative in $\Psi'$ that is preferred by $m \geq n+1$ agents to $CW$, which is impossible. Thus, $CW$ can never be eliminated, and will thus be elected under sincere voting.

We now argue that sincere voting is an ex-post perfect equilibrium. Fix an arbitrary preference profile and an arbitrary voter $i$. We show that given sincere behavior by all other voters, $i$ has no profitable deviation from sincere voting. Consequently, sincere voting is an ex-post perfect equilibrium.

Observe first that sincere voting is a best response if only two alternatives remain. Consider a voting stage where the decision is between the two subtrees $\Psi'$ and $\Psi''$ and assume that sincere voting is a best response in the subgame after this stage. Hence, if $\Psi'$ gains a majority at this stage, it follows from the first part that the final outcome will be the Condorcet winner among the alternatives in $\Psi'$, which we denote by $C'$. Similarly, if $\Psi''$ gains a majority, the final outcome will be the Condorcet winner among alternatives in $\Psi''$, denoted by $C''$.

To obtain a contradiction, suppose without loss of generality that $i$'s peak is $A \in \Psi'$ but that he is strictly better off voting for $\Psi''$. Then, there must be at least $n$ other voters with peak in $\Psi''$ and it must hold that $C'' \succ_i C'$. Because $\Psi' \cup \Psi''$ is also a tree (that has been approved at the previous stage) there exists an alternative $B$ that satisfies $B \in P_{AC'}$, $B \in P_{AC''}$ and $B \in P_{C'C''}$. Since $A, C' \in \Psi'$ and $\Psi'$ is a tree, it must also hold that

$B \in \Psi'$. Also, because alternative $A$ is $i$'s peak and because $B \in P_{AC''}$, single-peakedness implies $B \succeq_i C'' \succ_i C'$. Hence, $B \neq C'$.

We now consider two cases:

(1) Suppose that $B \notin \Psi''$. Since $\Psi''$ is a tree and $B \in P_{C'C''}$, $C' \notin \Psi''$. Also, for all $D \in \Psi''$, it must be the case that $B \in P_{C'D}$ (if not, then the concatenation of $P_{C'C''}$, $P_{C''D}$ and $P_{DC'}$ contains a cycle). By single-peakedness, every voter with a peak in $\Psi''$ prefers alternative $B$ to $C'$. Since at least $n$ other voters have a peak in $\Psi''$ and $B \succ_j C'$ for all such voters, we obtain a contradiction to the assumption that $C'$ is the Condorcet winner among alternatives in $\Psi'$.

(2) Suppose that $B \in \Psi''$.[14] Since $C''$ is the Condorcet winner among the alternatives in $\Psi''$, if $C'' \neq B$ then at least $n+1$ voters prefer $C''$ to $B$, and hence, by single-peakedness, they prefer $B$ to $C'$, contradicting the fact that $C'$ is the Condorcet winner among alternatives in $\Psi'$. Hence, $C'' = B$ and $C'' \in \Psi'$. Since $C'$ is the Condorcet winner in $\Psi'$, at least $n+1$ other voters prefer $C'$ to $C''$; since $C''$ is the Condorcet winner among alternatives in $\Psi''$, $C' \notin \Psi''$. Since at least $n$ other voters have a peak in $\Psi''$, there is a voter with peak in $\Psi''$ who prefers $C'$ to $C''$. Denote his peak by $D$. Then, there is an alternative $E \neq C', C''$ such that $E \in P_{DC'}$, $E \in P_{DC''}$ and $E \in P_{C'C''}$ (otherwise the concatenation of $P_{DC'}, P_{DC''}$ and $P_{C'C''}$ contains a cycle). Since $\Psi''$ is a tree, $E \in \Psi''$. Since $\Psi'$ is a tree and $C', C'' \in \Psi'$, we conclude $E \in \Psi'$. Therefore, $n+1$ voters prefer $C'$ to $E$ and hence $E$ to $C''$, contradicting the assumption that $C''$ is the Condorcet winner among alternatives in $\Psi''$. ■

Recall that if the set of alternatives $\mathcal{A}$ has cardinality $k$, then there are $k^{k-2}$ distinct trees on $\mathcal{A}$. For example, in the abortion law case discussed below there were 8 alternatives and hence $8^6 = 262.140$ trees on which preferences could have been, at least theoretically, single-peaked. Since in complex, real-life cases the tree structure is almost never made explicit, an important criterion for assessing the power of the subsequent empirical analysis is as follows: How arbitrary is the **analyst's** choice of a tree with respect to which preferences are potentially single-peaked? In other words, how much freedom do we theoretically have to choose a tree that fits our purpose in the case studies below? The rather surprising answer is the following:

**Proposition 1 (Trick [1989])** *Fix a profile of individual preferences such that each alternative in $\mathcal{A}$ is the peak of some voter. Then there exists at most one tree $\Psi$ such that this profile of preferences is single peaked on $\Psi$.*

[14] This case cannot occur in procedures where the binary decision is among disjoint subsets of alternatives as, e.g., in the successive procedure.

