Brexit: Dynamic Voting with an Irreversible Option

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Abstract

We analyze Brexit-like decisions in a polarized society. An electorate decides repeatedly between a reversible alternative (REMAIN) and an irreversible alternative (LEAVE). We compare strengths and weaknesses of several mechanisms that can be used in reality. Voting by supermajority dominates voting by simple majority. Decisions by simple majority and by a too small supermajority can perform very poorly under circumstances where it is socially optimal to never LEAVE, as they can exhibit equilibria where LEAVE is chosen very quickly. Mechanisms where LEAVE requires (super)majorities in two consecutive periods avoid this problem without relying on fine-tuning, but can lead to inefficient delays. If a final decision for either alternative requires winning by a certain margin, and if a new vote is triggered otherwise, both problems, choosing LEAVE too easily and inefficient delays, can often be avoided.

Keywords: Dynamic voting, Irreversible option, Option value, Supermajority rules

JEL-Codes: D72, D82, C72

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1. Introduction

On 23 June 2016 the UK voted by referendum to leave the European Union. The referendum required a simple majority (on a single day) in favor of LEAVE in order to force this option. LEAVE obtained 51.9% of the votes in that referendum, while REMAIN obtained 48.1%. The voting turnout was 72%. Large, important regions such as Scotland or Northern Ireland rejected LEAVE by relatively large majorities. Other Brexit-like decisions that were decided by simple majority were the 1995 Quebec-Referendum, and the 2014 Scottish Independence referendum. The 2006 Montenegrin independence referendum used a 55% supermajority rule.

While at the present time it is still unclear whether (and in which form) the Brexit-vote will be respected, we take decision making by referendum as given.

We introduce a tractable, dynamic framework for analyzing Brexit-like decisions in a polarized society. We uncover the various strategic issues behind such a decision, and explain why referendum decisions with long-term future consequences should not be taken by a simple majority at a single point in time. Our main findings are the following:

1. In environments where it is sometimes optimal to LEAVE, the irreversible option LEAVE is adopted too easily. Supermajority rules can then lead to better decisions.
2. In environments where it is optimal to never LEAVE, an equilibrium where LEAVE is never chosen can coexist with a welfare-inferior equilibrium where LEAVE is chosen extremely quickly. Requiring a supermajority for LEAVE can avoid the existence of such welfare-inferior equilibria.
3. There is an asymmetry in the potential welfare costs of using a too low and a too high supermajority rule: a too low supermajority rule can have a much higher cost. This is because, in scenarios where LEAVE should be adopted relatively easily, the adverse welfare consequences of a too large supermajority rule are mitigated by the fact that a part of the agents who prefer REMAIN from a short-term perspective vote LEAVE. By contrast, in scenarios where LEAVE should never be chosen, an equilibrium that exhibits myopic voting exists for any too low supermajority rule even if the future is extremely important.

We also describe alternative, potentially superior, mechanisms that are simple enough for actually being implemented in reality.

4. Requiring majorities in two consecutive periods avoids the problem of additional, welfare-inferior equilibria without relying on fine-tuning, but it can lead to inefficient delays.
5. Both problems, welfare-inferior equilibria and inefficient delays, can often be avoided if a final decision for either alternative requires winning by a certain margin, and if a new vote is triggered otherwise.

The main reason for the suboptimality of simple majority referenda such as Brexit is the large asymmetry in reversibility potential: LEAVE is widely believed to be irreversible for the foreseeable future, maybe a whole generation or more.[1] On the other hand, REMAIN defers the final decision and allows a new vote in the future—hence it is reversible. For example, the leader of the UK Independence party, Nigel Farage, a major Brexit proponent, warned shortly before the referendum that he would fight for a second referendum if the Remain campaign won by a narrow margin: "In a 52%-48% referendum this would be unfinished business by a long way."[2]

The asymmetric nature of the two main options, LEAVE and REMAIN, raises subtle questions about the properties of the decision mechanism: should Brexit-like decisions be taken by simple majority? This question was indeed recognized and discussed in the public debate. For instance, prominent economist Kenneth Rogoff argued that

> "[t]he real lunacy of the United Kingdom's vote to leave the European Union was not that British leaders dared to ask their populace to weigh the benefits of membership against the immigration pressures it presents. Rather, it was the absurdly low bar for exit, requiring only a simple majority. This isn't democracy; it is Russian roulette for republics. [...] Did the UK's population really know what they were

[1]See, for example, the discussion in the LSE-Blogpost "What if Britain rejoined the EU? Breaking up may be less hard than making up" by Iain Begg from 25 September 2018. He argues that "[f]or Remainers, the conclusions to draw are twofold. First, even if the country rapidly regrets its decision, there will be no going back to the deal the UK currently has. Second, returning to the EU on terms less palatable to UK voters will be hard to sell to them, rendering a future decision to rejoin politically implausible."

[2]"Nigel Farage wants second referendum if Remain campaign scrapes narrow win", Mirror, 16 May 2016.

> voting on? Absolutely not. Indeed, no one has any idea of the consequences [...] A country should not be making fundamental, irreversible changes based on a razor-thin minority that might prevail only during a brief window of emotion."[3]

In a similar vein, famed biologist Richard Dawkins explained that

> "[c]onstitutional amendments are—or should be—hard to achieve. In America, it takes a two-thirds majority in both houses of Congress. It's easy to see why the bar is set so high. Unlike ordinary lawmaking, constitutional changes are for keeps. Voters are fickle. Opinions change. We have no right to condemn future generations to abide, irrevocably, by the transient whims of the present. If ever a decision needed at least a two-thirds majority, it was Brexit."[4]

On the other hand, from an axiomatic (and static) perspective, there are reasons in favor of the simple majority rule (May (1952), Dasgupta and Maskin (2008)). Most notably, super-majority rules do not respect the "one person-one vote" principle: they allow a minority to impose the status quo on a majority that prefers change. In dynamic contexts where LEAVE is likely to be better from a social perspective at most periods, this can be particularly harmful if it allows the initial status quo, REMAIN, to prevail for a very long time.

In our model, a large electorate repeatedly decides between a reversible option (REMAIN) and an irreversible option (LEAVE).[5] If taken, the latter option ends the decision process. The short-term effects of the taken decision are clearer than its long-term consequences: an agent knows in each period whether he is a LEAVE-winner or a LEAVE-loser from a short-term perspective, but faces uncertainty about future payoffs. Fueled by the rise of the internet and social media, polarization became increasingly important during the last two decades (Sunstein, 2018). The existence of only two types of agents, winners and losers (without 50 shades of gray) models a simple notion of extreme polarization, but our results continue to hold in less polarized societies.

[3] "Britain's Democratic Failure", Project Syndicate, 26 June 2016.

[4] "Richard Dawkins: Brits have not spoken on Brexit", BBC Newsnight, 9 March 2017.

[5] While in the Brexit referendum, LEAVE was not a well-defined option, we abstract from this issue.

We allow the mass of LEAVE-winners to fluctuate over time, and we distinguish among two main scenarios: the environment is REMAIN-friendly (LEAVE-friendly) if the probability of a future majority of LEAVE-losers is higher (lower) that 50%.

We first assess optimal decisions from the perspective of a utilitarian social planner. In the benchmark case where both decisions are reversible, it is socially optimal to choose the option preferred by a majority from a short-term perspective.

Contrasting the benchmark case, when LEAVE is irrreversible REMAIN yields a positive **option value**. This implies that it is socially optimal to LEAVE only if there is a sufficiently large supermajority of LEAVE-winners. If it is likely that there is a majority of LEAVE-losers in future periods, and if the future is sufficiently important (i.e., discounting is not too high), it is then socially optimal to never LEAVE.

We next analyze decision making by voting referenda. An agent's preferred decision may differ from the planner's preferred decision for two reasons. Firstly, an agent who is either a current LEAVE-loser or a LEAVE-winner has a more extreme view about the short-term effects of the decision than the social planner who weighs its aggregate effects. Secondly, an agent who expects suboptimal future decisions underestimates the option value of REMAIN. These differences suggest the use of supermajority rules both in environments where it is socially optimal to never LEAVE, and in those where it is socially optimal to LEAVE under certain conditions.

Suppose first that it is socially optimal to never LEAVE and that simple majority is used. If an agent believes that other agents vote myopically, he might expect that LEAVE will nevertheless be soon chosen. But, this means that, today, he perceives the future consequences of LEAVE and of REMAIN as almost equal, and it may be thus individually optimal to also vote myopically. We show that a welfare-inferior equilibrium where LEAVE is quickly chosen even though it is socially optimal to never LEAVE can often coexist with a welfare-optimal equilibrium where LEAVE is never chosen. Interestingly, the welfare-inferior equilibrium can exist even if the future is arbitrarily important.

The welfare-inferior equilibrium generally ceases to exist if a sufficiently large supermajority is required for LEAVE. Intuitively, under a supermajority rule, LEAVE is, on average, chosen at a later point in time. This makes it harder to support the belief that the future consequences of LEAVE and of REMAIN are similar enough.

Suppose next that the mass of current LEAVE-winners determines which decision is socially optimal. We show that myopic voting constitutes then a unique equilibrium for any supermajority rule that is not too large. While the simple majority rule fails to reflect the option value of REMAIN, supermajority rules correct this failure by shifting the pivotal agent to be one who is less eager to LEAVE.

The above analysis relies on two main simplifying assumptions: very polarized short-term incentives, and uncorrelated short-term and long-term incentives. We discuss the robustness of our results to these assumptions in Appendix A. Specifically, we explain why supermajority rules become even more important if today's LEAVE-winners are more likely to be also tomorrow's LEAVE-winners (Extension I), and we introduce a weaker notion of polarization that is sufficient for our results (Extension II).

The coexistence of equilibria with very different welfare properties can be seen as a serious flaw of a voting mechanism. Even though this issue can be resolved by requiring a sufficiently large supermajority for LEAVE, the necessary fine-tuning and the significant non-neutrality of the procedure may render it both impractical and/or politically unfeasible.

Therefore, we discuss two alternative mechanisms that address the above mentioned problems without relying on fine-tuning in each application. In his critique of simple majority, Kenneth Rogoff proposed a mechanism that requires majorities in two consecutive periods in order to LEAVE:

> "What should the UK have done [...]? [...] for example, Brexit should have required, say, two popular votes spaced out over at least two years, followed by a 60% vote in the House of Commons. If Brexit still prevailed, at least we could know it was not just a one-time snapshot of a fragment of the population."[6]

Under such a mechanism, a decision for REMAIN today makes it impossible to LEAVE in the next period. This "cooling-off period" drives a wedge between the perceptions of REMAIN

[6]See "Britain's Democratic Failure", Project Syndicate, 26 June 2016.

and of LEAVE whenever it is socially optimal to never LEAVE. We show that this generally suffices to avoid equilibrium multiplicity. On the other hand, the inability to immediately LEAVE after a REMAIN decision may cause an inefficient delay in environments where it is socially optimal to sometimes LEAVE. We show that requiring two consecutive simple majorities often outperforms requiring two supermajorities. This is so because a higher majority increases the probability of an inefficient delay.

Finally, we propose a new mechanism that can avoid both equilibrium multiplicity and inefficient delays. As Nigel Farage's "unfinished business" quote suggests, a commitment to REMAIN might be possible if REMAIN wins by a large majority. We analyze mechanisms that reflect this property by ending the decision process as soon as either REMAIN or LEAVE wins by an a priori defined margin; if neither alternative wins by this margin, the vote is repeated in the next period. This means that final decisions for both REMAIN and LEAVE require a supermajority. Even though voting at each stage is binary, such a mechanism effectively creates three social alternatives: LEAVE forever, REMAIN for now but vote again next period, and REMAIN forever.

For any margin required by a final decision, the threat of ending the process with REMAIN avoids the existence of welfare-inferior equilibria if it is socially optimal to never LEAVE, and if discounting is not too high. At the same time, when it is socially optimal to LEAVE under certain conditions, the possible repetition of the vote in the next period avoids inefficient delays on the equilibrium path. Together with an optimally chosen supermajority rule, this mechanism implements the socially optimal alternative.

1.1. Related Literature

The classical literature on investment studies a single decision maker's trade-off between committing to an irreversible decision and waiting. See Dixit and Pindyck (1994) for a review of this literature. Our focus here is on a situation where a group of agents collectively takes such a decision through voting.

In the collective search literature a committee observes a stream of alternatives (candidates, proposals, ...) and search ends as soon as an alternative is accepted by a certain majority. Recent contributions to this literature (Albrecht et al., 2010, Compte and Jehiel, 2010, Moldovanu and Shi, 2013) focused on the nature of the accepted proposals for exogenously fixed decision rules.[7] While the payoff structure in that literature resembles the one in a bargaining problem where the accepted alternative generates a one-time payoff, our payoff structure resembles a repeated game where stage payoffs are generated in each period. An important implication of this difference is that, from a welfare point of view, it is optimal to immediately accept any sufficiently strong alternative in any of the collective search problems, whereas here it can be socially optimal to reject the irreversible decision forever, no matter how favorable it appears from a short-term perspective.

Albrecht et al. discuss the comparative statics of the optimal majority rule with respect to the discount factor: the optimal majority increases in the discount factor, and the unanimity rule is optimal if the committee is sufficiently large. Compte and Jehiel (2011) show that the optimal majority may also be intermediate.

Messner and Polborn (2012) is more closely related to our paper. In a two-period model with a payoff structure similar to ours, these authors compare the case where an irreversible project can only be implemented in the initial period to the one where it can be delayed to a second period. The (super)majority rule is designed by a utilitarian planner at an ex-ante stage where she is still uncertain about parameters of the payoff distribution. The authors show that deciding by a supermajority rule may be optimal, and that the option to wait can actually have a negative value (!).[8] By restricting attention to a two-period model without discounting, these authors implicitly assume that present and future are equally important. In contrast, we focus on how the relative importance of the future affects the performance of voting mechanisms. Allowing the future to be more important than the present (while avoiding last round effects) adds qualitatively new insights that also motivate our study of alternative voting mechanisms: these cannot be fruitfully studied in a two-period framework.

Strulovici (2010) introduced learning to dynamic collective decisions. He studies how long a

[7] This literature was initiated by Sakaguchi (1973) who analyzed equilibrium existence for two players under the unanimity rule. Kurano et al. (1980) extend the analysis to more players and general majority rules. Ferguson (2005) shows that multiple stationary cutoff equilibria can exist in these models.

[8] Gersbach (1993) makes a related point for an exogenously fixed voting rule (simple majority). In a two-period model, he identifies a specific scenario with three players and correlated values where the option to wait has a negative value.

committee learns about an ex-ante uncertain alternative until a majority settles on a decision. In a common values model, Chan et al. (2018) analyze the duration of learning for committees whose members are heterogeneous with respect to patience and error costs. Introducing such learning effects into our framework would constitute and interesting extension.

2. The Model

In each period $t = 1, 2, \ldots$ there are two possible decisions: REMAIN ($d_t = R$) and LEAVE ($d_t = L$). Once taken, LEAVE is irreversible. The stream of decisions $d_1, d_2, \ldots$ affects a continuum of risk-neutral agents having mass 1. In each period t, each agent i obtains a stage payoff of $\pi_t^i \in \{0, 1\}$ from LEAVE and a payoff of $\frac{1}{2}$ from REMAIN. If $\pi_t^i = 1$ (0), then agent i is a LEAVE-winner (loser) from a short-term perspective.[9] Agents discount future stage payoffs by a discount factor $\delta \in (0, 1)$.

The mass of LEAVE-winners, $p_t \in [0, 1]$, is independently distributed across periods according to a c.d.f F with p.d.f f and full support. Conditional on p_t, the payoffs π_t^i are independently distributed across periods and across agents:[10] Agent i is a LEAVE-winner with probability p_t and a LEAVE-loser with probability $1 - p_t$. Let $\bar{p} \equiv \mathbb{E}_{p_t}[p_t]$. We call the environment *LEAVE-friendly* if $\bar{p} > \frac{1}{2}$, *REMAIN-friendly* if $\bar{p} < \frac{1}{2}$, and *neutral* if $\bar{p} = \frac{1}{2}$.

The timing within period t is as follows: First, nature draws the mass of LEAVE-winners p_t, and reveals it publicly.[11] Second, nature privately reveals to each agent i whether he is a LEAVE-winner or loser. Third, if the previous decision was REMAIN, a new decision d_t is taken. Otherwise, $d_t = L$. Finally, stage payoffs realize.

The planner is utilitarian: her stage payoff is the average stage payoff across agents, i.e., it is p_t from LEAVE and $\frac{1}{2}$ from REMAIN, and she discounts future stage payoffs by δ.

[9] In Extension II in Appendix A we explain how our results extend to continuously distributed stage payoffs, i.e., less extreme notions of polarization.

[10] In Extension I in Appendix A we allow for a serial correlation across time.

[11] A motivation for the public observability of p_t could be that opinion polls are sufficiently accurate and reflect how the electorate assesses the aggregate short-term effects of LEAVE. Whether p_t is observable by the agents or not will only matter for our analysis of two-sided majority voting in Section 6.

3. Centralized Decision Making

A *cutoff policy* with cutoff $p \in \mathbb{R}$ selects LEAVE if $p_t \geq p$ and selects REMAIN otherwise.[12] If LEAVE is also reversible, the decisions in different periods are not connected. The optimal policy follows from comparing the current stage payoffs p_t and $\frac{1}{2}$, and a cutoff policy with cutoff $\frac{1}{2}$ is optimal. In other words, at each stage, it is optimal to take the decision preferred by a majority from a short-term perspective.