**Proof.** For the sake of completeness, we reproduce here the simple proof. Assume that the preferences are single-peaked on two distinct trees, $\Psi$ and $\Psi'$. Then $\Psi$ has an edge $e = (A, B)$ that is not contained in $\Psi'$. Consider then any node $C$ on the path between $A$ and $B$ in $\Psi'$ and its respective placement in $\Psi$. There must be such a node because, by assumption, $e = (A, B) \notin \Psi'$. There are two cases: either the path from $A$ to $C$ in $\Psi$ contains $B$, or the path from $B$ to $C$ in $\Psi$ contains $A$. In the first case, consider a voter $i$ that has a peak on $A$. Then, we must have $A \succ_i B \succ_i C$, which implies that $i$'s preferences cannot be single-peaked on $\Psi'$. The other case is similar, and this yields a contradiction. ■

# 5 Case Studies

In this section we apply our above analysis to two real cases from the German Bundestag and from the UK Parliament, respectively. The structure of the exercise – a revealed preference analysis – is as follows:

1. We first review the content of the proposed alternatives in each case, and suggest a logical tree structure that seem likely to induce single-peaked preferences.[15]

2. We then describe the employed binary, sequential voting procedures and their precise agendas. We also analyze whether, given the possible trees constructed in the previous, the agenda was convex.

3. Given the postulated tree (1.) and the real-life voting procedure and its agenda (2.), we next identify the small number of voting profiles – out of a much larger set of possible profiles – that are consistent with sincere voting in each instance. These are particular Yes-No sequences, one for each possible profile that is single-peaked according to the postulated tree (1.).

4. We then present in concise form the real-life, observed voting profiles and the voting outcome. Note that the voting profiles are available at the level of each individual legislator. Hence we can indeed draw conclusions at this level of behavior.

5. We compare the empirical data (4) to the predictions about sincere voting (3).

[15]Sometimes we test several alternatives.

6. Finally, we discuss the results and the underlying assumptions in each case. In particular, we note that sincere voting may have been induced by either equilibrium behavior under convex agendas, or by other features of the electoral system.

## 5.1 Abortion Law after the German Reunification

Prior to the 1992 reunification, abortions were strictly regulated in the Federal Republic of Germany, while the former Democratic Republic of Germany had a more liberal law. The German reunification treaty required new, uniform legal foundations. After a long debate, 7 proposals for a new formulation of the law were put up for vote in the Bundestag. The proposals covered a wide range of opinions and details, and there was considerable uncertainty about how many members of parliament supported each proposal.

It is important to note that, in ethical decisions, it is customary to free members of the Bundestag from party discipline. Our assumption of incomplete information becomes then salient: support for various alternatives crosses party lines, and members of the same party vote in favor of different alternatives, introducing real uncertainty about the outcome.[16]

Following the Standing Orders of the Bundestag, voting proceeded according to the successive voting procedure where single alternatives are put to vote, one after the other, until one is elected. The procedural agenda formation rule in those Standing Orders order implicitly assume that the issue is one-dimensional (i.e., the alternatives can be ordered on a line) and calls for voting on extreme alternatives first.

### 5.1.1 The proposed bills

The Elders' Council, headed by the Bundestag's president, suggested a very specific agenda. We very briefly describe here the proposed laws according to the order in which they were actually put up for vote, from **A** to **G**. The status quo is denoted by **H**.

- **A** The Greens' proposal was very liberal and basically allowed any abortion.
- **B** Similarly, the proposal by the Left party would allow any abortion, and there were several minor differences compared to proposal **A**.

[16]For example, in a recent case from 2018, Chancellor Merkel and a majority of legislators belonging to her governing party lost a landmark case that legalized gay marriage.

**C** This proposal, coming from a subgroup of very conservative parliamentarians was very restrictive: it allowed an abortion only if the life of the mother was otherwise at stake.

**D** The Liberals proposed that abortions should be legal in the first 12 weeks of pregnancy, but only if the mother takes part in pregnancy counseling. Moreover, the proposal demanded punishment for women aborting after the first 12 weeks.

**E** The Social Democrats suggested instead that any abortion within the first 12 weeks should be legal, but without enforcing punishments for later abortions.

**F** The main proposal brought forward by conservatives and supported by the leaders of the ruling CDU/CSU, allowed abortions only under restrictive regulations: even early abortions would remain legal only under medical and/or psycho-social indications.[17] Both woman and treating doctor would be punished for an abortion after the first 12 weeks.

**G** The so-called Group proposal was suggested by a group of legislators that crossed party lines: it was meant as a compromise between proposals **E** and **F**. An abortion within the first 12 weeks would not be punished. The woman needs to take part in a pregnancy counseling and the abortion must be performed by a doctor, but the ultimate decision stays with the woman.

**H** The status quo in the former Democratic Republic allowed an abortion in the first 12 weeks. In contrast, in the Federal Republic, an abortion required the presence of several "indications" that were not easy to fulfill.

### 5.1.2 Assumptions about preferences

The simplest structural hypothesis is that preferences were single peaked with respect to a linear order that goes from an emphasis on the free decisions for women on the one side, to an emphasis on the protection of unborn life on the other. Given the above described alternatives, there is only one order for which single-peaked preferences seem a more or less reasonable, approximate

[17]This effectively handed the final decision to the doctor, who also had to explain the decision in writing.

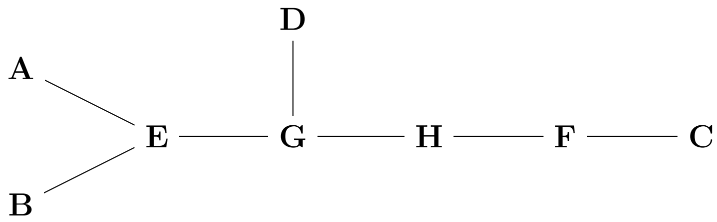


Figure 1: Preference tree

assumption: this is the linear order **A-B-E-G-D-H-F-C**, also suggested by Pappenberger and Wahl [1995].

Note that, however, alternative **D** was **not** a leaf for the tree **E-G-D-H-F** consisting of the alternatives remaining when **D** was put up for vote. In particular, if preferences were indeed linear, such an agenda would contradict the traditional rule to vote on extreme alternatives first. Therefore, neither our theoretical results nor the traditional custom in the Bundestag provide a foundation for using the particular agenda suggested by the Elders' Council. In particular, if any single-peaked preference with respect to the order **A-B-E-G-D-H-F-C** is feasible, then sincere voting in the game induced by the real-life agenda **ABCDEFGH** is not a robust equilibrium, and the Condorcet winner is not necessarily elected (see Kleiner and Moldovanu [2017].)

Thus, we look below for a tree structure that goes beyond a line. In order to estimate the true preferences of the legislators, Pappenberger and Wahl conducted a post-voting survey. Not surprisingly, out of 72 legislators who reported a complete preference ranking in this survey, only 4 of them reported a preference that is single-peaked according to the above linear order. On the other hand, it seems reasonable to assume that preferences were single-peaked with respect to the tree shown in Figure 1. Indeed, a majority of the available reported preferences of more than 70 legislators were indeed single-peaked with respect to this tree, and for no other possible tree more reported preference profiles would be single-peaked. Therefore, we use this tree for our analysis below.

#### 5.1.3 Analysis

Note first that, when they were put up for vote according to the agenda **ABCDEFGH,** all alternatives were leafs of the above described tree! Hence, the chosen agenda was convex and our theoretical result predicts that sincere voting constitutes a robust equilibrium, and that the Condorcet winner will be elected in this equilibrium.[18]

The following table summarizes the actual voting results in the sequence of binary votes:

| | **Yes** | **No** | **Abstain** | **Total** |
|---|---|---|---|---|
| **A** | 17 | 632 | 6 | 655 |
| **B** | 17 | 633 | 3 | 653 |
| **C** | 104 | 492 | 57 | 653 |
| **D** | 74 | 575 | 4 | 653 |
| **E** | 236 | 402 | 16 | 654 |
| **F** | 272 | 369 | 16 | 657 |
| **G** | **355** | **283** | **16** | **654** |

Alternative **G**, the compromise among the main alternatives supported by the big parties, was elected in the final vote.

We can now use the available records (see the archives of Deutscher Bundestag [1992]) of individual voting profiles to test our predictions. If our theory is correct, we should mainly observe voting profiles that are consistent with sincere voting according to single-peaked preferences on the assumed tree. In contrast, if we observe large numbers of voters using voting profiles that are inconsistent with sincere voting (for example, legislators voting Yes for the very liberal proposal **A**, but also voting Yes for the very conservative proposal **C**), we would have to reject the hypothesis that voting was sincere.

Abstract for the moment from abstentions, and assume that voters can only vote Yes or No at each stage. This yields $2^7 = 128$ possible individual voting profiles. In the successive procedure with a convex agenda, each alternative is a leaf of the tree remaining at the time it is voted upon. Therefore sincere voting prescribes to vote Yes if the current proposal is the most preferred among the remaining alternatives, and No otherwise. This feature shows that that the location of the peak completely determines the corresponding sincere voting strategy, i.e., this strategy is independent of how exactly alternatives are ranked below the peak.

The above considerations imply that, out of the 128 possible voting profiles, only 8 are consistent with sincere voting according to strict and single-peaked preferences on the above tree. This significant reduction in complexity well illustrates the empirical content of our proposed theory.

In reality, members of parliament can choose not to cast a vote on a specific proposal, or to formally abstain. 658 voters participated in at least one vote, while 638 voters participated in all votes in the sequence, and we

[18]Hence, the critique of this voting procedure that was put forward by some legislators has no theoretical foundation.

focus our analysis on the latter. More than 100 of these voters abstained at least once, which is why we include these voters in our analysis while treating an abstention as an expression of indifference.