If LEAVE is irreversible, the future consequences of the current decision matter. From the perspective of period t, the previous decision d_{t-1} affects which decisions are feasible in the current period. However, no other information about the past affects payoffs or actions in the current and in future periods. Therefore, we can denote the planner's value of entering period t with decision d by V_d^*. If the the previous decision was REMAIN, her payoff from LEAVE is $p_t + \delta V_L^*$, and her payoff from REMAIN is $\frac{1}{2} + \delta V_R^*$. The Bellman equations for the planner's optimal policy are

$$\begin{cases} V_L^* = \mathbb{E}_{p_t}[p_t + \delta V_L^*] \\ V_R^* = \mathbb{E}_{p_t}[\max\{p_t + \delta V_L^*, \frac{1}{2} + \delta V_R^*\}] \end{cases}.$$

Since the planner's stage payoff is bounded, and since the maximum is attained, the values V_d^*, $d \in \{R, L\}$, are uniquely defined. The difference $\Delta^* \equiv \delta(V_R^* - V_L^*)$ represents the *option value* of the reversible decision, REMAIN. It follows from the Bellman equations that the cutoff policy with cutoff

$$p^* \equiv \tfrac{1}{2} + \Delta^*$$

is optimal, where Δ^* is the unique solution to

$$\Delta^* = \delta \mathbb{E}_{p_t}[\max\{0, \Delta^* + \tfrac{1}{2} - p_t\}]. \tag{1}$$

The next Lemma establishes that $\Delta^* > 0$. Relative to the case where LEAVE is reversible, the planner is biased towards REMAIN: LEAVE is optimal only if there is a sufficiently large supermajority of LEAVE-winners.

[12] Allowing for cutoffs outside the interval $[0, 1]$ will be useful for expositional reasons (even though any cutoff $p > 1$, and any cutoff $p \leq 0$ describes the same policy, respectively).

Lemma 1. *Let* $p^* \equiv \frac{1}{2} + \Delta^*$ *and let*

$$\delta^* \equiv \frac{1}{2(1-\bar{p})}. \tag{2}$$

Then $p^* \in (\frac{1}{2}, 1)$ *for* $\delta < \delta^*$, $p^* = 1$ *for* $\delta = \delta^*$, *and* $p^* > 1$ *for* $\delta > \delta^*$.

If the environment is LEAVE-friendly or neutral, then $\delta^* \geq 1$ which implies that $p^* \in (\frac{1}{2}, 1)$ for any $\delta \in (0, 1)$. If the environment is REMAIN-friendly, $\delta^* \in (\frac{1}{2}, 1)$. Thus, if the future is important enough in a REMAIN-friendly environment, the bias is so large that it is never optimal to LEAVE.

It will be useful for our analysis below to also understand how the planner compares different, non-optimal cutoff policies. For any given cutoff p, consider the system of linear equations

$$\begin{cases} V_L(p) = \bar{p} + \delta V_L(p) \\ V_R(p) = F(p)(\frac{1}{2} + \delta V_R(p)) + (1 - F(p))(\mathbb{E}_{p_t}[p_t | p_t \geq p] + \delta V_L(p)) \end{cases}. \tag{3}$$

The system possesses a unique solution.[13] $\delta V_d(p)$ describes the planner's continuation value from decision $d \in \{L, R\}$ if future decisions are taken according to cutoff policy p. It follows from (3) that

$$\Delta(p) \equiv \delta(V_R(p) - V_L(p)) = \frac{\delta}{1 - \delta F(p)} \int_0^p (\tfrac{1}{2} - p_t) \mathrm{d}F(p_t). \tag{4}$$

The utilitarian welfare from cutoff policy p is $V_R(p)$. Since the consequences of LEAVE are exogenous, maximizing $V_R(p)$ is equivalent to maximizing the future advantage of REMAIN over LEAVE, $\Delta(p)$.

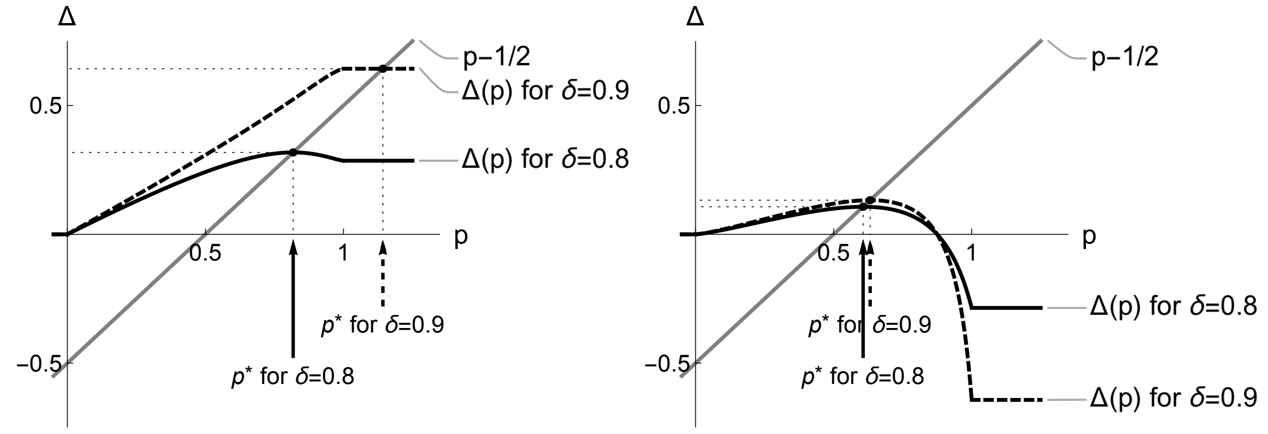
Lemma 2. *(a)* $\Delta(p) \geq \Delta(p')$ *if and only if the planner weakly prefers cutoff* p *over cutoff* p'.

(b) $\Delta(p)$ *is piecewise constant on* $(-\infty, 0]$ *and on* $[1, \infty)$. *On the interval* $[0, 1]$, $\Delta(p)$ *is single-peaked with peak at* p^* *if* $p^* < 1$, *and strictly increasing if* $p^* \geq 1$.

[13] The system can be rewritten as

$$\begin{pmatrix} 1-\delta & 0 \\ -\delta(1-F(p)) & 1-\delta F(p) \end{pmatrix} \begin{pmatrix} V_L(p) \\ V_R(p) \end{pmatrix} = \begin{pmatrix} \bar{p} \\ F(p)\frac{1}{2} + (1-F(p))\mathbb{E}_{p_t}[p_t | p_t \geq p] \end{pmatrix}.$$

Since $\delta \in (0, 1)$ implies that the matrix has for any given $p \in \mathbb{R}$ full rank, a unique solution exists.



(a) REMAIN-friendly environment $[\gamma = \frac{3}{4}]$ (b) LEAVE-friendly environment $[\gamma = \frac{4}{3}]$

Figure 1: Future advantage of REMAIN and current advantage of LEAVE

(c) $\Delta(p^*) = p^* - \frac{1}{2}$. *Moreover,* $\Delta(p) > p - \frac{1}{2}$ *for* $p < p^*$ *and* $\Delta(p) < p - \frac{1}{2}$ *for* $p > p^*$.

Example. For illustrations, we use the power distribution functions

$$F_\gamma(p_t) \equiv \begin{cases} p_t^\gamma & \text{if } \gamma \in [1, \infty) \\ 1 - (1 - p_t)^{1/\gamma} & \text{if } \gamma \in (0, 1) \end{cases}.$$

The expected average stage payoff is given by $\bar{p}_\gamma \equiv \frac{1}{2} + \frac{1}{2}\frac{\gamma-1}{\gamma+1}$. The environment is LEAVE-friendly for $\gamma > 1$, neutral for $\gamma = 1$ and REMAIN-friendly for $\gamma \in (0, 1)$.

Figure 1 illustrates $\Delta(p)$ and the optimal policy p^*. The left (right) panel is for a REMAIN-friendly (LEAVE-friendly) power distribution function. In each panel, the black solid curve depicts $\Delta(p)$ for $\delta = 0.8$ and the black dashed curve depicts $\Delta(p)$ for $\delta = 0.9$. The gray curve displays $p - \frac{1}{2}$. This is the advantage that LEAVE brings in the current period when $p_t = p$. The black dots indicate (p^*, Δ^*). By Lemma 2 (a), p^* is a maximizer of $\Delta(p)$. By Lemma 2 (c), (p^*, Δ^*) is the unique intersection point of the black and the gray curve.

4. Voting by (Super)Majority

We next consider decisions taken by (super)majority voting with a cutoff $\kappa \in [\frac{1}{2}, 1)$. In every period t such that the previous decision was REMAIN, all agents simultaneously learn their stage payoffs and then simultaneously vote for LEAVE or for REMAIN. The decision is LEAVE if the mass of LEAVE-votes, l_t, is at least κ.

4.1. Equilibrium notion

We focus here on Markov strategies that only condition on payoff-relevant information.[14] An equilibrium is a profile of symmetric Markov strategies such that, after any history where the previous decision was REMAIN, no agent has an incentive to unilaterally deviate under the assumption that his vote is pivotal. *Pivotal voting* also corresponds here to *sincere voting*: an agent votes sincerely if he votes for the decision he prefers, taking as given the future decisions implied by the Markov strategy used by the other agents.[15]

Under majority voting with cutoff κ, an agent only conditions his behavior on whether he currently is a LEAVE-winner.[16] Markov strategies are then described by $(\lambda(0), \lambda(1)) \in [0,1]^2$ where $\lambda(\pi_t^i)$ denotes the probability with which an agent of type $\pi_t^i \in \{0,1\}$ votes for LEAVE.[17]

If all agents vote according to the Markov strategy $(\lambda(0), \lambda(1))$, the decision is

$$d(p_t) = \begin{cases} L & \text{if } p_t\lambda(1) + (1-p_t)\lambda(0) \geq \kappa \\ R & \text{if } p_t\lambda(1) + (1-p_t)\lambda(0) < \kappa \end{cases}. \tag{5}$$

Consider the system of linear equations

$$\begin{cases} V_L = \mathbb{E}_{p_t}[\mathbb{E}_{\pi_t^i|p_t}[\pi_t^i + \delta V_L]] \\ V_R = \mathbb{E}_{p_t}[\mathbb{E}_{\pi_t^i|p_t}[\mathbf{1}_{d(p_t)=L}(\pi_t^i + \delta V_L) + \mathbf{1}_{d(p_t)=R}(\frac{1}{2} + \delta V_R)]] \end{cases} \tag{6}$$

implied by policy (5). It possesses a unique solution.[18] δV_d describes the continuation value

[14]Our notion of payoff-relevance can informally be described as follows: An agent does not condition his behavior on parts of his private information/public history about which he does not care if he believes that his vote is pivotal, and that other agents do not condition their behavior upon this information.

[15]Our equilibrium notion is a refinement of Markov perfect equilibrium. In the version of our model with a large finite electorate, pivotality considerations are effective, and the same policies are essentially implementable in both versions of the model. The continuum version simplifies the exposition.

[16]If i is pivotal, the decision is LEAVE (REMAIN) if he votes LEAVE (REMAIN). The payoffs he assigns to the two options depend on his current stage payoff from LEAVE, π_t^i, and on the continuation values. If other agents do not condition on the public history, these values do not depend on it, and π_t^i is the only payoff-relevant information.

[17]Allowing for mixed strategies ensures equilibrium existence. In the most interesting cases, at least one equilibrium in pure strategies exists, and our main results concern pure strategy equilibria. Technically, our formulation assumes that the mass of LEAVE-votes is $l_t = p_t\lambda(1) + (1-p_t)\lambda(0)$. This corresponds to the limit mass that would occur for finite electorates with an increasing number of agents voting according to $(\lambda(0), \lambda(1))$.

[18]The system can be rewritten as

$$\begin{pmatrix} 1-\delta & 0 \\ -\delta\mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=L}] & 1-\delta\mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}] \end{pmatrix} \begin{pmatrix} V_L \\ V_R \end{pmatrix} = \begin{pmatrix} \mathbb{E}_{p_t}[p_t] \\ \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=L}p_t + \mathbf{1}_{d(p_t)=R}\frac{1}{2}] \end{pmatrix}.$$

from decision $d \in \{R, L\}$. This continuation value is common to all agents due to the serial independence of individual stage payoffs. The continuation values affect voting incentives only through the induced future advantage of REMAIN, $\Delta \equiv \delta(V_R - V_L)$. If LEAVE is also reversible, then $V_R = V_L$ and $\Delta = 0$.

Agent i's payoff from LEAVE (REMAIN) in period t is $\pi_t^i + \delta V_L$ $(\frac{1}{2} + \delta V_R)$. The Markov strategy profile $(\lambda(0), \lambda(1))$ forms an equilibrium if it is optimal under pivotal voting given the future of advantage of REMAIN, Δ, that it generates, i.e., if, and only if,

$$\begin{cases} \lambda(\pi_t^i) = 1 & \text{if } \pi_t^i - \frac{1}{2} > \Delta \\ \lambda(\pi_t^i) \in [0,1] & \text{if } \pi_t^i - \frac{1}{2} = \Delta \\ \lambda(\pi_t^i) = 0 & \text{if } \pi_t^i - \frac{1}{2} < \Delta \end{cases}, \pi_t^i \in \{0, 1\}. \tag{7}$$

A policy is *implementable by κ-majority voting* if κ-majority voting possesses an equilibrium that induces this policy. A policy is *uniquely implemented by κ-majority voting* if it is the only policy that is implementable by κ-majority voting.

4.2. Equilibrium Analysis

Consider first the case where LEAVE is also reversible. The decision is LEAVE if the mass of LEAVE-winners, p_t, is at least κ. The optimal policy is uniquely implemented by the simple majority rule where $\kappa = \frac{1}{2}$.

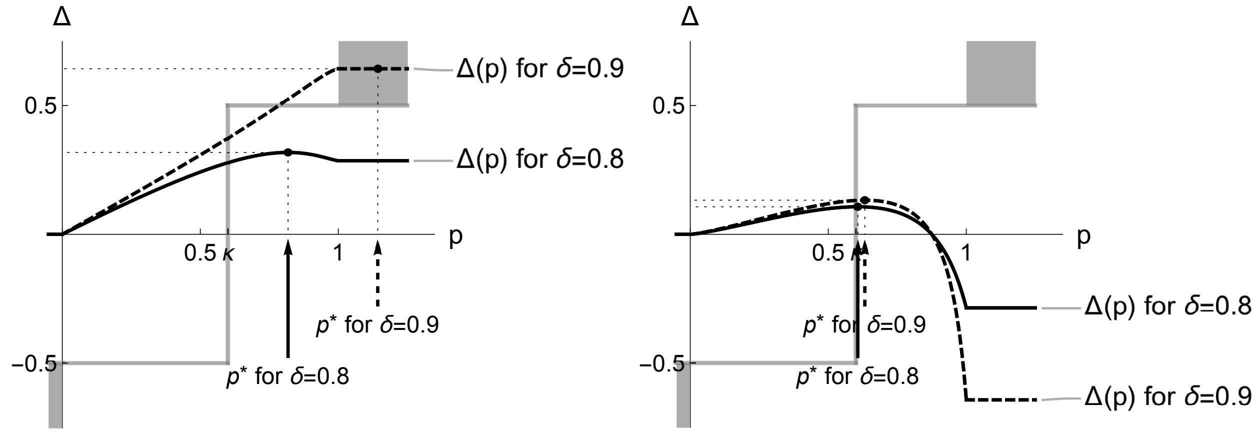
If LEAVE is irreversible, LEAVE-winners have, for any given Δ, a stronger incentive to vote for LEAVE than LEAVE-losers, and we obtain:

Lemma 3. *Only cutoff policies are implementable by κ-majority voting. If cutoff policy p is implemented, then $\Delta = \Delta(p)$ with $\Delta(p)$ as introduced in (4):*

$$\Delta(p) = \frac{\delta}{1 - \delta F(p)} \int_0^p (\tfrac{1}{2} - p_t) dF(p_t).$$

We next motivate the conditions under that a cutoff policy p is implementable by a given (super)majority rule with cutoff κ: Fix any p and assume that the agents believe that future decisions are taken according to cutoff policy p. Suppose first that $\Delta(p) \in (-\frac{1}{2}, \frac{1}{2})$. Then, for

Since $\delta \in (0, 1)$ implies that the matrix has full rank, a unique solution exists.



(a) REMAIN-friendly environment [$\gamma = \frac{3}{4}$] (b) LEAVE-friendly environment [$\gamma = \frac{4}{3}$]

Figure 2: Implementability conditions under κ-majority voting [$\kappa = 0.6$]

Implementable polices: Policies where the gray correspondence and the black curve intersects. All cutoffs $p > 1$ and all cutoffs $p \leq 0$ describe the same policy, respectively.

each agent, the decision's short-term effect is more important than its long-term consequences. Voting is myopic, and the cutoff policy $p = \kappa$ is induced. Hence, the cutoff policy p with $\Delta(p) \in (-\frac{1}{2}, \frac{1}{2})$ is implementable if and only if $p = \kappa$. Suppose next that $\Delta(p) > \frac{1}{2}$. Then, REMAIN is associated with a large future advantage that dominates any short-term effects. All agents vote REMAIN, and the induced decision is described by any cutoff policy $p > 1$. A cutoff policy p with $\Delta(p) > \frac{1}{2}$ is implementable if and only if $p > 1$. Lastly, suppose that $\Delta(p) = \frac{1}{2}$. Then LEAVE-winners are indifferent, and may vote for LEAVE with any probability $\lambda(1) \in [0, 1]$, whereas LEAVE-losers vote for REMAIN. The cutoff policy p with $\Delta(p) = \frac{1}{2}$ is implementable if and only if $p \geq \kappa$.[19]

Example. Figure 2a (Figure 2b) illustrates the implementability conditions for a REMAIN-friendly (LEAVE-friendly) environment. The black curves show $\Delta(p)$ for the same environment and discount factors as in Figure 1. For any Δ, the gray correspondence displays the cutoff policies p that are consistent with pivotal voting under majority rule κ. The intersections of the black curves and the gray correspondence describe the implementable policies.

We address now our main questions: How does the importance of the future, relative to the

[19]The case where $\Delta(p) \leq -\frac{1}{2}$ leads to analogous implementability conditions.

present, affect the set of implementable cutoff policies? Which (super)majority rule is optimal?