**Our main finding is that 601 out of 638 profiles, a vast majority, used a strategy that is consistent with sincere voting!**

As explained above, given the observed data, we can infer the most preferred alternative of each legislator: for example, a legislator voting Yes at the first vote has a peak on **A**, a legislator who votes Yes for the first time at the second vote has a peak on **B**, and so on. Based on the record of voting profiles, the following table shows, for each alternative, how many legislators had a peak on that alternative.[19]

| **Peak** | **A** | **B** | **C** | **D** | **E** | **F** | **G** | **H** | **other profiles** |
|---|---|---|---|---|---|---|---|---|---|
| **Number (min)** | 2 | 0 | 93 | 71 | 206 | 126 | 30 | 1 | 37 |
| **Number (max)** | 6 | 2 | 144 | 73 | 225 | 179 | 48 | 7 | 37 |

Although only a small minority of voters had a peak on the elected alternative **G**, it turns out that, under the above inferred possible distributions of peaks, this alternative is indeed the Condorcet winner.

Thus, our analysis also implies that the Bundestag's president and the Elders' Council managed to intuitively choose an agenda that made strategic voting unnecessary, and that ensured the election of the Condorcet winner - a compromise alternative that did not have much direct support. In other words, the employed agenda consistently extended the traditional "extremes first" doctrine from a line to a more complex tree that remained implicit in the process- no mean feat in this complex situation.

## 5.2 The Brexit Voting Marathon

A voting marathon consisting of a sequence of eight binary votes was conducted by the British Parliament between March 12 and March 14, 2019. At stake was the shape and even the future of Brexit - UK's separation form the European Union - that was supposed to formally take place just two weeks later, on March 29, 2019.

We precisely describe below the complex, non-convex agendas that were used. It is worth mentioning here that at least one explicit deviation from

[19] Due to abstentions, we cannot precisely identify the peak of some legislators. We therefore display for each alternative a lower and an upper bound on the number of legislators that have a peak on this alternative.

convexity was desired, as it was actually part of premier's May strategy in order to get her deal through. Here is what the Economist wrote about it:[20]

> [...] Mrs May's plan is to hold yet another vote on her deal and to cudgel Brexiteers into supporting it by threatening them with a long extension that she says risks the cancellation of Brexit altogether. At the same time she will twist the arms of moderates by pointing out that a no-deal Brexit could still happen, because avoiding it depends on the agreement of the EU, which is losing patience. It is a desperate tactic from a prime minister who has lost her authority. It forces MPs to choose between options they find wretched when they are convinced that better alternatives are available. [...]

Prior to the voting marathon, May's negotiated Brexit deal with the EU has been rejected by a very large margin of 230 votes on January 15, 2019.[21] Nevertheless, it was put to vote again, **before** the more "extreme" alternatives such as a no-deal Brexit or a new referendum (or, say, an arrangement whereby the UK remains in the EU common market and customs union) were formally discarded.[22] As the Economist explains, her hope was that both Leavers and Remainers would finally unite behind her deal because each group perceived one of the remaining, extreme alternatives still on the table (and thus also a "lottery" among them) as catastrophic from their point of view.[23] Such an agenda, where a compromise is voted upon before the extremes, clearly violates convexity.

The UK parliament has 649 members. Since Sinn Fein's 7 MPs do not take their seats, a majority of 322 was needed to pass legislation. The Tory (Conservative) government, supported by the North-Irish DUP had a very thin, theoretical majority of 324, but was facing many rebel members in favor of a hard Brexit. Thus, the outcomes of various votes were highly uncertain and the entire situation was rather dramatic.

[20] "Whatever next?" Lead Article, *The Economist* March 16th 2019, page 11.

[21] This was the largest defeat for a sitting government in history.

[22] The same strategy has been pursued by May' successor, Boris Johnson. It was repeatedly countered by a majority in Parliament who refused to vote for a deal while a no-deal Brexit was still an option (the Benn and Letwin amendments).

[23] Zeckhauser [1969] shows that introducing lotteries may destroy single-peakedness. Lotteries become relevant when the agenda is not convex because the anticipated outcome depends then on beliefs about others' preferences.

### 5.2.1 The Motions, Agendas and Outcomes

The UK Parliament used a relatively complicated sequential, binary agenda that mixed elements of the Amendment Procedure (AP) and the Successive Procedure (SP). This was necessary because some of the bills (such as May's negotiated deal) were complete pieces of legislation, while others were only partial amendments.

**The First Voting Sequence** The first sequence of votes involved decisions about alternative courses of actions up to the official Brexit date on March 29, 2019. It consisted of 4 binary votes involving 5 alternatives:[24]

0 We denote by 0 a no deal Brexit on March 29. Implicitly, this was the legal status quo unless further action was taken, and this was mentioned as such in May's motion 1.

1 May's deal with the EU.

2 May's no Brexit without a deal on March 29.

3 Malthouse: An alternative to May's deal (1) that would execute Brexit on March 29.[25]

4 Spelman: No Brexit without deal, ever (amendment to 2).

The voting agenda is illustrated in Figure 2. The first vote was on May's motion 1, according to SP: voting would have stopped in case of acceptance. But motion 1 was defeated by 391 to 242 votes,[26] and a more traditional sequence according to AP followed. First, the Spelman amendment 4 narrowly passed. In other words, the original motion 2 was defeated against the amended version by 312 to 308 votes. Hence, motion 2 amended by 4, denoted here by $2_4$, became the standing motion. Then, the Malthouse proposal 3 was defeated by 374 to 164 votes. Finally, the still standing motion $2_4$ passed against the status quo 0 by 312 to 278 votes.