Proposition 1. *Consider a REMAIN-friendly environment. Let δ^* as introduced in (2): $\delta^* = \frac{1}{2(1-\bar{p})}$. Define $\delta^M \in (\delta^*, \infty)$ by*

$$\delta^M \equiv \frac{1}{2\int_0^{1/2}(1-p_t)dF(p_t)}.$$

Case i: $\delta \in (0, \delta^*)$**.** *(Super)majority rule $\kappa \in [\frac{1}{2}, 1)$ uniquely implements cutoff policy κ. Only the supermajority rule $\kappa = p^*$ implements the optimal policy.*

Case ii: $\delta \in (\delta^*, \delta^M] \cap (\delta^*, 1)$**.** *Consider $\kappa \in [\frac{1}{2}, \Delta^{-1}(\frac{1}{2})]$ and note that $\Delta^{-1}(\frac{1}{2}) \in (\frac{1}{2}, 1)$. The cutoff policy $p^* > 1$ which corresponds to never choosing LEAVE, and the cutoff policy κ are both implementable by (super)majority rule κ. Any supermajority rule $\kappa \in (\Delta^{-1}(\frac{1}{2}), 1)$ uniquely implements the optimal policy.*[20]

Case iii: $\delta \in (\delta^M, 1)$**.** *Any (super)majority rule $\kappa \in [\frac{1}{2}, 1)$ uniquely implements the optimal policy.*

In Case i, the short-term effects dominate any long-term effects, and voting is myopic for any (super)majority rule. Myopic voting fails to reflect the option value of REMAIN, and a supermajority becomes optimal.

In Cases ii and iii, the option value of REMAIN is large, and it is optimal to REMAIN under any circumstances. If agents believe that LEAVE will not be chosen in the future, even LEAVE-winners are willing to forego the current advantage of LEAVE. For any (super)majority rule, there exists an equilibrium where all agents vote REMAIN, and the optimal policy is implemented. This welfare-optimal equilibrium can coexist with a welfare-inferior equilibrium where agents vote myopically: If they believe that REMAIN today will nevertheless lead to LEAVE quite soon, the perceived future advantage of REMAIN can be small enough to turn myopic voting into optimal. Conversely, the expectation of myopic voting in the future can support the belief that REMAIN today leads to LEAVE quite soon.

[20] If $\delta = \delta^*$, $\Delta^{-1}(\frac{1}{2}) = 1$. Then, for any $\kappa \in [\frac{1}{2}, 1)$, the optimal policy and cutoff policy κ are both implementable by majority rule κ. We omit the proof for this non-generic case.

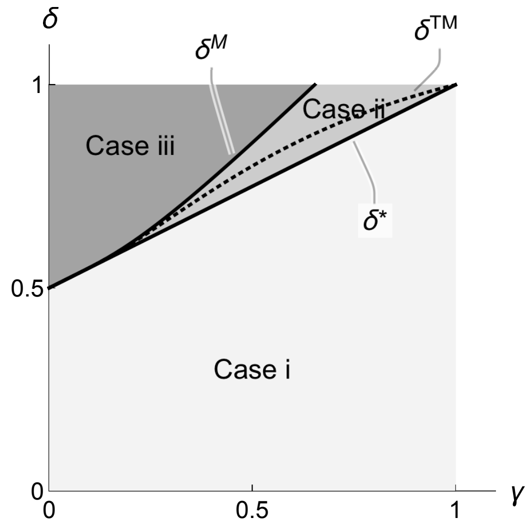


Figure 3: δ-regions in Proposition 1 for power distribution functions

In Case ii, a myopic equilibrium exists for the simple majority rule, but it can be avoided by using a sufficiently large supermajority rule. Intuitively, a large enough supermajority implies that LEAVE is chosen later, and a belief that REMAIN today soon leads to LEAVE cannot be supported. While the necessary supermajority may be large, smaller supermajorities are still useful since they improve the policy induced by the welfare-inferior equilibrium. In Case iii, the future is so important that, even under the simple majority rule, the belief about myopic voting in the future does not rationalize myopic voting today.

Example. In Case i, the implementability conditions look like those for $\delta = 0.8$ in Figure 2a; in Case ii, they look like those for $\delta = 0.9$;[21] in Case iii, the shape of $\Delta(p)$ is similar to that in Case ii, but $\Delta(p)$ is so steep that it exceeds $\frac{1}{2}$ already at $p = \frac{1}{2}$. Figure 3 illustrates the δ-intervals where the three cases apply. If γ is close enough to 1, then Case iii never applies for $\delta \in (0, 1)$; i.e., $\delta^M \geq 1$ is possible.

Corollary 1. *There exist REMAIN-friendly environments such that, for all* $\delta \in (0, 1)$*, either Case i or Case ii of Proposition 1 applies. For such environments, the simple majority rule possesses an equilibrium that leads to a welfare loss relative to the optimal supermajority rule,*

[21] The figure shows that a third equilibrium can exist in Case ii when $\kappa < \Delta^{-1}(\frac{1}{2})$. In this equilibrium, LEAVE-winners only vote with a certain probability for LEAVE, and an intermediate cutoff policy is implemented. This renders this equilibrium welfare-superior relative to the "welfare-inferior pure strategy equilibrium", but welfare-inferior relative to the "welfare-optimal pure strategy equilibrium".

$V_R(p^*) - V_R(\frac{1}{2})$*, that grows without bound as* $\delta \to 1$*.*

In LEAVE-friendly and in neutral environments there always exist circumstances under which it is optimal to LEAVE. Supermajority rules are then necessary to incorporate the option value of REMAIN.

Proposition 2. *Consider a LEAVE-friendly or neutral environment.*

Case i: $\delta \in (0, \frac{1}{2\bar{p}})$**.** *(Super)majority rule* $\kappa \in [\frac{1}{2}, 1)$ *uniquely implements cutoff policy* κ*. Only the supermajority rule* $\kappa = p^*$ *implements the optimal policy.*[22]

Case ii: $\delta \in (\frac{1}{2\bar{p}}, 1)$**.** *A (super)majority rule* $\kappa \in [\frac{1}{2}, \Delta^{-1}(-\frac{1}{2})]$ *uniquely implements cutoff policy* κ*. Any supermajority rule* $\kappa \in (\Delta^{-1}(-\frac{1}{2}), 1)$ *uniquely implements the cutoff policy* $\Delta^{-1}(-\frac{1}{2}) \in (p^*, 1)$*. Only the supermajority rule* $\kappa = p^*$ *implements the optimal policy.*

A qualitative difference relative to Case i in Proposition 1 occurs only if the future is very important, and if a supermajority rule that is significantly larger than the optimal rule is used. Then, even though LEAVE is likely to be individually (and socially) optimal in the future, agents expect that it will take very long until LEAVE will be chosen after REMAIN. This gives LEAVE-losers an incentive to vote for LEAVE. In the unique equilibrium, each LEAVE-loser votes with a certain probability for LEAVE, limiting the welfare consequences of choosing a too large supermajority rule:

Corollary 2. *In any LEAVE-friendly environment, a larger than optimal supermajority rule leads to a welfare loss that converges to a value smaller than* 1 *as* $\delta \to \infty$*.*

Example. In Case i, the implementability conditions look like those for $\delta = 0.8$ in Figure 2b; in Case ii, they look like those for $\delta = 0.9$. In LEAVE-friendly environments $\Delta(p)$ can be negative. If δ is close enough to 1, then, for p close to 1, $\Delta(p)$ is smaller than $-\frac{1}{2}$ and decreases without bound as $\delta \to 1$. As $\Delta(p)$ is a positive linear transformation of welfare, this property

[22] For $\delta = \frac{1}{2\bar{p}}$ the same result as in Case i applies, but the proof differs slightly. We omit the proof for this case.

renders the potential welfare loss from a suboptimal cutoff policy relative to the optimal cutoff policy unbounded.

In Propositions 1 and 2 we have shown that some supermajority rule is (weakly) optimal in all circumstances. Obviously, some supermajority rule will also be ex-ante optimal if the majority rule is chosen before the distribution function F is known. Corollaries 1 and 2 point to an asymmetry in the welfare costs resulting from either a too low and a too high supermajority rule.

5. Voting by Consecutive (Super)Majority

The existence of the relatively plausible equilibrium in which agents vote myopically can be a serious flaw in situations where it is optimal to never LEAVE. In the Brexit problem, a proponent of LEAVE who wants to "plant" beliefs leading to the myopic equilibrium could use the following kind of rhetoric: "If we do not LEAVE now, we will try again until we succeed. But if we LEAVE already now, we do not have to let the refugees in. So it is important that you vote for LEAVE already now." Nigel Farage's "threats" of coming back until Brexit does finally happen can be interpreted in this vein.[23]

We now show that mechanisms that require majorities in multiple, consecutive periods (as proposed, e.g., by Kenneth Rogoff) avoid equilibrium multiplicity without relying on a fine-tuning of the required supermajority.

Suppose now that LEAVE requires a κ-majority, $\kappa \in [\frac{1}{2}, 1)$, in **two** consecutive periods: In every period t such that LEAVE has not been yet chosen, agents vote either for LEAVE or for REMAIN. The decision is LEAVE if the mass of LEAVE-votes in both periods $t-1$ and t, l_{t-1} and l_t, is at least κ. We call this mechanism *voting by consecutive κ-majority*, and assume that $l_0 \geq \kappa$, so that LEAVE is possible in the initial period $t = 1$.

The problem can be now re-interpreted as one with three possible decisions: REMAIN (R), PROVISIONAL LEAVE (PL), and LEAVE (L). After a REMAIN, the effective choice

[23] See, for example, Nigel Farage's "unfinished business" comment that we quoted in the introduction. In June 2017, he argued that "All I can say is that if Brexit doesn't mean Brexit, I intend to don khaki and head for the frontline again." See "Nigel Farage in BREXIT PROMISE: 'I'll do khaki and head for the frontline again'", Express, June 25, 2017.

is between two alternatives: REMAIN and PROVISIONAL LEAVE; after a PROVISIONAL LEAVE, the effective choice is among REMAIN and LEAVE; after a LEAVE, there is no decision anymore. Notice that in periods where PROVISIONAL LEAVE is chosen each agent still gets a stage payoff of $\frac{1}{2}$ since the irreversible alternative is executed only after LEAVE is chosen.

The identity of the previous decision (R or PL) becomes now payoff-relevant, but the exact mass of LEAVE-votes in the preceding period, and the stage payoff from LEAVE in periods where LEAVE is not possible are not. Markov strategies are thus described by vectors $(\lambda_R, \lambda_{PL}(0), \lambda_{PL}(1)) \in [0,1]^3$ where λ_R describes the probability with which each agent votes for LEAVE if $d_{t-1} = R$ and where $\lambda_{PL}(\pi_t^i)$ describes the probability with which an agent of type $\pi_t^i \in \{0,1\}$ votes for LEAVE if $d_{t-1} = PL$.

If all agents vote according to the Markov strategy $(\lambda_R, \lambda_{PL}(0), \lambda_{PL}(1))$ and if the previous decision was $d_{t-1} \in \{PL, R\}$, the current decision is $d_t = d(p_t, d_{t-1})$ with

$$d(p_t, PL) = \begin{cases} L & \text{if } p_t\lambda_{PL}(1) + (1-p_t)\lambda_{PL}(0) \geq \kappa \\ R & \text{if } p_t\lambda_{PL}(1) + (1-p_t)\lambda_{PL}(0) < \kappa \end{cases} \text{ and } d(p_t, R) = \begin{cases} PL & \text{if } \lambda_R \geq \kappa \\ R & \text{if } \lambda_R < \kappa \end{cases}. \quad (8)$$

Consider then the system of linear equations

$$\begin{cases} V_L = \mathbb{E}_{p_t}[\mathbb{E}_{\pi_t^i|p_t}[\pi_t^i + \delta V_L]] \\ V_{PL} = \mathbb{E}_{p_t}[\mathbb{E}_{\pi_t^i|p_t}[\mathbf{1}_{d(p_t,PL)=L}(\pi_t^i + \delta V_L) + \mathbf{1}_{d(p_t,PL)=R}(\frac{1}{2} + \delta V_R)]] \\ V_R = \mathbb{E}_{p_t}[\mathbb{E}_{\pi_t^i|p_t}[\mathbf{1}_{d(p_t,R)=PL}(\frac{1}{2} + \delta V_{PL}) + \mathbf{1}_{d(p_t,R)=R}(\frac{1}{2} + \delta V_R)]] \end{cases} \quad (9)$$

implied by policy (8). It has a unique solution.[24] δV_d describes here the continuation value, common to all agents, from decision $d \in \{R, PL, L\}$.

The Markov strategy $(\lambda_R, \lambda_{PL}(0), \lambda_{PL}(1))$ specifies an equilibrium if it is optimal under

[24]The system can be rewritten as

$$\begin{pmatrix} 1-\delta & 0 & 0 \\ -\delta\mathbb{E}_{p_t}[\mathbf{1}_{d(p_t,PL)=L}] & 1 & -\delta\mathbb{E}_{p_t}[\mathbf{1}_{d(p_t,PL)=R}] \\ 0 & -\delta\mathbb{E}_{p_t}[\mathbf{1}_{d(p_t,R)=PL}] & 1-\delta\mathbb{E}_{p_t}[\mathbf{1}_{d(p_t,R)=R}] \end{pmatrix} \begin{pmatrix} V_L \\ V_{PL} \\ V_R \end{pmatrix} = \begin{pmatrix} \mathbb{E}_{p_t}[p_t] \\ \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t,PL)=L}p_t \\ +\mathbf{1}_{d(p_t,PL)=R}\frac{1}{2}] \\ \frac{1}{2} \end{pmatrix}.$$

Also in this case $\delta \in (0,1)$ implies that the matrix has full rank, such that a unique solution exists.

pivotality considerations for the continuation values it generates, i.e., if, and only if,

$$\begin{cases} \lambda_R = 1 & \text{if } V_{PL} > V_R \\ \lambda_R \in [0,1] & \text{if } V_{PL} = V_R \\ \lambda_R = 0 & \text{if } V_{PL} < V_R \end{cases} \text{ and } \begin{cases} \lambda_{PL}(\pi_t^i) = 1 & \text{if } \pi_t^i + \delta V_L > \frac{1}{2} + \delta V_R \\ \lambda_{PL}(\pi_t^i) \in [0,1] & \text{if } \pi_t^i + \delta V_L = \frac{1}{2} + \delta V_R \\ \lambda_{PL}(\pi_t^i) = 0 & \text{if } \pi_t^i + \delta V_L < \frac{1}{2} + \delta V_R \end{cases}, \pi_t^i \in \{0,1\}. \tag{10}$$

Proposition 3. *Let δ^* as introduced in (2): $\delta^* = \frac{1}{2(1-\bar{p})}$.*

(a) Suppose that $\delta > \delta^*$. For any κ, consecutive κ-majority voting uniquely implements the optimal policy.

(b) Suppose that $\delta < \delta^*$. Then the optimal policy is not implementable by a consecutive κ-majority voting.

Recall that $\delta > \delta^*$ can only hold in REMAIN-friendly environments. Conversely, if the environment is LEAVE-friendly or neutral, $\delta < \delta^*$ is satisfied for any $\delta \in (0,1)$.

Part (a) of the above result implies that, for any κ, κ-majority voting and consecutive κ-majority voting both implement the optimal policy in REMAIN-friendly environments where $\delta > \delta^*$. But, the consecutive κ-majority voting avoids the existence of additional, welfare-inferior equilibria. The intuition is as follows: even though each LEAVE-winner prefers the payoff stream from "REMAIN forever", $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \dots)$, over the payoff stream from LEAVE, $(1, \bar{p}, \bar{p}, \dots)$, voting for LEAVE could nevertheless be optimal for a LEAVE-winner if he believed that REMAIN leads to an average future stage payoff smaller than $\frac{1}{2}$. This can induce the coexistence of equilibria where LEAVE is chosen relatively quickly and where it is never chosen. In order to obtain an analogous multiplicity under consecutive majority voting, agents must believe that REMAIN leads to an average future stage payoff smaller than $\frac{1}{2}$. However, in situations where LEAVE is not currently possible ($d_{t-1} = R$), all agents decide between making LEAVE possible in the next period ($d_t = PL$) and delaying this decision for one period, while realizing a stage payoff of $\frac{1}{2}$ in the next period ($d_t = R$). Due to the impossibility of immediate LEAVE, voting incentives are solely driven by the future consequences of the decision. Thus, if agents believe that the average future stage payoff from making LEAVE possible in the next

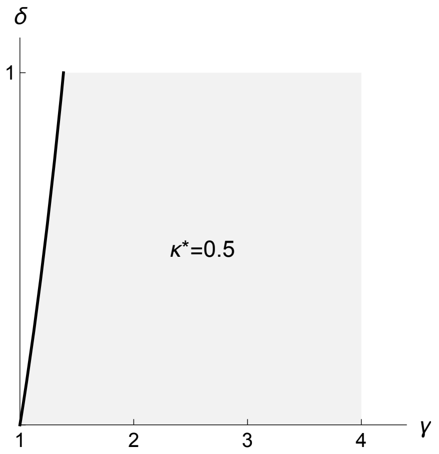


Figure 4: δ-region in Proposition 4 for LEAVE-friendly power distribution functions

period is smaller than $\frac{1}{2}$, they will all vote to delay the decision. As delaying the decision forever results in an average future stage payoff of $\frac{1}{2}$, the average future stage payoff of REMAIN can never fall short of $\frac{1}{2}$. This implies that, whenever the policy preferred by a current LEAVE-winner is "REMAIN forever", there exists no equilibrium where LEAVE is ever chosen.

Part (b) of the Proposition shows that using consecutive κ-majority voting also comes with a cost: even with fine-tuning, it leads to inefficient delays whenever it is sometimes optimal to LEAVE. The possibility of inefficient delays determines the optimal majority rule in LEAVE-friendly environments.

Proposition 4. *Suppose that*

$$\delta \leq \frac{\bar{p}-\frac{1}{2}}{\int_0^{1/2} F(p_t)\,dp_t}.$$

Then, among all policies that are implementable by consecutive κ-majority voting, $\kappa = \frac{1}{2}$ uniquely implements the best policy.

The proof parallels our analysis of (super)majority voting. Intuitively, the delay that comes along with REMAIN makes LEAVE relatively more attractive from a social perspective. The option value of REMAIN may realize, at the earliest, in two periods whereas delay leads in LEAVE-friendly environments to an expected loss in the next period. Due to this trade-off, it can be optimal to LEAVE, even if LEAVE has a disadvantage in the current period.

Example. For LEAVE-friendly power distributions, the gray region in Figure 4 illustrates the part of the parameter space where the simple majority rule is optimal. We know from Proposition 3 (a) that the simple majority rule is (weakly) optimal for REMAIN-friendly environments with $\delta > \delta^*$. This suggests that the role of supermajority rules is much smaller under consecutive majority requirements.

6. Voting by Two-Sided Supermajority

In his "unfinished business"-quote, Nigel Farage suggested that a commitment to REMAIN would be possible if REMAIN wins by a sufficiently large margin. In this section, we propose a voting mechanism that incorporates such a feature.

Suppose that the decision process ends if **either** LEAVE or REMAIN gains a κ-majority, $\kappa \in (\frac{1}{2}, 1)$: the decision is LEAVE (L) if the mass of LEAVE-votes l_t is at least κ, and it is REMAIN FOREVER ($R\infty$) if l_t is at most $1-\kappa$; if $l_t \in (1-\kappa, \kappa)$, the decision is REMAIN (for now) with a new vote in the next period (R). We call this mechanism *two-sided κ-majority voting.*[25]

There are now two different scenarios where an agent can be pivotal. He can be pivotal for LEAVE reaching either the κ-majority, or the $(1-\kappa)$-majority. In the former case, he effectively decides between LEAVE and REMAIN, whereas in the latter case he effectively decides between REMAIN and REMAIN FOREVER. An agent's incentives depend on the probability μ with which he believes to be pivotal for LEAVE reaching the κ-majority (conditional on being pivotal).

Under consistent beliefs,[26] the current mass of LEAVE-winners, p_t, becomes payoff-relevant:[27] Even if an agent believes that others do not condition on p_t, this number is still informative about the mass of LEAVE-votes and thus about his belief. A Markov strategy is now described by $(\lambda(0, p_t), \lambda(1, p_t))$. $\lambda(\pi_t^i, p_t)$ denotes the probability with which an agent of type π_t^i votes for LEAVE if the current mass of LEAVE-winners is p_t. A belief system is described by $\mu(p_t)$.

[25] A related mechanism is used in papal enclaves: Voting is repeated until a candidate obtains a certain supermajority. If no candidate is elected in an initial phase, there is a runoff election between the candidates who received the most votes where a two-third majority is still required.

[26] In Appendix B we discuss the beliefs derived from Bayesian updating for the version of our model with a large finite number of agents, and motivate a notion of consistent beliefs for the continuum version.

[27] Recall that we assumed p_t to be publicly observable. See Footnote 11 for a motivation of this assumption.

$\mu(p_t)$ denotes the belief that the agents hold if the current mass of LEAVE-winners is p_t. A belief system $\mu(p_t)$ is *consistent* with the Markov strategy $(\lambda(0, p_t), \lambda(1, p_t))$ if, for each p_t,

$$\begin{cases} \mu(p_t) = 1 & \text{if } l_t > \frac{1}{2} \\ \mu(p_t) \in [0, 1] & \text{if } l_t = \frac{1}{2} \\ \mu(p_t) = 0 & \text{if } l_t < \frac{1}{2} \end{cases} \quad \text{with } l_t = p_t\lambda(1, p_t) + (1 - p_t)\lambda(0, p_t). \tag{11}$$

We employ the following extended equilibrium notion: The Markov strategy $(\lambda(0, p_t), \lambda(1, p_t))$ together with the belief system $\mu(p_t)$ forms an equilibrium if (i) given the belief system, the Markov strategy is optimal under pivotal voting for the continuation values it generates, and (ii) the belief system is consistent with the Markov strategy.[28]

Proposition 5. *Let*

$$\delta^{TM} \equiv \frac{1}{1+F(1/2)(1-2\bar{p})}$$

and note that $\delta^{TM} \in (\delta^*, \min\{\delta^M, 1\})$*. Suppose that the environment is REMAIN-friendly and that* $\delta > \delta^{TM}$*. For any* κ*, two-sided* κ*-majority voting uniquely implements the optimal policy.*

The Proposition establishes that, in contrast to majority voting, an equilibrium must now implement the optimal policy when it is optimal to never LEAVE, and when the future is sufficiently important. An intuition for the qualitative difference to voting by majority is as follows: Assume, by contradiction, that there exists an equilibrium of two-sided κ-majority voting where LEAVE is chosen with positive probability. For any p_t where LEAVE is chosen, belief consistency implies that each agent believes to be pivotal for LEAVE reaching the κ-majority. An agent effectively decides then between LEAVE and REMAIN, i.e., he faces a trade-off that is similar to that under normal majority voting: LEAVE-losers have a strict incentive to vote for REMAIN, but LEAVE-winners can have an incentive to vote LEAVE if they expect the future consequences of REMAIN and LEAVE to be similar. However, since LEAVE-losers vote REMAIN for any p_t and for any belief, the 50%-majority will be missed at least in periods where a majority of agents are LEAVE-losers. This is where the contrast to

[28]Note that, for majority voting and for consecutive majority voting, the belief system is trivial since there is always only a single event in which the agent is pivotal. Hence, the extended equilibrium notion reduces to our original equilibrium notion for those voting mechanisms.

normal majority voting kicks in. For such periods, consistency of beliefs implies that each agent believes to be pivotal for REMAIN, or for REMAIN FOREVER. As all agents prefer REMAIN FOREVER over REMAIN if they believe that REMAIN leads to suboptimal decisions in the future, REMAIN eventually leads with a probability of at least $F(\frac{1}{2})$ to REMAIN FOREVER. This drives a wedge between individual assessments of the future consequences of REMAIN and LEAVE. If the future is sufficiently important, this wedge is large enough, and LEAVE-winners always prefer REMAIN over LEAVE, contradicting the choice of LEAVE in some periods.

Example. The dotted curve in Figure 3 depicts δ^{TM} for REMAIN-friendly power distribution functions. Proposition 5 applies for any parameter constellation above this curve. In the part of the "Case ii" labeled region that lies above the dotted curve, uniquely implementing the optimal policy by κ-majority voting requires a fine-tuned supermajority rule, whereas the optimal policy is uniquely implemented by any two-sided κ-majority voting rule.

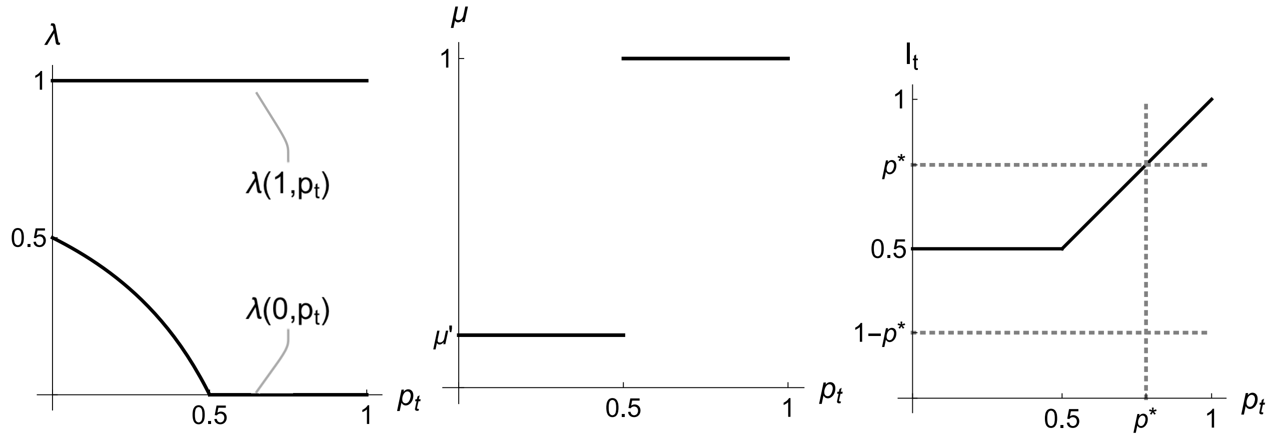
Recall that a commitment to REMAIN FOREVER is only possible here if REMAIN gains a sufficiently large margin of victory; i.e., two-sided κ-majority voting might only be feasible for values of κ that exceed a certain bound. Interestingly, the implications of the Proposition do not depend on how large the value of this bound is.

Like consecutive majority voting, two-sided supermajority voting can circumvent the existence of welfare-inferior equilibria without relying on fine-tuning. Yet two-sided supermajority voting might be easier to implement politically since it treats LEAVE and REMAIN more symmetrically. Two-sided supermajority voting is also capable of avoiding inefficient delays in environments where it is optimal to sometimes LEAVE:

Proposition 6. *Let δ^* as introduced in (2): $\delta^* = \frac{1}{2(1-\bar{p})}$. If $\delta < \delta^*$, two-sided p^*-majority voting implements the optimal policy.*

Note that the same supermajority rule is here optimal as under majority voting.

Under the assumption that future decisions are optimal, all agents prefer REMAIN over REMAIN FOREVER, but only LEAVE-winners prefer LEAVE over REMAIN. Each LEAVE-winner always votes for LEAVE, whereas the voting incentives of a LEAVE-loser depend on the belief system $\mu(p_t)$. If $p_t > \frac{1}{2}$, a LEAVE-loser believes to be pivotal for LEAVE reaching the



(a) Markov strategy: probabilities of voting for LEAVE

(b) Belief system: probability to be pivotal for LEAVE reaching the κ-majority

(c) Mass of LEAVE-votes

Figure 5: The equilibrium behind Proposition 6 [$\gamma = \frac{3}{4}$, $\delta = 0.77$, $\kappa = p^*$]

κ-majority. Voting for REMAIN is optimal (assuming $\delta < \delta^*$) for the same reasons as under normal κ-majority voting. If $p_t < \frac{1}{2}$ and if the voting behavior of LEAVE-losers is such that $l_t = \frac{1}{2}$, any belief is consistent with the voting behavior. Since a LEAVE-loser prefers REMAIN over REMAIN FOREVER, and prefers REMAIN over LEAVE, there exists an intermediate belief $\mu' \in (0, 1)$ under which such an agent is indeed indifferent between voting for REMAIN and for LEAVE, rendering mixing optimal.

Example. Figure 5 illustrates the equilibrium behind Proposition 6. Figure 5a shows the probabilities with which LEAVE-winners and LEAVE-losers vote for LEAVE; Figure 5b illustrates the belief system; Figure 5c shows the mass of LEAVE-votes. If $p_t < \frac{1}{2}$, LEAVE-winners vote for LEAVE and LEAVE-losers mix in a way such that the mass of LEAVE-votes is exactly $\frac{1}{2}$; if $p_t \geq \frac{1}{2}$, voting is myopic. This means that the decision is never REMAIN FOREVER on the equilibrium path. Since LEAVE obtains the κ-majority only if $p_t \geq p^*$, the optimal policy is implemented.

Remark 1 (Essentiality of mixing)**.** *Mixing is an essential part of any equilibrium where agents strictly prefer REMAIN over REMAIN FOREVER. Suppose, by contradiction, that an equilibrium exists where, for $p_t < \frac{1}{2}$, LEAVE-losers vote for REMAIN with certainty. Then, for any $p_t < \frac{1}{2}$, each agent would expect that LEAVE misses the simple majority. Consistency of beliefs requires thus that agents believe to be pivotal for REMAIN reaching the κ-majority. Since such* *a belief endows each agent with a strict incentive to vote for LEAVE (because of his preference for REMAIN over REMAIN FOREVER), we obtain a contradiction to LEAVE-losers voting for REMAIN with certainty.*

Remark 2 (Equilibrium multiplicity)**.** *Despite the previous remark, two-sided p^*-majority voting does not uniquely implement the optimal policy under the assumptions of Proposition 6. There exist additional equilibria where agents are indifferent between REMAIN and REMAIN FOREVER. In such an equilibrium, an agent believes that he is never pivotal for LEAVE reaching the κ-majority, and LEAVE is never chosen on the equilibrium path.*

Yet the multiplicity of equilibrium in this context does not pose a real economic problem. If we consider the version of the model with a large finite electorate, and if we introduce an arbitrarily small, exogenous probability with which an agent votes myopically, the above kind of equilibrium immediately ceases to exist.

7. Conclusion

We studied a dynamic voting scenario where one alternative is irreversible. Except for a special case where any majority rule implements the welfare-optimal policy, voting by simple majority cannot take into account the option value attached to the reversible alternative. Supermajorities biased in favor of the latter are necessary to implement an optimal policy in environments where opinions fluctuate over time. But, even such a bias cannot always avoid the existence of myopic voting equilibria with inferior welfare properties. We also proposed two other simple voting procedures that can be implemented in practice and that can eliminate inferior equilibria: requiring majorities in consecutive periods in order to choose the irreversible option, or, more symmetrically, requiring that no decision is final unless if it obtains a high enough margin of support in the population.

Appendix A. Extensions

Extension I: Inter-Temporal Correlations

In reality, today's LEAVE-winners are more likely than today's LEAVE-losers to be tomorrow's LEAVE-winners. In order to study voting in such scenarios, we introduce serial correlation of individual stage payoffs in a simple way that leaves the optimal policy unchanged: Let $q_1(p_{t-1}, p_t)$ ($q_0(p_{t-1}, p_t)$) denote the probability with which a period $t-1$ LEAVE-winner (LEAVE-loser) becomes a LEAVE-winner in period t, conditional on p_{t-1} and p_t. We impose the following two assumptions:

Assumption 1. $q_1(p_{t-1}, p_t) \geq q_0(p_{t-1}, p_t)$.

Assumption 2. $p_{t-1}q_1(p_{t-1}, p_t) + (1-p_{t-1})q_0(p_{t-1}, p_t) = p_t$.

Assumption 1 introduces our notion of positive serial correlation. Assumption 2 implies that the mass of LEAVE-winners in period t is still described by p_t. The optimal policy is therefore unaffected.

Consider again κ-majority voting. Since the current mass of LEAVE-winners affects the updated probability to be a LEAVE-winner in the next period, p_t becomes payoff-relevant. If all agents vote according to the Markov strategy $\lambda(\pi_t^i, p_t)$, the decision is

$$d(p_t) = \begin{cases} L & \text{if } p_t\lambda(1, p_t) + (1-p_t)\lambda(0, p_t) \geq \kappa \\ R & \text{if } p_t\lambda(1, p_t) + (1-p_t)\lambda(0, p_t) < \kappa \end{cases}. \tag{A.1}$$

Consider the system of linear equations

$$\begin{cases} V_L(\pi_{t-1}^i, p_{t-1}) = \mathbb{E}_{p_t}[\mathbb{E}_{\pi_t^i|\pi_{t-1}^i, p_{t-1}, p_t}[\pi_t^i + \delta V_L(\pi_t^i, p_t)]] \\ V_R(\pi_{t-1}^i, p_{t-1}) = \mathbb{E}_{p_t}[\mathbb{E}_{\pi_t^i|\pi_{t-1}^i, p_{t-1}, p_t}[\mathbf{1}_{d(p_t)=L}(\pi_t^i + \delta V_L(\pi_t^i, p_t)) + \mathbf{1}_{d(p_t)=R}(\frac{1}{2} + \delta V_R(\pi_t^i, p_t))]] \end{cases} \tag{A.2}$$

implied by policy (A.1). It possesses a unique solution.[29] $\delta V_d(\pi_t^i, p_t)$ describes the continuation value of an agent of type π_t^i from decision $d \in \{R, L\}$. The continuation values affect voting

[29] The same reasoning as in Footnote 18 applies.

incentives only through the future advantage implied for REMAIN,

$$\Delta(\pi_t^i, p_t) \equiv \delta(V_R(\pi_t^i, p_t) - V_L(\pi_t^i, p_t)).$$

A Markov strategy is optimal under pivotal voting if and only if it satisfies a condition that is analogous to (7).

The future advantage of REMAIN is now a more complicated object than its analog in the original model. Yet, the structure imposed by Assumptions 1 and 2 implies that there is a clear relation between the values that this advantage assumes in both versions of the model:

Lemma 4. *Fix any policy $d(p_t)$ and let $\Delta(\pi_t^i, p_t)$ be the future advantage of REMAIN generated by $d(p_t)$. Then,*

$$\Delta(1, p_t) \leq \Delta \leq \Delta(0, p_t)$$

where Δ is the future advantage of REMAIN implied by $d(p_t)$ in our original model.

Intuitively, the Lemma says that the positive serial correlation reinforces incentives: Holding the expectation about future decisions fixed, LEAVE-winners get more eager to vote for LEAVE, while LEAVE-losers get more eager to vote for REMAIN. This has two immediate implications:

1) Whenever myopic voting is the unique equilibrium in our base model, it is the unique equilibrium that implements a cutoff policy in the modified model.[30] Put differently, whenever it is optimal to sometimes LEAVE, a supermajority plays the same role as in our base model.

2) If it is optimal to REMAIN forever, supermajority rules can play an even bigger role. In the base model, the optimal policy was generally implementable by the simple majority rule but not necessarily uniquely implementable. In the equilibrium that implements the optimal policy, all agents vote for REMAIN irrespective of their short-term incentives. Supermajority rules were only useful for improving upon the welfare-inferior equilibrium or for circumventing its existence. In the modified model, the stronger incentive of LEAVE-winners to vote for LEAVE can render myopic voting the unique equilibrium.[31] Any supermajority rule improves then upon

[30]Because of the dependence of Markov strategies on p_t, it is not immediately obvious that only cutoff policies are implementable in our modified model.

[31]An example can be constructed in the following way: Fix a REMAIN-friendly environment. For $\delta = \delta^* + \epsilon$

the policy implemented by the unique equilibrium. Interestingly, by a reasoning similar to the one in Corollary 1, myopic voting can constitute the *unique* equilibrium even if all agents agree that it would be better to REMAIN forever, and even if they perceive the long-term consequences of the decision as arbitrarily more important than its short-term consequences.