We suggest that the five motions in this part of the voting marathon can be arranged on a tree as shown in Figure 3. We assume below that

[24] Many other proposals were ultimately not put to a vote - the ultimate agenda setting power lied with the powerful Speaker John Bercow.

[25] This was procedurally presented as an amendment to 2, but logically represented an altogether independent course of action.

[26] Note that this was tighter than the original defeat by 230 votes. An even tighter outcome was obtained at a later, third vote on the same issue. Thus, May's strategy, described by the Economist, might have worked to some extent.

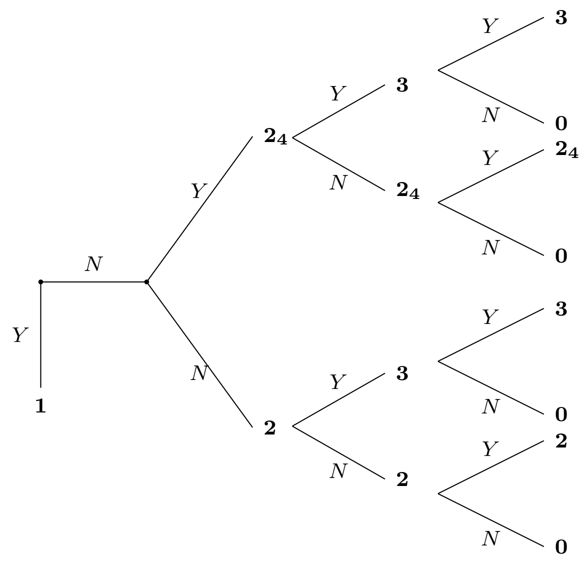


Figure 2: Voting procedure

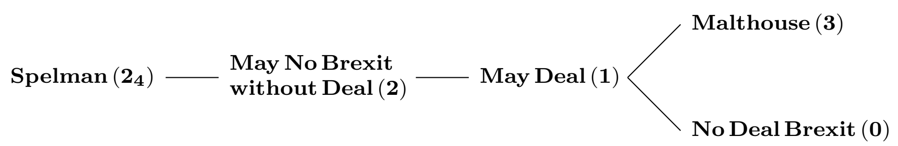


Figure 3: Preference tree underlying the first votes on Brexit

preferences were single-peaked on this tree, and check whether the actually observed voting profiles are consistent with sincere voting according to such preferences **given** the employed, non-convex agenda. In the concluding Section we discuss the sincerity assumption in this context.

Out of the 120 possible orders over the five alternatives, only 24 are compatible with single-peakedness on the above tree. Moreover, while there are $2^4 = 16$ possible Yes-No sequences on the 4 votes actually taken, only 10 of them can be induced by single-peaked preferences on the tree according to sincere voting given the agenda. Thus, we expect to observe at most 10 basic voting patterns among the 649 individual voting records.

Let us also briefly comment on the construction of the tree: By Cayley's formula there are $5^3 = 125$ potential trees here. Since all motions were proposed by a member of parliament and since the motions selected by the speaker Bercow were those that seemed to have some chance of success, we assume that the requirement for Trick's Theorem was satisfied.[27] In other words, if actual preferences were indeed single-peaked on a tree, this tree is unique.

The following table summarizes the most frequently observed profiles and the single-peaked preference order on the tree that would generate each of the observed profiles given sincere voting, and given the agenda used.[28] We denote indifference between alternatives 1 and 2 by $1 \sim 2$, and the notation $1 \succ (2, 3)$ summarizes that the preference could be either $1 \succ 2 \succ 3$ or $1 \succ 3 \succ 2$.

It follows from the above table that, with the exception of one profile that was observed just five times (YYNY), all common profiles are indeed consistent with our assumption that voting was sincere according to single-peaked preferences on the constructed tree.[29] Under this assumption, alternative $2_4$ that was selected was indeed the Condorcet winner because it won the direct vote against alternative 2, the only other close contender.

**The Second Voting Sequence** The second sequence of votes can be seen as determining how to precisely continue the process, and how to implement the previous decision of not leaving the EU without a deal by March 29,

[27] In particular, two Labor voters voted against the Spelman amendment at the second vote, but for May's motion 2 amended by 4 at the last vote. Assuming sincere voting, their peak was on alternative 2.

[28] We show all vote profiles that were cast by at least 5 voters.

[29] After alternative 1 was defeated by a large majority, the problematic profile YYNY is consistent with single-peaked preferences with a peak on $2_4$. Out of the rare profiles that were used by 25 voters and that we didn't list, 14 voters cast profiles that are inconsistent with our assumption.

| Profile | Observations | Implied single-peaked ranking |
|---|---|---|
| NYNY | 310 | $2_4 \succ 2 \succ 1 \succ (3,0)$ |
| YNYN | 94 | $1 \succ (3,0,2) \succ 2_4$ |
| YNAN | 68 | $1 \succ (2,0) \succ 3 \sim 2_4$ |
| NNYN | 65 | $2 \succ 1 \succ (3,0) \succ 2_4$, $3 \succ 1 \succ (2,0) \succ 2_4$, $0 \succ 1 \succ (3,2) \succ 2_4$ |
| YNNN | 32 | $1 \succ (0,2) \succ 2_4 \succ 3$ |
| YANN | 16 | $1 \succ 0 \succ 2 \sim 2_4 \succ 3$ |
| NNAN | 11 | $0 \succ 1 \succ 2 \succ 2_4 \sim 3$, $2 \succ 1 \succ 0 \succ 2_4 \sim 3$ |
| AAAA | 11 | $1 \sim 2 \sim 2_4 \sim 3 \sim 0$ |
| YAAN | 7 | $1 \succ 0 \succ 2 \sim 2_4 \sim 3$ |
| YYNY | 5 | None |
| NNNN | 5 | $0 \succ 1 \succ 2 \succ 2_4 \succ 3$, $2 \succ 1 \succ 0 \succ 2_4 \succ 3$ |
| Others | 25 | Diverse (including peaks on 2) |

Table 1: Individual vote profiles for the first sequence of Brexit votes.

2019. The motions were:

5 Corbyn: extend Article 50[30] + new Brexit approach (amendment to 8)

6 Wollaston: Hold a new referendum (amendment to 8)

7 Benn: Hold indicative votes (amendment to 8)[31]

8 May: Motion to delay the Brexit date.

9 We denote by 9 the status quo, a no deal Brexit on March 29. Although Parliament has just excluded a no-deal Brexit "forever", without further legislative steps, including the approval of the EU, a Brexit on March 29 was still the legal default.[32]

The voting agenda is depicted in Figure 4. The agenda for this sequence was again a combination of SP and AP, but with a more pronounced SP component.

[30]This was the legal step announcing the intention to leave the EU, including the deadline of March 29.

[31]The purpose was to find a deal that can be approved by a majority. For simplicity we ignore here the Powell amendment to this amendment, which would hold indicative votes while specifying a precise Brexit date of June 30.

[32]This has also been emphasized by the EU's leadership in the summit that followed the defeat of May's deal. The legal conundrum stemming from this status quo continued also after Brexit's delay and Johnson's premiership.

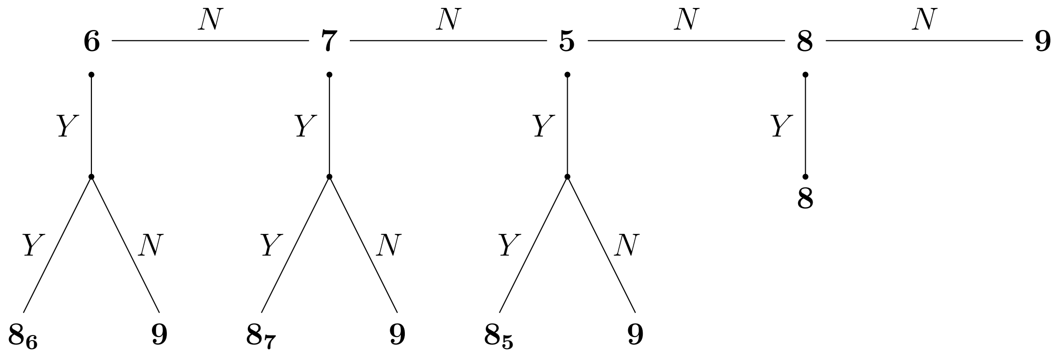


Figure 4: Agenda

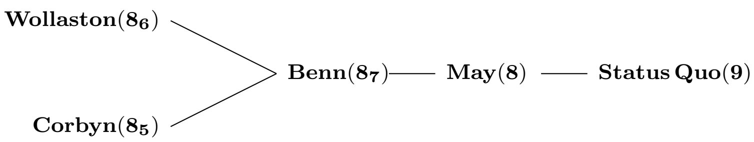


Figure 5: Preference tree underlying the second Brexit vote

May's basic motion 8 asked for a delay in the Brexit process that would give the parliament more time to approve a deal. The first vote was on amendment Wollaston 6 (new referendum). If accepted, the only other vote would be on May's motion 8 amended by 6, denoted by $8_6$, pitted against the status quo. Wolllaston was defeated by 85 to 334 votes. The second vote was on Benn's amendment 7. If accepted, the only other vote would be on motion $8_7$ pitted against the status quo. Benn's amendment was narrowly defeated by 312 to 314 votes. The third vote was on Corbyn's amendment 5. If accepted, the only other vote would be motion $8_5$ pitted against the status quo. Corbyn's amendment lost by 302 to 318 votes. Finally, as none of the amendments was successful, the un-amended motion 8 was pitted against the status quo, and passed by 413 to 202 votes.

We now assume that preferences for the second sequence were single-peaked on the tree shown in Figure 5. Analogously to the tree of the first sequence, only ten voting profiles are consistent with sincere voting according to single-peaked preferences on the tree in the agenda employed.