Extension II: Less Polarization

We assumed above that an agent's stage payoff from LEAVE can adopt only two values, $\pi_t^i = 1$ and $\pi_t^i = 0$. The electorate was polarized in the sense that the median agent's payoff was 1 for $p_t > \frac{1}{2}$ and 0 for $p_t < \frac{1}{2}$. We now relax the assumption of a completely polarized electorate, and we generalize our base model in two steps: Firstly, the mass of LEAVE-winners is now distributed on the interval $[\alpha, 1-\alpha]$ with $\alpha \in (0, \frac{1}{2})$. Secondly, we introduce unpolarized agents with intermediate stage payoffs $\pi_t^i \in [0,1]$ such that the average stage payoff from LEAVE across agents, p_t, does not change. This is done as follows: suppose that, conditional on p_t, π_t^i is 1 with probability $p_t - \alpha$, 0 with probability $(1-\alpha) - p_t$, and it is drawn with probability 2α according to a c.d.f G with a p.d.f g that is symmetric around $\frac{1}{2}$, and support $[0,1]$. Everything else stays as in our base model. The electorate is completely polarized for $\alpha = 0$ and it converges to an unpolarized electorate as $\alpha \to \frac{1}{2}$.

Consider κ-majority voting. A $(1-\kappa)$-fraction of agents can enforce REMAIN. That is, the decision is effectively taken by the $(1-\kappa)$-quantile agent. Because of our continuum electorate assumption, this agent's stage payoff from LEAVE is a deterministic function of p_t. We denote the $(1-\kappa)$-quantile agent's stage payoff by $\pi^{(1-\kappa)}(p_t)$. See Appendix B for a derivation of the functional form of $\pi^{(1-\kappa)}$. The equilibrium behavior follows from comparing the $(1-\kappa)$-quantile agent's advantage of LEAVE in the current period, $\pi^{(1-\kappa)}(p_t) - \frac{1}{2}$, with the future advantage of REMAIN. Only cutoff policies can be induced.[32] Each agent's future advantage of REMAIN from cutoff policy p is still $\Delta(p)$ as defined in (4).[33] An interior cutoff $p \in (\alpha, 1-\alpha)$ is implementable if $\pi^{(1-\kappa)}(p) - \frac{1}{2} = \Delta(p)$. For the implementability of non-interior cutoffs we

with $\epsilon > 0$, Δ is larger than $\frac{1}{2}$ and converges to $\frac{1}{2}$ as $\epsilon \to 0$. By choosing the function q_1 that determines the serial correlation such that $\Delta(1, p_t)$ is bounded away from Δ as $\epsilon \to 0$, we obtain the result.

[32] This follows from the fact that, as in our base model, $\pi^{(1-\kappa)}(p_t)$ is non-decreasing in p_t, and continuation values do not depend on p_t as Markov strategies condition only π_t^i.

[33] The only effect of the support change is that $\Delta(p)$ is now piecewise constant on $(-\infty, \alpha]$ and on $[1-\alpha, \infty)$.

obtain conditions that are analogous to those in our base model. Since $\pi^{(1-\kappa)}(p_t)$ is weakly decreasing in κ, the majority rule κ serves as an instrument for increasing the induced cutoff by making the pivotal agent less eager to LEAVE. This allows us to give a graphical intuition for equilibrium behavior and for the relations to our base model.

Example 1: Unpolarized agents with uniformly distributed stage payoffs. Suppose that $g(\pi_t^i) = 1$. The black curves in Figure A.6 display $\Delta(p)$ in a REMAIN-friendly environment for various discount factors.[34] All cutoff policies p such that $\Delta(p)$ falls into the gray correspondence in the left panel (the right panel) are implementable by the simple majority rule (by a supermajority rule with $\kappa \approx 0.59$). The optimal policy p^* is still determined by the unique intersection of $p - \frac{1}{2}$ and $\Delta(p)$. The black dot in each panel indicates (p^*, Δ^*).[35]

Suppose first that δ is sufficiently small such that $\Delta(p) \in [0, \frac{1}{2})$ for all p (Figure A.6a). An interior cutoff $p^* \in (\frac{1}{2}, 1-\alpha)$ is then optimal. Since $\pi^{(1/2)}(p) - \frac{1}{2} > p - \frac{1}{2}$ for all $p \in (\frac{1}{2}, 1-\alpha]$, the simple majority uniquely implements a cutoff that is too small from a social perspective, whereas the right supermajority rule uniquely implements the optimal policy (see the right panel).

Suppose next that δ is large enough such that $\Delta(1-\alpha) > \frac{1}{2}$ but small enough such that the simple majority rule gives rise to multiple equilibria (Figure A.6b). It is then possible to uniquely implement the optimal policy by choosing a sufficiently large supermajority rule (as displayed in the right panel).

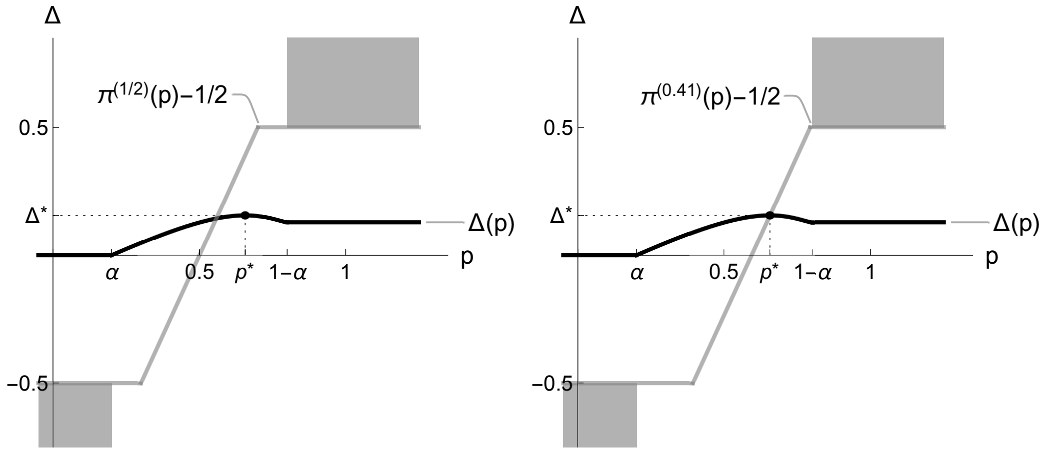
Finally, for very large δ (see Figure A.6c) it can happen that the majority rule does not matter. The simple majority rule and any supermajority rule uniquely implement then the optimal policy.

Qualitatively, the three cases and the role supermajority rules play in these cases are analogous to the three cases in Proposition 1.

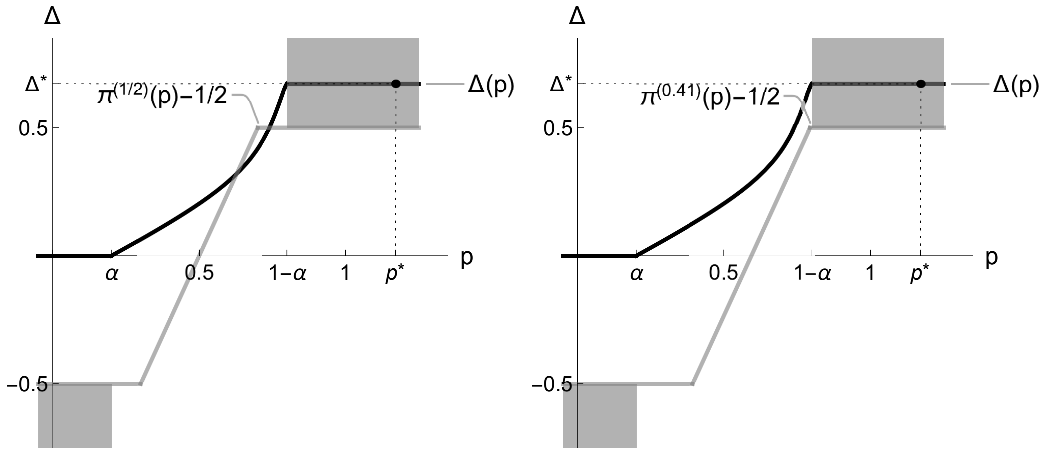
In Example 1, for any $p_t > \frac{1}{2}$, the median agent is more eager to LEAVE than the planner:

[34] The effects in a LEAVE-friendly environment are essentially analogous to those that we will explain for the REMAIN-friendly environment and a sufficiently small discount factor (see Figure A.6a).

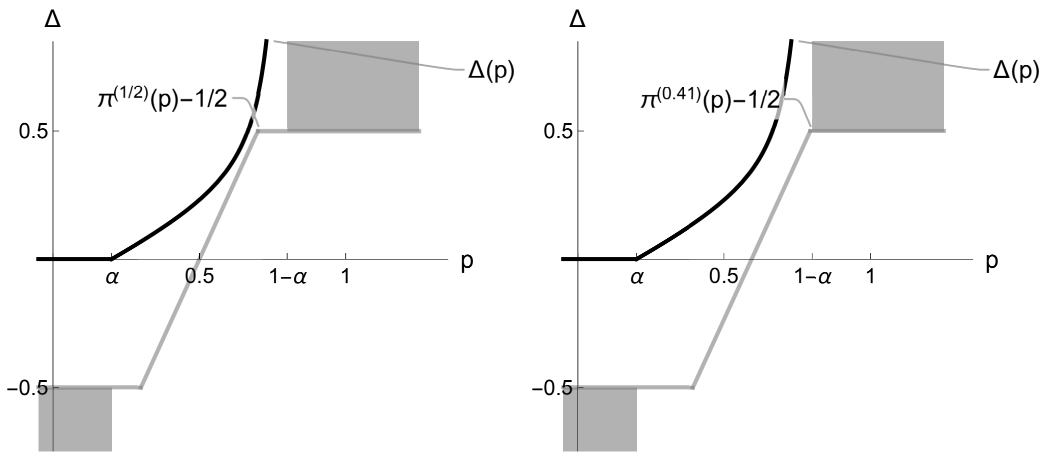
[35] Note that in Figure A.6c, this dot falls outside the displayed area.



(a) $\kappa = \frac{1}{2}$ (left panel) and $\kappa = \kappa^* \approx 0.59$ (right panel) for $\delta = 0.75$



(b) $\kappa = \frac{1}{2}$ (left panel) and $\kappa \approx 0.59$ (right panel) for $\delta = 0.94$



(c) $\kappa = \frac{1}{2}$ (left panel) and $\kappa \approx 0.59$ (right panel) for $\delta = 0.99$

Figure A.6: Implementability conditions under κ-majority voting in the model with less polarization $[\alpha = \frac{1}{5}, F(p_t) = F_{3/4}(\frac{p_t - \alpha}{1 - 2\alpha}),\ g(\pi_t^i) = 1]$

Implementable polices: Policies where the gray correspondence and the black curve intersects. All cutoffs $p > 1$, and all cutoffs $p \leq 0$ describe the same policy, respectively.

he assigns to any $p_t \in (\frac{1}{2}, 1-\alpha]$ a higher current advantage of LEAVE; i.e.,

$$\pi^{(1/2)}(p_t) - \tfrac{1}{2} > p_t - \tfrac{1}{2}. \tag{A.3}$$

In our base model, this condition was implied by our extreme notion of polarization: The median agent had a stage payoff of 1 whenever the mass of LEAVE-winners exceeded $\frac{1}{2}$. Any weaker notion of polarization that implies (A.3) for $p_t \in (\frac{1}{2}, 1-\alpha]$ is sufficient for obtaining the optimality of a supermajority rule. Part (a) of the subsequent Lemma identifies such a notion.

Lemma 5. *(a) Suppose that the density g is quasi-convex. Then, for any degree of polarization and for all $p_t \in (\frac{1}{2}, 1-\alpha]$, the median agent is more eager to LEAVE than the planner.*

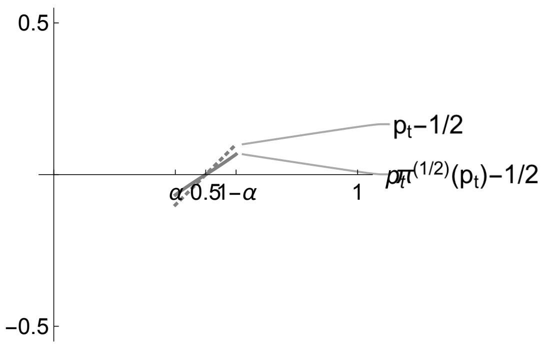
(b) Suppose that g is strictly quasi-concave. Then, if the degree of polarization is sufficiently low, there exist $p_t \in (\frac{1}{2}, 1-\alpha]$ such that the planner is more eager to LEAVE than the median agent.

In Example 1, we discussed a boundary case: g was linear (i.e., it was quasi-convex and quasi-concave, but not strictly quasi-concave). The subsequent example illustrates how our results are affected if g is strictly quasi-concave.

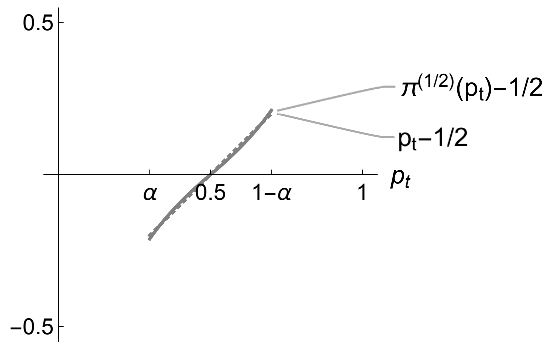
Example 2: Unpolarized agents with stage payoffs that are distributed according to a symmetric triangular distribution. Suppose $g(\pi_t^i) = \min\{4\pi_t^i, 4(1-\pi_t^i)\}$. For four different degrees of polarization in the electorate, Figure A.7 shows how the median agent's current advantage of LEAVE (solid curves) compares to that of the planner (dotted curves).

Figure A.7a shows that with 20% polarized agents the planner is always more eager to LEAVE than the median agent. As a consequence, the simple majority rule is optimal as a corner solution. With 40% polarized agents (Figure A.7b), the planner is more eager to LEAVE for small p_t, but the median agent is more eager to LEAVE for large p_t.

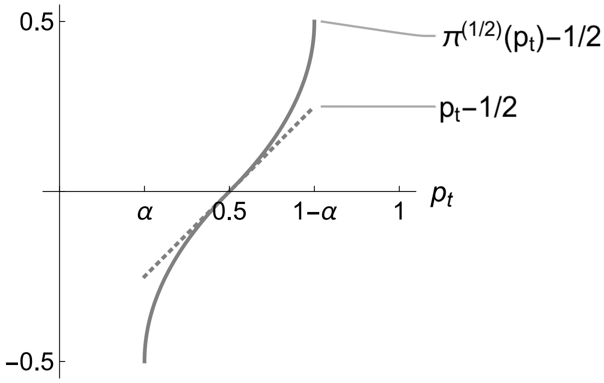
Finally, with 50% or more polarized agents, the median agent is, for all p_t, more eager to LEAVE than the planner (Figures A.7c and A.7d). The median agent's stage payoff from LEAVE behaves in all important respects like those in Example 1. This becomes particularly



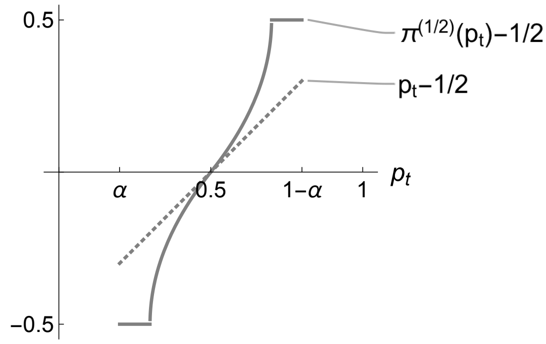
(a) 20% polarized agents [$\alpha = 0.4$]



(b) 40% polarized agents [$\alpha = 0.3$]



(c) 50% polarized agents [$\alpha = 0.25$]



(d) 60% polarized agents [$\alpha = 0.2$]

Figure A.7: Advantage of LEAVE in the current period of the median agent and of the planner [$g(\pi_t^i) = \min\{4\pi_t^i, 4(1-\pi_t^i)\}$]

apparent by comparing the gray solid curve in Figure A.7d with that in the left panels of Figure A.6. The same reasoning as in our discussion of Example 1 applies. In particular, a supermajority rule is generally optimal, and equilibrium multiplicity can occur for the simple majority rule, but can be avoided by choosing a sufficiently large supermajority rule.

Appendix B. Proofs

Proofs for Section 3

Proof of Lemma 1

(1) is equivalent to $\phi(p^*, \delta) = 0$, where the auxiliary function $\phi : \mathbb{R} \times (0,1) \to \mathbb{R}$ is defined by

$$\phi(p, \delta) \equiv p - \tfrac{1}{2} - \delta \int_0^p (p - p_t) \mathrm{d}F(p_t). \tag{B.1}$$

For fixed δ, p^* is the unique root of $\phi(p, \delta)$. Two properties of $\phi(p, \delta)$ are important. Firstly, since $\phi_p(p, \delta) = 1 - \delta F(p) > 0$, we obtain, for fixed δ, that $\phi(p, \delta) < 0$ implies $p^* > p$ and that $\phi(p, \delta) > 0$ implies $p^* < p$. Secondly, p^* is strictly increasing in δ since $\phi(0, \delta) = -\frac{1}{2}$ for all δ, and since $\phi_p(p, \delta) > 0$ and $\phi_{p\delta}(p, \delta) < 0$ for $p > 0$.

Because

$$\phi(\tfrac{1}{2}, \delta) = -\delta \int_0^{1/2} (\tfrac{1}{2} - p_t) \mathrm{d}F(p_t) < 0$$

we obtain $p^* > \frac{1}{2}$. Since $\phi(1, \delta) = 1 - \frac{1}{2} - \delta(1 - \bar{p})$, we obtain that $\phi(1, \delta) > 0$ for $\bar{p} \geq \frac{1}{2}$. Hence, by the Intermediate Value Theorem, $p^* < 1$ for any LEAVE-friendly or neutral environment. If the environment is REMAIN-friendly, the sign of $\phi(1, \delta)$ depends on the discount factor. Since $\delta = \frac{1}{2(1-\bar{p})}$ solves $\phi(1, \delta) = 0$, we obtain the wished result.