Table 2 summarizes all common profiles and the single-peaked preference orders that would generate each of these observed profiles given sincere voting and given the agenda.

All common profiles are indeed consistent with our assumptions,[33] but

[33]Among the rare profiles cast by 23 voters, only 5 voters behaved inconsistently with

| Profile | Observed Number | Implied single-peaked preference relation |
|---|---|---|
| AYYY | 202 | $8_6 \sim 8_7 \succ 8_5 \succ 8 \succ 9$ |
| NNNN | 200 | $9 \succ 8 \succ 8_7 \succ (8_5, 8_6)$ |
| NNNY | 103 | Any with peak on 8 |
| YYYY | 83 | $8_6 \succ 8_7 \succ 8_5 \succ 8 \succ 9$ |
| AAAA | 14 | $8_6 \sim 8_7 \sim 8_5 \sim 8 \sim 9$ |
| NYYY | 10 | $8_7 \succ (8_6, 8_5) \succ 8 \succ 9,\ 8_7 \succ 8_5 \succ 8 \succ (8_6, 9)$ |
| NNNA | 8 | $9 \sim 8 \succ 8_7 \succ (8_5, 8_6)$ |
| NYNY | 6 | $8_7 \succ 8 \succ (8_5, 8_6, 9), \quad 8_7 \succ 8_6 \succ 8 \succ (8_5, 9)$ |
| Others | 23 | Diverse |

Table 2: Individual vote profiles for the second sequence of Brexit votes.

the identification of the Condorcet winner is here more complex: either alternative $8_7$ (Benn) or alternative 8 (May) could have been it. Alternative 8 very narrowly won against $8_7$ by 314 to 312 votes, suggesting at first sight that 8 was the Condorcet winner. But, note that at that point in the voting sequence, alternative $8_5$ (Corbyn) was still in play. For a voter with a peak on $8_5$, sincere voting prescribes a vote **against** $8_7$ even though he/she prefers $8_7$ to 8. Since we do not have direct information on how many voters had a peak on $8_5$, it is not completely clear which alternative was the Condorcet winner. On the other hand, the second vote in the sequence clearly pitted $8_7$ vs. 8, so a home-style argument a la Fenno (see discussion below) might actually speak here **against** sincere voting and thus reinforce the view that alternative 8 (May) was the Condorcet winner.

The identification difficulty described above is typical of non-convex agendas.

# 6 Discussion

The employed agendas in the Brexit case were not convex, partly by design. Thus, sincere voting need not constitute a strategic equilibrium. Nevertheless, we have shown that sincere voting based on single-peaked preferences on a tree yields relatively precise predictions that agree well with the data also in this case. Why would legislators vote here sincerely?

An important force behind sincere, straightforward voting is the need to explain behavior and to make it transparent to constituents (see Fenno [1978]).[34] For example, we observe a high correlation between MP's hawk-

our assumption.

[34]But recall that in non-convex procedures sincere voting might not always be the sim-

ish voting behavior on Brexit and the percentage in favor of Leave in their constituency at the 2016 Referendum. Thus, an MP from a strong Leave constituency may find it difficult, if not impossible, to opportunistically vote Yes on a soft-Brexit alternative even if it yields some strategic gain. This disciplining effect seems to be particularly relevant in the UK, where each member of parliament is individually elected (first past the post) in relatively small constituencies of about 70-80.000 people each.[35]

This should be contrasted with Germany, where a majority of legislators are elected on state-wide party lists (proportional representation), and are therefore not directly accountable to a local community. Moreover, the directly elected legislators represent much larger, and possibly more diverse constituencies of about 250.000 people each. Thus, if sincere, transparent voting is a desideratum, a carefully designed agenda seems relatively more important in Germany than in the UK.

But, even if sincere voting is being enforced via motives and institutions that lie outside the immediate scope of this paper, we strongly believe that having content-based agenda formation rules that induce convexity ensure a much smoother process both at the agenda setting stage and at the voting stage, and we recommend their use. A steady use of well-designed, convex agendas - that do not serve special interests and that tend to elect the Condorcet winner - establishes sincere voting as the *modus operandi* for members of parliaments and frees them from the need to strategically asses each instance anew. As we saw above, Premier May's strategy of using a non-convex agenda designed to create uncertainty has badly backfired, and she lost her job.

We conclude by noting that our general method of inquiry can be extended to obtain a more robust inference of preferences even for non-convex agendas. Rather than assuming sincere voting one could compute equilibrium strategies and use these to infer preferences. However, as explained above, equilibrium computation is very complex, and inferences are then particularly sensitive to the exact (non-observable) beliefs held by voters. For future work, we propose instead to determine the strategies that survive iterated elimination of weakly dominated strategies and to base inference on these strategies. Such an inference can yield bounds on the number of voters with each possible preference profile.

---

plest behavior to explain one's constituents!

[35]For example, Prime Minister Boris Johnson was elected to the relevant Parliament by gathering just 29.000 votes in his constituency

# References

[1987] Austen-Smith, D. (1987). “Sophisticated Sincerity: Voting over Endogenous Agendas”, *American Political Science Review* **81**(4), 1323–1330.

[2004] Ausubel, L. M. (2004). “An Efficient Ascending-Bid Auction for Multiple Objects”, *American Economic Review*, **94**(5), 1452–1475.

[2017] Barbera, S. and Gerber, A. (2017). “Sequential Voting and Agenda Manipulation”, *Theoretical Economics*, **12**(1), 211–247.

[1962] Berge, C. (1962). *“The Theory of Graphs and Its Applications”*, London: Methuen.

[1948] Black, D. (1948). “On the Rationale of Group Decision-Making”, *Journal of Political Economy*, **56**(1), 23–34.

[1988] Demange, G. (1982). “Single-Peaked Orders on a Tree”, *Mathematical Social Sciences*, **3**(4), 389–396.

[1992] Deutscher Bundestag, *Plenarprotokoll der 99. Sitzung*, 25.06.2019, https://bundestag.de.

[1969] Farquharson, R. (1969). “*The Theory of Voting*”, New Haven: Yale University Press.

[1978] Fenno, R. F. (1978). “*Home Style: House Members in Their Districts*”, Boston: Little, Brown.

[2017] Gershkov, A., Moldovanu, B. and Shi, X. (2017). “Optimal Voting Rules”, *Review of Economic Studies*, **84**(2), 688–717.

[2019] Gershkov, A., Kleiner, A., Moldovanu, B., and Shi, X. (2019). “The Art of Compromising: Voting With Interdependent Values and the Flag of the Weimar Republic”, *discussion paper*, University of Bonn.

[2010] Groseclose, T., and Milyo, J. (2010). “Sincere Versus Sophisticated Voting in Congress: Theory and Evidence”, *The Journal of Politics*, **72**(1), 60–73.

[2019] *”The Hansard”*, (Official Report of the UK Parliament) March 12–14, 2019, https://archives.parliament.uk/online-resources/parliamentary-debates-hansard/.

[2017] Kleiner, A. and Moldovanu, B. (2017). "Content-Based Agendas and Qualified Majorities in Sequential, Binary Voting", *American Economic Review*, **107**(6), 1477–1506.

[2020] Kleiner, A. and Moldovanu, B. (2020). "The Failure of a Nazi "Killer" Amendment", *Public Choice*, **183**(1), 133–149.

[1994] Ladha, K. K. (1994). "Coalitions in Congressional Voting", *Public Choice*, **78**(1), 43–63.

[1993] Leininger, W. (1993). "The Fatal Vote", *Finanzarchiv*, **50**, 1–20.

[1978] McKelvey, R. D. and Niemi, R. G. (1978). "A Multi-Stage Game Representation of Sophisticated Voting for Binary Procedures", *Journal of Economic Theory*, **18**, 1–22.

[1977] Miller, N. R. (1977). "Graph Theoretic Approaches to the Theory of Voting", *American Journal of Political Science*, **21**(4), 769–803.

[1979] Moulin, H. (1979). "Dominance Solvable Voting Schemes", *Econometrica*, **47**(6), 1337–1351.

[2003] von Oertzen, J. (2003). "Komplexe Abstimmungssituationen in Deutschen Bundestag: Ein Verfahren zur Sicherung parlamentarischer Legitimität", *Zeitschtift f. Parlamentsfragen*, **34**(3), 453–477.

[1988] Ordeshook, P. C. and Palfrey, T. R. (1988). "Agendas, Strategic Voting, and Signaling with Incomplete Information", *American Journal of Political Science*, **32**(2), 441–463.

[1987] Ordeshook, P. C. and Schwartz, T. (1987). "Agendas and The Control of Political Outcomes", *American Political Science Review*, **81**(1). 179–200.

[1995] Pappenberger, K. and Wahl, J. (1995). "Der manipulierte Bundestag? Die Abstimmung über §218 StGb im Deutschen Bundestag am 25. Juni 1992 aus dem Blickwinkel der Social Choice Theorie", *Politische Vierteljahrschrift*, **36**(4), 630–654.

[1992] Pappi, F.U. (1992). "Die Abstimmungsreihenfolge der Anträge zum Parlaments- und Regierungssitz am 20. Juni 1991 im Deutschen Bundestag", *Zeitschtift f. Parlamentsfragen*, **23**, 403–412.

[2005] Poole, K. T. (2005). "*Spatial Models of Parliamentary Voting*", Cambridge University Press.

[2000] Poole, K. T. and Rosenthal, H. (2000). "*Congress: A Political-Economic History of Roll Call Voting*", Oxford University Press.

[2000] Rasch, B. E. (2000). "Parliamentary Floor Voting Procedures and Agenda Setting in Europe", *Legislative Studies Quarterly*, **25**(1), 3–23.

[1989] Trick, M. A. (1989). "Recognizing Single-Peaked Preferences on a Tree", *Mathematical Social Sciences*, **17**(3), 329–334.

[1969] Zeckhauser, R. (1969). "Majority Rule With Lotteries Over Alternatives", *Quarterly Journal of Economics*, **83**(4), 696–703.