Proof of Lemma 2

(a) The planner's ex-ante expected payoff from cutoff policy p is $V_R(p)$. Since the function V_L does not depend on p, $\Delta(p)$ is a positive linear transformation of $V_R(p)$.

(b) It is obvious from (4) that Δ is continuous on $\mathbb{R}$ and piecewise constant on $(-\infty, 0]$ and on $[1, \infty)$. For all $p \in (0, 1)$, we have

$$\begin{aligned}
\Delta'(p) &= \frac{\delta^2 f(p)}{(1 - \delta F(p))^2} \int_0^p (\tfrac{1}{2} - p_t) \mathrm{d}F(p_t) + \frac{\delta}{1 - \delta F(p)} (\tfrac{1}{2} - p) f(p) \\
&= -\frac{\delta f(p)}{(1 - \delta F(p))^2} \Big(-\delta \int_0^p (\tfrac{1}{2} - p_t) \mathrm{d}F(p_t) - (1 - \delta F(p))(\tfrac{1}{2} - p) \Big) \\
&= -\frac{\delta f(p)}{(1 - \delta F(p))^2} \Big(p - \tfrac{1}{2} - \delta \int_0^p (p - p_t) \mathrm{d}F(p_t) \Big) \\
&\overset{(B.1)}{=} -\frac{\delta f(p)}{(1 - \delta F(p))^2} \phi(p, \delta).
\end{aligned}$$

By the reasoning in the proof of Lemma 1, for any fixed δ, $\phi(p,\delta)$ is strictly increasing in p with a root at p^*. If $p^* \geq 1$, we obtain $\Delta'(p) > 0$ for all $p \in (0,1)$. If $p^* < 1$, we obtain $\Delta'(p) > 0$ for $p \in (0, p^*)$, and $\Delta'(p) < 0$ for $p \in (p^*, 1)$.

(c) We have

$$\begin{aligned}
\Delta(p) - (p - \tfrac{1}{2}) &\overset{(4)}{=} \frac{1}{1-\delta F(p)}\left(\delta \int_0^p (\tfrac{1}{2} - p_t)\mathrm{d}F(p_t) - (1-\delta F(p))(p-\tfrac{1}{2})\right) \\
&= -\frac{1}{1-\delta F(p)}\left(p - \tfrac{1}{2} - \delta \int_0^p (p - p_t)\mathrm{d}F(p_t)\right) \\
&\overset{(B.1)}{=} -\frac{1}{1-\delta F(p)}\phi(p,\delta).
\end{aligned}$$

We obtain $\Delta(p) > p - \frac{1}{2}$ for $p < p^*$, $\Delta(p^*) = p^* - \frac{1}{2}$, and $\Delta(p) < p - \frac{1}{2}$ for $p > p^*$.

Proofs for Section 4

Proof of Lemma 3

Suppose that $(\lambda(0), \lambda(1))$ constitutes an equilibrium under κ-majority voting. By (7), $\lambda(1) \geq \lambda(0)$. Since this implies that $p_t\lambda(1) + (1-p_t)\lambda(0)$ is non-decreasing in p_t, (5) defines a cutoff policy.

Suppose $d(p_t)$ is a cutoff policy with cutoff p. By using the structure of this policy and $\mathbb{E}_{\pi_t^i|p_t}[\pi_t^i] = p_t$ in system (6), system (6) simplifies to the system (3). Hence, $\Delta = \Delta(p)$.

Auxiliary Results for the Proof of Proposition 1

We next establish properties which $\Delta(p)$ has in REMAIN-friendly environments that are crucial for the proof of Proposition 1.

Lemma B.1. *Fix any REMAIN-friendly environment.*

(a) Assume that $\delta < \delta^*$. Then $\Delta(p) \in [0, \frac{1}{2})$ for all p.

(b) Assume that $\delta > \delta^*$. Then there exists a unique value $\Delta^{-1}(\frac{1}{2}) \in [\frac{1}{2}, 1)$ for $\delta \in (\delta^*, \delta^M]$ and a unique value $\Delta^{-1}(\frac{1}{2}) \in (0, \frac{1}{2})$ for $\delta > \delta^M$. Moreover, $\Delta(p) \in [0, \frac{1}{2})$ for $p < \Delta^{-1}(\frac{1}{2})$, and $\Delta(p) > \frac{1}{2}$ for $p > \Delta^{-1}(\frac{1}{2})$.

Proof. We divide the argument in three steps. (a) follows from Steps 1 and 2 and (b) from Steps 1 and 3.

Step 1. By Lemma 2 (b), $\min_p \Delta(p) = \min\{\Delta(0), \Delta(1)\}$. Since $\Delta(0) = 0$, $\Delta(1) = \frac{\delta}{1-\delta}(\frac{1}{2} - \bar{p})$, and $\bar{p} < \frac{1}{2}$ we obtain that $\min_p \Delta(p) = 0$ in any REMAIN-friendly environment.

Step 2. Suppose that $\delta < \delta^*$. By Lemma 2 (a), $\Delta(p) \leq \Delta(p^*)$. Since by Lemma 2 (c), $\Delta(p^*) = p^* - \frac{1}{2}$, and by Lemma 1, $p^* < 1$, we obtain $\Delta(p) < \frac{1}{2}$.

Step 3. Suppose that $\delta > \delta^*$. Then, $p^* > 1$ by Lemma 1. By Lemma 2 (b) and (c), this implies that $\Delta(p^*) > \frac{1}{2}$, and that $\Delta(p)$ is strictly increasing on $[0, 1]$, with $\Delta(1) = \Delta(p^*)$. Since $\Delta(0) = 0$ and since $\Delta(p)$ is continuous, we can apply the Intermediate Value Theorem to obtain a unique value $\Delta^{-1}(\frac{1}{2}) \in (0, 1)$. Since $\Delta(p)$ is strictly increasing on $[0, 1]$ and weakly increasing on $\mathbb{R}$, $\Delta(p) < \frac{1}{2}$ for $p < \Delta^{-1}(\frac{1}{2})$ and $\Delta(p) > \frac{1}{2}$ for $p > \Delta^{-1}(\frac{1}{2})$. Finally,

$$\begin{aligned} \Delta^{-1}(\tfrac{1}{2}) < \tfrac{1}{2} \Leftrightarrow \Delta(\tfrac{1}{2}) > \tfrac{1}{2} \quad &\Leftrightarrow \quad \frac{\delta}{1 - \delta F(\frac{1}{2})} \int_0^{1/2} (\tfrac{1}{2} - p_t) \mathrm{d}F(p_t) > \tfrac{1}{2} \\ &\Leftrightarrow \quad \delta 2 \int_0^{1/2} (\tfrac{1}{2} - p_t) \mathrm{d}F(p_t) > 1 - \delta F(\tfrac{1}{2}) \\ &\Leftrightarrow \quad \delta 2 \int_0^{1/2} (1 - p_t) \mathrm{d}F(p_t) > 1 \Leftrightarrow \delta > \delta^M. \end{aligned}$$

□

Proof of Proposition 1

Note that

$$\delta^* = \frac{1}{2\int_0^1 (1 - p_t) \mathrm{d}F(p_t)} > \delta^M = \frac{1}{2\int_0^{1/2} (1 - p_t) \mathrm{d}F(p_t)}.$$

Case i: $\delta \in (0, \delta^*)$. Suppose that $(\lambda(0), \lambda(1))$ constitutes an equilibrium. By Lemma 3, $\Delta = \Delta(p)$ for some p. Since, by Lemma B.1 (a) $\Delta(p) \in [0, \frac{1}{2})$ for all p, $(\lambda(0), \lambda(1))$ is by (7) optimal for the Δ it implies if and only if $(\lambda(0), \lambda(1)) = (0, 1)$. In other words, myopic voting constitutes the unique equilibrium. Since, by (5), myopic voting induces cutoff policy κ, we obtain the result.

Case ii and iii: $\delta \in (\delta^*, 1)$.[36]

Step 1: For any κ, always voting for REMAIN, $(\lambda(0), \lambda(1)) = (0, 0)$, constitutes an equilibrium. By (5), any cutoff policy $p' > 1$ describes the policy implemented by $(\lambda(0), \lambda(1)) = (0, 0)$. Fix any such p'. By Lemma 3, $\Delta = \Delta(p')$. Since $1 < p'$ and since $\Delta^{-1}(\frac{1}{2}) < 1$ by Lemma B.1

[36]The subsequent Steps 1–3 do not differentiate between $\delta \leq \delta^M$ and $\delta > \delta^M$. The set $[\frac{1}{2}, \Delta^{-1}(\frac{1}{2})]$ is empty if $\delta > \delta^M$.

(b), we obtain that $\Delta(p') > \frac{1}{2}$. By (7) this implies that $(\lambda(0), \lambda(1)) = (0, 0)$ is optimal under pivotality considerations for the Δ it induces.

Step 2: If $\kappa \in [\frac{1}{2}, \Delta^{-1}(\frac{1}{2})]$, myopic voting, $(\lambda(0), \lambda(1)) = (0, 1)$, constitutes an equilibrium. By (5), myopic voting induces cutoff policy $p = \kappa$. By Lemma 3, $\Delta = \Delta(\kappa)$. Since $\kappa \leq \Delta^{-1}(\frac{1}{2})$ and Lemma B.1 (b) imply that $\Delta(\kappa) \in [0, \frac{1}{2}]$, we obtain by (7) that myopic voting is optimal under pivotality considerations for the Δ it induces.

Step 3: If $\kappa \in (\Delta^{-1}(\frac{1}{2}), 1)$, the equilibrium from Step 1 is unique. Assume to the contrary that $(\lambda(0), \lambda(1)) \neq (0, 0)$ constitutes a further equilibrium. By Lemma 3, $\Delta = \Delta(p)$ for some p. Since $\Delta(p) \geq 0$ by Lemma B.1 (b), $\lambda(0) = 0$. This implies $p \geq \kappa$ by (5). However, since $\kappa > \Delta^{-1}(\frac{1}{2})$ and Lemma B.1 (b) imply $\Delta(p) > \frac{1}{2}$, pivotality implies also $\lambda(1) = 0$ by (7), yielding a contradiction.

Step 4: The statements about the optimal majority rule. By Lemma 1, it is optimal to never LEAVE for $\delta > \delta^*$. By Step 1, the optimal policy is implementable by κ-majority voting with any κ. By Steps 2 and 3, the optimal policy is uniquely implemented by κ-majority voting only if $\kappa > \Delta^{-1}(\frac{1}{2})$. Recall that this condition is satisfied for all $\kappa \in [\frac{1}{2}, 1)$ if $\delta > \delta^M$.

Proof of Corollary 1

Note that $\delta^M \geq 1$ is equivalent to

$$2 \int_0^{1/2} (1 - p_t) f(p_t) \mathrm{d}p_t \leq 1. \tag{B.2}$$

Consider REMAIN-friendly power distribution functions F_γ, $\gamma \in (0, 1)$. Since $F'_\gamma(p_t) = \frac{1}{\gamma}(1 - p_t)^{\frac{1}{\gamma} - 1}$, the left-hand side of (B.2) can be written as

$$2 \int_0^{1/2} \tfrac{1}{\gamma}(1 - p_t)^{\frac{1}{\gamma}} \mathrm{d}p_t = 2 \left[\frac{-1}{1 + \gamma} (1 - p_t)^{\frac{1}{\gamma} + 1} \right]_{p_t = 0}^{p_t = 1/2} = \frac{1}{1 + \gamma} (2 - (\tfrac{1}{2})^{\frac{1}{\gamma}}).$$

Since this expression is continuous in γ and converges to $\frac{3}{4}$ as $\gamma \to 1$, we obtain that $\delta^M \geq 1$ for any γ sufficiently close to 1.

Suppose the environment is REMAIN-friendly and that $\delta^M \geq 1$. Then, by Proposition 1, for any $\delta > \delta^*$, the cutoff policy $\frac{1}{2}$ is implementable by the simple majority rule. By the same Proposition, any sufficiently large supermajority rule uniquely implements the optimal policy

$p^* > 1$. Hence, the simple majority rule can lead to a welfare loss of

$$V_R(p^*) - V_R(\tfrac{1}{2}) = \tfrac{1}{\delta}(\Delta(p^*) - \Delta(\tfrac{1}{2}))$$

where the equality follows because $V_L(p^*) = V_L(\frac{1}{2})$ by the irreversibility of LEAVE. For $\delta \leq \delta^M$, Lemma B.1 (b) implies that $\Delta(\frac{1}{2}) \leq \frac{1}{2}$. On the other hand, (4) and $p^* > 1$ imply that $\Delta(p^*) = \frac{\delta}{1-\delta}(\frac{1}{2} - \bar{p})$. Since $\frac{1}{2} - \bar{p} > 0$ in any REMAIN-friendly environment, $\Delta(p^*)$ grows without bound as $\delta \to 1$. Hence, $V_R(p^*) - V_R(\frac{1}{2})$ grows without bound as $\delta \to 1$.

Auxiliary Results for the Proof of Proposition 2

We next establish properties of $\Delta(p)$ in LEAVE-friendly environments that are crucial for the proof of Proposition 2.

Lemma B.2. *Fix any LEAVE-friendly or neutral environment.*

(a) Assume that $\delta < \frac{1}{2\bar{p}}$. Then $\Delta(p) \in (-\frac{1}{2}, \frac{1}{2})$ for all p.

(b) Assume that $\delta > \frac{1}{2\bar{p}}$. Then there exists a unique value $\Delta^{-1}(-\frac{1}{2}) \in (p^*, 1)$. Moreover, $\Delta(p) \in (-\frac{1}{2}, \frac{1}{2})$ for $p < \Delta^{-1}(-\frac{1}{2})$ and $\Delta(p) < -\frac{1}{2}$ for $p > \Delta^{-1}(-\frac{1}{2})$.

Proof. (a) follows from Steps 1 and 2.1, (b) from Steps 1 and 2.2.

Step 1. By Lemma 2 (a), $\Delta(p) \leq \Delta(p^*)$. Since $\Delta(p^*) = p^* - \frac{1}{2}$, and since, by Lemma 1, $p^* \in (\frac{1}{2}, 1)$ for any LEAVE-friendly or neutral environment, we obtain $\Delta(p) < \frac{1}{2}$.

Step 2. By Lemma 2 (b), $\min_p \Delta(p) = \min\{\Delta(0), \Delta(1)\}$. Since (4) implies $\Delta(0) = 0$ and $\Delta(1) = \frac{\delta}{1-\delta}(\frac{1}{2} - \bar{p})$, and since $\bar{p} \geq \frac{1}{2}$ in any LEAVE-friendly or neutral environment, we obtain that $\min_p \Delta(p) = \Delta(1)$. Thus,

$$\min_p \Delta(p) - (-\tfrac{1}{2}) = \frac{\delta}{1-\delta}(\tfrac{1}{2} - \bar{p}) - (-\tfrac{1}{2}) = \frac{\bar{p}}{1-\delta}(\frac{1}{2\bar{p}} - \delta). \tag{B.3}$$

Step 2.1. Suppose that $\delta < \frac{1}{2\bar{p}}$. Then, (B.3) implies $\Delta(p) > -\frac{1}{2}$.

Step 2.2. Suppose that $\delta > \frac{1}{2\bar{p}}$. Then, (B.3) and $\min_p \Delta(p) = \Delta(1)$ imply $\Delta(1) < -\frac{1}{2}$. In Step 1, we have already observed that $\Delta(p^*) \in (0, \frac{1}{2})$ with $p^* \in (0, 1)$. Since $\Delta(p)$ is continuous, we can apply the Intermediate Value Theorem to obtain a unique value $\Delta^{-1}(-\frac{1}{2}) \in (p^*, 1)$.

Furthermore, it follows from Lemma 2 (b) that $\Delta(p) > -\frac{1}{2}$ for $p < \Delta^{-1}(-\frac{1}{2})$ and $\Delta(p) < -\frac{1}{2}$ for $p > \Delta^{-1}(-\frac{1}{2})$. □

Proof of Proposition 2

Case i: $\delta \in (0, \frac{1}{2\bar{p}})$**.** Since, by Lemma B.2, this condition implies $\Delta(p) \in (-\frac{1}{2}, \frac{1}{2})$ for all p, a reasoning analogous to that in Case i of Proposition 1 applies.

Case ii: $\delta \in (\frac{1}{2\bar{p}}, 1)$**.** We now show that a unique equilibrium exists for any $\kappa \in [\frac{1}{2}, 1)$. The reasoning in the subsequent steps proves that the implemented policy is as asserted in the Proposition.

Step 1: For any $\kappa \in [\frac{1}{2}, \Delta^{-1}(-\frac{1}{2})]$*, myopic voting,* $(\lambda(0), \lambda(1)) = (0, 1)$*, constitutes an equilibrium.* By (5), myopic voting induces cutoff policy $p = \kappa$. By Lemma 3, $\Delta = \Delta(\kappa)$. Lemma B.2 (b), $\delta > \frac{1}{2\bar{p}}$ and $\kappa \in [\frac{1}{2}, \Delta^{-1}(-\frac{1}{2})]$ imply $\Delta(\kappa) \in [-\frac{1}{2}, \frac{1}{2})$. Thus, by (7), myopic voting is optimal under pivotality considerations for the Δ it induces.

Step 2: For any $\kappa \in [\frac{1}{2}, \Delta^{-1}(-\frac{1}{2})]$*, no other equilibrium exists.* Assume to the contrary that $(\lambda(0), \lambda(1)) \neq (0, 1)$ constitutes a further equilibrium. By Lemma 3, $\Delta = \Delta(p)$ for some p. Since, by Lemma B.2 (b), $\Delta(p) < \frac{1}{2}$, pivotality implies $\lambda(1) = 1$ by (7). Thus, $\lambda(0) > 0$ must be true. By (5), $\lambda(1) = 1$ and $\lambda(0) > 0$ imply that a cutoff $p < \kappa$ is induced. Since $\kappa \leq \Delta^{-1}(-\frac{1}{2})$ by our supposition, $p < \Delta^{-1}(-\frac{1}{2})$. Since this implies for $\delta > \frac{1}{2\bar{p}}$ that $\Delta(p) > -\frac{1}{2}$ by Lemma B.2 (b), only $\lambda(0) = 0$ is optimal under pivotality by (7). This yields a contradiction.

Step 3: For any $\kappa \in (\Delta^{-1}(-\frac{1}{2}), 1)$*,*

$$(\lambda(0), \lambda(1)) = (\tfrac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}, 1)$$

constitutes an equilibrium. By (5), the strategy profile $(\frac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}, 1)$ induces the cutoff policy $\Delta^{-1}(-\frac{1}{2})$. By Lemma 3, $\Delta = \Delta(\Delta^{-1}(-\frac{1}{2})) = -\frac{1}{2}$. Hence, by (7), $(\frac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}, 1)$ is optimal under pivotality for the Δ it induces.

Step 4: For any $\kappa \in (\Delta^{-1}(-\frac{1}{2}), 1)$*, no other equilibrium exists.* Assume to the contrary that

$$(\lambda(0), \lambda(1)) \neq (\tfrac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}, 1)$$

constitutes a further equilibrium. By Lemma 3, $\Delta = \Delta(p)$ for some p. Since $\Delta(p) < \frac{1}{2}$ by Lemma B.2 (b), pivotality considerations imply $\lambda(1) = 1$ by (7). Thus, $\lambda(0) \neq \frac{\kappa - \Delta^{-1}(-1/2)}{1 - \Delta^{-1}(-1/2)}$. We distinguish two cases:

Suppose first that $\lambda(0) < \frac{\kappa - \Delta^{-1}(-1/2)}{1-\Delta^{-1}(-1/2)}$. Together with $\lambda(1) = 1$ and (5), this implies that a cutoff $p > \Delta^{-1}(-\frac{1}{2})$ is induced. By Lemma B.2 (b), this implies that $\Delta(p) < -\frac{1}{2}$ for $\delta > \frac{1}{2\bar{p}}$. Thus, by (7), only $\lambda(0) = 1$ is optimal under pivotality, yielding a contradiction.

Suppose next that $\lambda(0) > \frac{\kappa - \Delta^{-1}(-1/2)}{1-\Delta^{-1}(-1/2)}$. Then a cutoff $p < \Delta^{-1}(-\frac{1}{2})$ is induced, and only $\lambda(0) = 0$ is optimal, yielding again a contradiction.

Proof of Corollary 2

Fix any LEAVE-friendly environment and consider $\delta > \frac{1}{2\bar{p}}$. By Proposition 2, a policy is implementable by some majority rule κ if and only if it is a cutoff policy $p \in [\frac{1}{2}, \Delta^{-1}(-\frac{1}{2})]$. By Lemma B.2 (b),

$$\min_{p \in [1/2, \Delta^{-1}(-1/2)]} \Delta(p) = -\tfrac{1}{2} \quad \text{and} \quad \max_{p \in [1/2, \Delta^{-1}(-1/2)]} \Delta(p) < \tfrac{1}{2}.$$

Since the welfare loss from cutoff policy p is

$$V_R(p^*) - V_R(p) = \tfrac{1}{\delta}(\Delta(p^*) - \Delta(p)),$$

the maximal welfare loss is strictly smaller than $\frac{1}{\delta}$, and it is bounded by 1 for $\delta \to 1$.

Proofs for Section 5

Recall that we consider in this section the version of the model where votes are for REMAIN or for LEAVE, but where, overall, three decisions are possible, REMAIN (R), PROVISIONAL LEAVE (PL) and LEAVE (L).

Proof of Proposition 3

(a) Suppose that $\delta > \delta^*$, and note that the environment must be REMAIN-friendly. By Lemma 1 it is optimal to never LEAVE. Assume to the contrary that an equilibrium $(\lambda_R, \lambda_{PL}(0), \lambda_{PL}(1))$ with $\lambda_{PL}(1) > 0$ or with $\lambda_{PL}(0) > 0$ exists. By (10), optimality under pivotality considerations requires

$$1 + \delta V_L \geq \tfrac{1}{2} + \delta V_R. \tag{B.4}$$

Any $\lambda_R \in \arg\max_{\lambda\in[0,1]}(1-\lambda)V_R + \lambda V_{PL}$ is optimal under pivotality by (10). Thus, by the third equation in (9), $V_R = \frac{1}{2} + \delta \max\{V_R, V_{PL}\}$. Since this implies $V_R \geq \frac{1}{2} + \delta V_R$, we get $V_R \geq \frac{1}{1-\delta}\frac{1}{2}$. Because of this, and because the first equation in (9) implies $V_L = \frac{1}{1-\delta}\bar{p}$, a necessary condition for (B.4) is

$$\begin{aligned} & 1 + \delta(\tfrac{1}{1-\delta}\bar{p}) \geq \tfrac{1}{2} + \delta(\tfrac{1}{1-\delta}\tfrac{1}{2}) \\ \Leftrightarrow \quad & \tfrac{1}{2} \geq \tfrac{\delta}{1-\delta}(\tfrac{1}{2} - \bar{p}). \end{aligned} \tag{B.5}$$

By Lemma 1, $\delta > \delta^*$ implies $p^* > 1$. By Lemma 2 (c), this implies $\Delta(p^*) > \frac{1}{2}$. Since, by (4), $\Delta(p^*) = \frac{\delta}{1-\delta}(\frac{1}{2} - \bar{p})$, we obtain $\frac{1}{2} < \frac{\delta}{1-\delta}(\frac{1}{2} - \bar{p})$, yielding a contradiction to (B.5). Hence, there exists no equilibrium where LEAVE is ever selected; that is, if an equilibrium exists, it implements the optimal policy. It can easily be verified that, for any $\lambda_R \in [0,1]$, $(\lambda_R, 0, 0)$ constitutes an equilibrium.

(b) Suppose that $\delta < \delta^*$. Then, by Lemma 1, a cutoff policy $p^* \in (\frac{1}{2}, 1)$ is optimal. In other words, it is optimal to LEAVE in every period with a positive probability that is strictly less than 1. Since a decision not to LEAVE in period t implies that LEAVE is not possible in period $t+1$, we obtain that the the optimal policy is not implementable under consecutive majority voting.

Derivation of the Constraint Optimal Policy

We now derive the optimal policy subject to the constraint that $d_t \in \{R, PL\}$ if $d_{t-1} = R$, and that $d_t \in \{R, L\}$ if $d_{t-1} = PL$. From the perspective of period t, the previous decision d_{t-1} determines which decisions are feasible in the current period. However, no other information about the past affects payoffs or actions in the current and in future periods. Therefore, we can denote the planner's value of entering period t with decision d by V_d^{**}, $d \in \{R, PL, L\}$. The Bellman equations for the planner's optimal policy are

$$\begin{cases} V_L^{**} = \mathbb{E}_{p_t}[p_t + \delta V_L^{**}] \\ V_{PL}^{**} = \mathbb{E}_{p_t}[\max\{p_t + \delta V_L^{**}, \frac{1}{2} + \delta V_R^{**}\}] \\ V_R^{**} = \mathbb{E}_{p_t}[\max\{\frac{1}{2} + \delta V_{PL}^{**}, \frac{1}{2} + \delta V_R^{**}\}] \end{cases} . \tag{B.6}$$

Since the planner's stage payoff is bounded, and since the maximum of the program is attained, the value V_d^{**}, $d \in \{R, PL, L\}$, is unique.

We note that $V_{PL}^{**} \geq V_R^{**}$. If $V_{PL}^{**} < V_R^{**}$ was true, we would have $V_R^{**} = \frac{1}{2} + \delta V_R^{**}$ by the third equation of (B.6), and $V_{PL}^{**} = \mathbb{E}_{p_t}[\max\{p_t + \delta V_L^{**}, V_R^{**}\}]$ by using this property in the second equation of (B.6). As this yields a contradiction to $V_{PL}^{**} < V_R^{**}$, we can conclude that there exists an optimal policy under which $d_t = R$ implies $d_{t+1} = PL$.

A *policy with cutoff* p *and delay after REMAIN* is a policy where $d_{t-1} = R$ implies $d_t = PL$, and where $d_{t-1} = PL$ implies $d_t = R$ if $p_t < p$ and $d_t = L$ if $p_t \geq p$. By the reasoning in the preceding paragraph, and by the second equation of (B.6), the policy with cutoff

$$p^{**} \equiv \tfrac{1}{2} + \delta(V_R^{**} - V_L^{**}) \tag{B.7}$$

and delay after REMAIN is optimal. It follows from the Bellman equations and fom $V_{PL}^{**} \geq V_R^{**}$ that $V_R^{**} - V_L^{**}$ is the unique solution to

$$\begin{aligned} V_R^{**} - V_L^{**} &= (\tfrac{1}{2} + \delta V_{PL}^{**}) - (\bar{p} + \delta V_L^{**}) \\ &= \tfrac{1}{2} - \bar{p} + \delta \mathbb{E}_{p_t}[\max\{0, \tfrac{1}{2} - p_t + \delta(V_R^{**} - V_L^{**})\}]. \end{aligned}$$

By using (B.7) and rearranging, the last equation can be written as

$$(p^{**} + \delta\bar{p}) - (\tfrac{1}{2} + \delta\tfrac{1}{2}) = \delta^2 \mathbb{E}_{p_t}[\max\{0, p^{**} - p_t\}].$$

The above equation is equivalent to $\tilde{\phi}(p, \delta) = 0$, where the auxiliary function $\tilde{\phi} : \mathbb{R} \times (0, 1) \to \mathbb{R}$ is defined by

$$\tilde{\phi}(p, \delta) \equiv (p - \tfrac{1}{2}) + \delta(\bar{p} - \tfrac{1}{2}) - \delta^2 \int_0^p (p - p_t) \mathrm{d}F(p_t). \tag{B.8}$$

Lemma B.3. *Fix* δ *and let* p^{**} *denote the unique root of* $\tilde{\phi}(p, \delta)$*. If*

$$\delta \leq \frac{\bar{p} - \frac{1}{2}}{\int_0^{1/2} F(p_t)\, dp_t}$$

then, $p^{**} \in (0, \frac{1}{2}]$.

Proof. Note that $\tilde{\phi}_p(p, \delta) = 1 - \delta^2 F(p) > 0$. Thus, $p^{**} > 0$ is equivalent to $\tilde{\phi}(0, \delta) < 0$. By

(B.8), $\tilde{\phi}(0,\delta) = -\frac{1}{2} + \delta(\bar{p} - \frac{1}{2}) < 0$. Likewise, $p^{**} \leq \frac{1}{2}$ is equivalent to $\tilde{\phi}(\frac{1}{2},\delta) \geq 0$. Because

$$\begin{aligned}\tilde{\phi}(\tfrac{1}{2},\delta) \quad &= \quad \delta(\bar{p} - \tfrac{1}{2}) - \delta^2 \int_0^{1/2} (\tfrac{1}{2} - p_t)\mathrm{d}F(p_t) \\ &\overset{\text{integration by parts}}{=} \quad \delta(\bar{p} - \tfrac{1}{2}) - \delta^2 \int_0^{1/2} F(p_t)\mathrm{d}p_t,\end{aligned}$$

we obtain that $\tilde{\phi}(\frac{1}{2},\delta) \geq 0$ if, and only if, $\delta \leq \frac{\bar{p}-\frac{1}{2}}{\int_0^{1/2} F(p_t)\mathrm{d}p_t}$. ☐

Comparison of Non-Optimal Cutoff Policies with Delay After REMAIN

We next describe how the planner compares non-optimal cutoff policies with delay after REMAIN. For given p, consider the system of linear equations

$$\begin{cases} \tilde{V}_L(p) = \bar{p} + \delta\tilde{V}_L(p) \\ \tilde{V}_{PL}(p) = F(p)(\frac{1}{2} + \delta\tilde{V}_R(p)) + (1 - F(p))(\mathbb{E}_{p_t}[p_t|p_t \geq p] + \delta\tilde{V}_L(p)) \\ \tilde{V}_R(p) = \frac{1}{2} + \delta\tilde{V}_{PL}(p) \end{cases} . \tag{B.9}$$

It has a unique solution.[37] $\delta\tilde{V}_d(p)$ describes the planner's continuation value from decision $d \in \{R, PL, L\}$ if future decisions are taken according to the policy with cutoff p and delay after REMAIN. Then

$$\tilde{\Delta}(p) \equiv \delta(\tilde{V}_R(p) - \tilde{V}_L(p)) \tag{B.10}$$

describes the future advantage of $d_t = R$ over $d_t = L$ that is induced by the policy with cutoff p and delay after REMAIN. We have

$$\begin{aligned}\tilde{\Delta}(p) \quad &\overset{(B.9)}{=} \quad \delta(\tfrac{1}{2} - \bar{p}) + \delta^2(\tilde{V}_{PL}(p) - \tilde{V}_L(p)) \\ &\overset{(B.9)}{=} \quad \delta(\tfrac{1}{2} - \bar{p}) + \delta^2 F(p)(\tfrac{1}{2} + \delta\tilde{V}_R(p) - \mathbb{E}_{p_t}[p_t|p_t \leq p] - \delta\tilde{V}_L(p)) \\ &\overset{(B.10)}{=} \quad \delta(\tfrac{1}{2} - \bar{p}) + \delta^2 \int_0^p (\tfrac{1}{2} - p_t)\mathrm{d}F(p_t) + \delta^2 F(p)\tilde{\Delta}(p).\end{aligned}$$

[37] The system can be rewritten as

$$\begin{pmatrix} 1-\delta & 0 & 0 \\ -\delta(1-F(p)) & 1 & -\delta F(p) \\ 0 & -\delta & 1 \end{pmatrix} \begin{pmatrix} \tilde{V}_L(p) \\ \tilde{V}_{PL}(p) \\ \tilde{V}_R(p) \end{pmatrix} = \begin{pmatrix} \bar{p} \\ F(p)\frac{1}{2} + (1-F(p))\mathbb{E}_{p_t}[p_t|p_t \geq p] \\ \frac{1}{2} \end{pmatrix}.$$

Also in this case $\delta \in (0,1)$ implies that the matrix has full rank, such that a unique solution exists.

Hence,

$$\tilde{\Delta}(p) = \frac{\delta}{1-\delta^2 F(p)}\Big((\tfrac{1}{2}-\bar{p}) + \delta\int_0^p (\tfrac{1}{2}-p_t)\mathrm{d}F(p_t)\Big). \tag{B.11}$$

Lemma B.4. *(a) $\tilde{\Delta}(p) \geq \tilde{\Delta}(p')$ if, and only if, the planner weakly prefers the policy with cutoff p and delay after REMAIN over the policy with cutoff p' and delay after REMAIN.*

(b) $\tilde{\Delta}(p)$ is piecewise constant on $(-\infty,0]$ and on $[1,\infty)$. On the interval $[0,1]$, $\tilde{\Delta}(p)$ is strictly increasing for $p \leq p^{**}$, and strictly decreasing for $p \geq p^{**}$, with $\tilde{\Delta}(p^{**}) = p^{**}-\frac{1}{2}$. Moreover, $\tilde{\Delta}(p) > p-\frac{1}{2}$ for $p < p^{**}$ and $\tilde{\Delta}(p) < p-\frac{1}{2}$ for $p > p^{**}$.

Proof. (a) Since we assumed $l_0 \geq \kappa$, the planner's ex ante expected payoff from the policy with cutoff p and delay after REMAIN is $\tilde{V}_{PL}(p)$. Since $\tilde{V}_R(p)$ is a positive linear transformation of $\tilde{V}_{PL}(p)$ by the third equation in (B.9), and since the function $\tilde{V}_L$ does not depend on p by the first equation in (B.9), it follows from (B.10) that $\tilde{\Delta}(p)$ is a positive linear transformation of $\tilde{V}_{PL}(p)$.

(b) It is obvious from (B.11) that $\tilde{\Delta}(p)$ is continuous on $\mathbb{R}$ and constant on $(-\infty,0]$ and on $[1,\infty)$. For all $p \in (0,1)$, we have

$$\begin{aligned}
\tilde{\Delta}'(p) &= \frac{\delta^3 f(p)}{(1-\delta^2F(p))^2}\Big((\tfrac{1}{2}-\bar{p}) + \delta\int_0^p(\tfrac{1}{2}-p_t)\mathrm{d}F(p_t)\Big) + \frac{\delta^2}{1-\delta^2F(p)}(\tfrac{1}{2}-p)f(p)\\
&= -\frac{\delta^2 f(p)}{(1-\delta^2F(p))^2}\Big((p-\tfrac{1}{2}) + \delta(\bar{p}-\tfrac{1}{2}) - \delta^2\int_0^p (p-p_t)\mathrm{d}F(p_t)\Big)\\
&\overset{(B.8)}{=} -\frac{\delta^2 f(p)}{(1-\delta^2F(p))^2}\tilde{\phi}(p,\delta)
\end{aligned}$$

Since $\tilde{\phi}(p,\delta)$ is strictly increasing in p with a root at p^{**}, we obtain that $\tilde{\Delta}'(p) > 0$ for $p \in (0,p^{**})$ and that $\tilde{\Delta}'(p) < 0$ for $p \in (p^{**},1)$.

We have

$$\begin{aligned}
\tilde{\Delta}(p) - (p-\tfrac{1}{2}) &= \frac{1}{1-\delta^2F(p)}\Big(\delta(\tfrac{1}{2}-\bar{p}) + \delta^2\int_0^p(\tfrac{1}{2}-p_t)\mathrm{d}F(p_t) - (1-\delta^2F(p))(p-\tfrac{1}{2})\Big)\\
&= -\frac{1}{1-\delta^2F(p)}\Big((p-\tfrac{1}{2}) + \delta(\bar{p}-\tfrac{1}{2}) - \delta^2\int_0^p(p-p_t)\mathrm{d}F(p_t)\Big)\\
&\overset{(B.8)}{=} -\frac{1}{1-\delta^2F(p)}\tilde{\phi}(p,\delta).
\end{aligned}$$

Since $\tilde{\phi}(p,\delta)$ is strictly increasing in δ with a root at p^{**}, we obtain that $\tilde{\Delta}(p) > p-\frac{1}{2}$ for

$p < p^{**}$, that $\tilde{\Delta}(p^{**}) = p^{**} - \frac{1}{2}$, and that $\tilde{\Delta}(p) < p - \frac{1}{2}$ for $p > p^{**}$. □

Equilibrium Characterization for Consecutive κ-Majority Voting

We next establish properties of $\tilde{\Delta}(p)$ that are crucial for the equilibrium characterization.

Lemma B.5. *Suppose that*

$$\delta \leq \frac{\bar{p} - \frac{1}{2}}{\int_0^{1/2} F(p_t) dp_t}.$$

(a) Assume that $\delta < \frac{1}{2\bar{p}}$. Then $\tilde{\Delta}(p) \in (-\frac{1}{2}, 0]$ for all p.

(b) Assume that $\delta = \frac{1}{2\bar{p}}$. Then $\tilde{\Delta}^{-1}(-\frac{1}{2}) = 1$ and $\tilde{\Delta}(p) \in (-\frac{1}{2}, 0]$ for $p < 1$.

(c) Assume that $\delta > \frac{1}{2\bar{p}}$. Then $\tilde{\Delta}^{-1}(-\frac{1}{2})$ is uniquely defined and $\tilde{\Delta}^{-1}(-\frac{1}{2}) \in (p^{**}, 1)$. Moreover, $\tilde{\Delta}(p) \in (-\frac{1}{2}, 0]$ for $p < \tilde{\Delta}^{-1}(-\frac{1}{2})$, and $\tilde{\Delta}(p) < -\frac{1}{2}$ for $p > \tilde{\Delta}^{-1}(-\frac{1}{2})$.

Proof. *Step 1: Upper bound on $\tilde{\Delta}(p)$.* By Lemma B.4 (a), $\tilde{\Delta}(p) \leq \tilde{\Delta}(p^{**})$. Since $\tilde{\Delta}(p^{**}) = p^{**} - \frac{1}{2}$ and since $p^{**} \leq \frac{1}{2}$ for

$$\delta \leq \frac{\bar{p} - \frac{1}{2}}{\int_0^{1/2} F(p_t) dp_t},$$

we obtain that $\tilde{\Delta}(p) \leq 0$.

Step 2: Lower bound on $\tilde{\Delta}(p)$. By Lemma B.4 (b),

$$\min_p \tilde{\Delta}(p) = \min\{\tilde{\Delta}(0), \tilde{\Delta}(1)\}.$$

By (B.11),

$$\tilde{\Delta}(0) = \delta(\tfrac{1}{2} - \bar{p}) > -\tfrac{1}{2},$$

and

$$\tilde{\Delta}(1) = \frac{\delta}{1-\delta^2}((\tfrac{1}{2} - \bar{p}) + \delta(\tfrac{1}{2} - \bar{p})) = \frac{\delta}{1-\delta}(\tfrac{1}{2} - \bar{p}) = -\tfrac{1}{2} + \frac{\bar{p}}{1-\delta}(\frac{1}{2\bar{p}} - \delta).$$

We distinguish three cases:

Case i: $\delta < \frac{1}{2\bar{p}}$. Then, $\tilde{\Delta}(1) > -\frac{1}{2}$ such that $\min_p \tilde{\Delta}(p) > -\frac{1}{2}$.

Case ii: $\delta > \frac{1}{2\bar{p}}$. Then, $\tilde{\Delta}(1) < -\frac{1}{2}$. Since $\tilde{\Delta}(p)$ is continuous and since, by Lemma B.4 (b), $\tilde{\Delta}(p)$ is strictly increasing on $[0, p^{**}]$ and strictly decreasing on $[p^{**}, 1]$, we can apply the Intermediate Value Theorem to obtain a unique value $\tilde{\Delta}^{-1}(-\frac{1}{2}) \in (p^{**}, 1)$. Furthermore, it follows from Lemma B.4 (b) that $\tilde{\Delta}(p) \in (-\frac{1}{2}, 0]$ for $p < \tilde{\Delta}^{-1}(-\frac{1}{2})$ and $\tilde{\Delta}(p) < -\frac{1}{2}$ for $p > \tilde{\Delta}^{-1}(-\frac{1}{2})$.

Case iii: $\delta = \frac{1}{2\bar{p}}$. Then, $\tilde{\Delta}(1) = -\frac{1}{2}$. By a reasoning similar to that in the previous case, we obtain that $\tilde{\Delta}(p) > -\frac{1}{2}$ for $p < 1$ and that $\tilde{\Delta}(p) = -\frac{1}{2}$ for $p \geq 1$. □

We next establish that, under the supposition of Proposition 4, consecutive κ-majority voting has a unique equilibrium.

Lemma B.6. *Suppose that*

$$\delta \leq \frac{\bar{p} - \frac{1}{2}}{\int_0^{1/2} F(p_t)\, dp_t}.$$

Then, there exists a unique equilibrium $(\lambda_R, \lambda_{PL}(0), \lambda_{PL}(1))$ *under consecutive* κ*-majority voting:*

Case i: $\delta \in (0, \frac{1}{2\bar{p}})$**.** $(\lambda_R, \lambda_{PL}(0), \lambda_{PL}(1)) = (1, 0, 1)$.

Case ii: $\delta \in [\frac{1}{2\bar{p}}, 1)$ **and** $\kappa \leq \tilde{\Delta}^{-1}(-\frac{1}{2})$**.** $(\lambda_R, \lambda_{PL}(0), \lambda_{PL}(1)) = (1, 0, 1)$.

Case iii: $\delta \in (\frac{1}{2\bar{p}}, 1)$ **and** $\kappa > \tilde{\Delta}^{-1}(-\frac{1}{2})$**.** $(\lambda_R, \lambda_{PL}(0), \lambda_{PL}(1)) = (1, \frac{\kappa - \tilde{\Delta}^{-1}(-\frac{1}{2})}{1 - \tilde{\Delta}^{-1}(-\frac{1}{2})}, 1)$.[38]

Proof. *Step 1: For any* $(\lambda_R, \lambda_{PL}(0), \lambda_{PL}(1))$, $V_L = V_L^{**}$, $V_{PL} \leq V_{PL}^{**}$ *and* $V_R \leq V_R^{**}$. By using that $\mathbb{E}_{\pi_t^i|p_t}[\pi_t^i] = p_t$, the system (9) becomes

$$\begin{cases} V_L = \mathbb{E}_{p_t}[p_t + \delta V_L] \\ V_{PL} = \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t,PL)=L}(p_t + \delta V_L) + \mathbf{1}_{d(p_t,PL)=R}(\frac{1}{2} + \delta V_R)] \\ V_R = \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t,R)=PL}(\frac{1}{2} + \delta V_{PL}) + \mathbf{1}_{d(p_t,R)=R}(\frac{1}{2} + \delta V_R)] \end{cases} . \tag{B.12}$$

Since this system reduces to the system of Bellman equations when the optimal policy is used, V_d^{**} is an upper bound of V_d for any $d \in \{L, PL, R\}$. $V_L = V_L^{**}$ follows from the irreversibility of LEAVE.

Step 2: For any $\lambda_{PL}(0)$ *and any* λ_R*, only* $\lambda_{PL}(1) = 1$ *is optimal under pivotality considerations for the continuation values that the Markov strategy* $(\lambda_R, \lambda_{PL}(0), \lambda_{PL}(1))$ *induces.* By Step 1, $\delta(V_R - V_L) \leq \delta(V_R^{**} - V_L^{**})$. Since, by (B.7), $\delta(V_R^{**} - V_L^{**}) = p^{**} - \frac{1}{2}$, and since, by

[38]Note that, by Lemma B.5 (b), $\delta = \frac{1}{2\bar{p}}$ cannot hold together with $\kappa > \tilde{\Delta}^{-1}(-\frac{1}{2})$.

Lemma B.3, $\delta \le \frac{\bar{p}-\frac{1}{2}}{\int_0^{1/2} F(p_t)\mathrm{d}p_t}$ implies $p^{**} \le \frac{1}{2}$, we get $\delta(V_R - V_L) \le 0$. Thus, by (10), only $\lambda_{PL}(1) = 1$ is optimal under pivotality considerations.

Step 3: For any $\lambda_{PL}(0)$, only $\lambda_R = 1$ is optimal under pivotality considerations for the continuation values that the Markov strategy $(\lambda_R, \lambda_{PL}(0), 1)$ induces. Suppose that $\lambda_{PL}(1) = 1$. Assume to the contrary that some $\lambda_R < 1$ is optimal under pivotality considerations. By (10), optimality under pivotality considerations requires that either $V_R > V_{PL}$ in combination with $\lambda_R = 0$ or $V_R = V_{PL}$ in combination with $\lambda_R \in [0, 1)$. In either case, by the third equation in (9),

$$V_R = \tfrac{1}{2} + \delta V_R. \tag{B.13}$$

We will use this property to obtain a contradiction to $V_R \ge V_{PL}$.

By (8), the policy with cutoff

$$p(\lambda_{PL}(0)) \equiv \begin{cases} 0 & \text{if } \lambda_{PL}(0) \ge \kappa \\ \frac{\kappa - \lambda_{PL}(0)}{1-\lambda_{PL}(0)} & \text{if } \lambda_{PL}(0) < \kappa \end{cases} \tag{B.14}$$

and delay after REMAIN is induced. By plugging this policy in the second equation of (B.12), we obtain

$$\begin{aligned} V_{PL} &= F(p(\lambda_{PL}(0)))(\tfrac{1}{2} + \delta V_R) + (1 - F(p(\lambda_{PL}(0))))(\mathbb{E}_{p_t}[p_t | p_t \ge p(\lambda_{PL}(0))] + \delta V_L) \\ &\ge F(p(\lambda_{PL}(0)))(\tfrac{1}{2} + \delta V_R) + (1 - F(p(\lambda_{PL}(0))))(\bar{p} + \delta V_R^{**} + \tfrac{1}{2} - p^{**}) \\ &> F(p(\lambda_{PL}(0)))(\tfrac{1}{2} + \delta V_R) + (1 - F(p(\lambda_{PL}(0))))(\delta V_R^{**} + \tfrac{1}{2}) \\ &\ge \tfrac{1}{2} + \delta V_R \\ &= V_R. \end{aligned}$$

The first inequality follows from $\mathbb{E}_{p_t}[p_t | p_t \ge p(\lambda_{PL}(0))] \ge \bar{p}$, $\delta V_L = \delta V_L^{**}$, and $\delta V_L^{**} = \delta V_R^{**} + \frac{1}{2} - p^{**}$. The second inequality follows from two observations: first, $1 - F(p(\lambda_{PL}(0))) > 0$ for any $\lambda_{PL}(0) \in [0, 1]$; second, $\delta \le \frac{\bar{p}-\frac{1}{2}}{\int_0^{1/2} F(p_t)\mathrm{d}p_t}$ requires $\bar{p} > \frac{1}{2}$ and, by Lemma B.3, it implies $p^{**} \le \frac{1}{2}$. The third inequality follows since $V_R^{**} \ge V_R$. (B.13) implies the second equality. Since we have therewith obtained a contradiction to $V_R \ge V_{PL}$, we can conclude that only $\lambda_R = 1$ is optimal under pivotality considerations.

Step 4: Only

$$\lambda_{PL}(0) \;=\; \begin{cases} 0 & \text{if } \kappa \leq \tilde{\Delta}^{-1}(-\frac{1}{2}) \\ \frac{\kappa-\tilde{\Delta}^{-1}(-\frac{1}{2})}{1-\tilde{\Delta}^{-1}(-\frac{1}{2})} & \text{if } \kappa > \tilde{\Delta}^{-1}(-\frac{1}{2}) \end{cases}$$

is optimal under pivotality considerations for the continuation values that the Markov strategy $(1, \lambda_{PL}(0), 1)$ *induces.* Suppose that $\lambda_{PL}(1) = 1$ and $\lambda_R = 1$. Then, the policy with cutoff $p(\lambda_{PL}(0))$ and delay after REMAIN is induced. By plugging the structure of cutoff policies with delay after REMAIN in system (B.12), system (B.12) simplifies to the system (B.9). Thus, $\delta(V_R - V_L) = \tilde{\Delta}(p(\lambda_{PL}(0)))$. By (10), $\lambda_{PL}(0)$ is optimal under pivotality considerations if, and only if,

$$\begin{cases} \lambda_{PL}(0) = 1 & \text{if } \tilde{\Delta}(p(\lambda_{PL}(0))) < -\frac{1}{2} \\ \lambda_{PL}(0) \in [0,1] & \text{if } \tilde{\Delta}(p(\lambda_{PL}(0))) = -\frac{1}{2} \\ \lambda_{PL}(0) = 0 & \text{if } \tilde{\Delta}(p(\lambda_{PL}(0))) > -\frac{1}{2} \end{cases} . \tag{B.15}$$

We distinguish now the three cases:

Case i: $\delta \in (0, \frac{1}{2\bar{p}})$. Then, Lemma B.5 (a) implies $\tilde{\Delta}(p(\lambda_{PL}(0))) > -\frac{1}{2}$ such that $\lambda_{PL}(0) = 0$ by (B.15).

Case ii: $\delta \in [\frac{1}{2\bar{p}}, 1)$ *and* $\kappa \leq \tilde{\Delta}^{-1}(-\frac{1}{2})$. Since (B.14) implies $p(\lambda_{PL}(0)) \leq \kappa$, $p(\lambda_{PL}(0)) \leq \tilde{\Delta}^{-1}(-\frac{1}{2})$ (with a strict inequality if $\tilde{\Delta}^{-1}(-\frac{1}{2}) = 1$). It follows from Lemma B.5 (b) and (c) that $\tilde{\Delta}(p(\lambda_{PL}(0))) > -\frac{1}{2}$. Thus, by (B.15), $\lambda_{PL}(0) = 0$.

Case iii: $\delta \in (\frac{1}{2\bar{p}}, 1)$ *and* $\kappa > \tilde{\Delta}^{-1}(-\frac{1}{2})$. We prove that $\tilde{\Delta}(p(\lambda_{PL}(0))) = -\frac{1}{2}$ must hold.

Assume first to the contrary that $\tilde{\Delta}(p(\lambda_{PL}(0))) > -\frac{1}{2}$. Then, by (B.15), only $\lambda_{PL}(0) = 0$ is optimal under pivotality considerations. It follows from (B.14) that $p(\lambda_{PL}(0)) = \kappa$. However, since Lemma B.5 (c) implies that $\tilde{\Delta}(\kappa) < -\frac{1}{2}$ for $\kappa > \tilde{\Delta}^{-1}(-\frac{1}{2})$, we obtain a contradiction.

Assume next that $\tilde{\Delta}(p(\lambda_{PL}(0))) < -\frac{1}{2}$. Then, by (B.15), $\lambda_{PL}(0) = 1$. It follows from (B.14) that $p(\lambda_{PL}(0)) = 0$. However, since Lemma B.5 (c) implies that $\tilde{\Delta}(0) > -\frac{1}{2}$, we obtain a contradiction.

Hence, $p(\lambda_{PL}(0)) = \tilde{\Delta}^{-1}(-\frac{1}{2})$ must hold. Because, by Lemma B.5 (c), $\tilde{\Delta}^{-1}(-\frac{1}{2}) \in (p^{**}, 1)$ and, by Lemma B.3, $p^{**} > 0$, we obtain from (B.14) that $\frac{\kappa-\lambda_{PL}(0)}{1-\lambda_{PL}(0)} = \Delta^{-1}(-\frac{1}{2})$. This yields, $\lambda_{PL}(0) = \frac{\kappa-\tilde{\Delta}^{-1}(-\frac{1}{2})}{1-\tilde{\Delta}^{-1}(-\frac{1}{2})}$.

The four steps above establish that a unique candidate for an equilibrium exists, and that this candidate is indeed an equilibrium. □

Proof of Proposition 4

It follows from the equilibrium characterization above that consecutive κ-majority voting uniquely implements the policy with cutoff κ and delay after REMAIN if $\delta < \frac{1}{2\bar{p}}$, and the policy with cutoff $\min\{\kappa, \tilde{\Delta}^{-1}(-\frac{1}{2})\}$ and delay after REMAIN if $\delta \geq \frac{1}{2\bar{p}}$. In any case, $\kappa = \frac{1}{2}$ uniquely implements the smallest implementable cutoff.

It follows from our analysis of the constraint optimal policy p^{**} that, in our case of interest with $\delta \leq \frac{\bar{p}-\frac{1}{2}}{\int_0^{1/2} F(p_t)\mathrm{d}p_t}$, $p^{**} \in (0, \frac{1}{2}]$ (Lemma B.3). Since, by Lemma B.5 (b) and (c), $p^{**} < \tilde{\Delta}^{-1}(-\frac{1}{2})$, the smallest implementable cutoff is larger than p^{**}.

By our analysis of the comparison of non-optimal cutoff policies with delay after REMAIN, $\tilde{\Delta}(p)$ is strictly decreasing on $[p^{**}, 1]$ (Lemma B.4 (b)), and the planner weakly prefers the policy with cutoff p and delay after REMAIN over the policy with cutoff p' and delay after REMAIN if, and only if, $\tilde{\Delta}(p) \geq \tilde{\Delta}(p')$ (Lemma B.4 (a)). This allows us to conclude that the cutoff policy with delay after REMAIN implemented by $\kappa = \frac{1}{2}$ is the best implementable cutoff policy with delay after REMAIN.

Proofs for Section 6

Recall that, in this section, we consider the version of the model where votes can be for REMAIN or for LEAVE, but where three decisions are possible in each period, LEAVE (L), REMAIN (R) and REMAIN forever ($R\infty$).

Sensible Beliefs under Voting by Two-Sided Supermajority

In our model with a continuum of agents, each pivotality event occurs with probability 0 and Bayes Law is not applicable. In order to obtain an understanding of what reasonable beliefs are, we consider now the version of our model with a large, finite number of n agents, and we derive how the Bayesian belief behave in the limit. This will motivate our notion of sensible beliefs in our model with a continuum of agents.

Let $m_n^L \equiv \lceil \kappa n \rceil - 1$ and $m_n^R \equiv \lceil (1-\kappa)n \rceil - 1$. If m_n^L other agents vote for LEAVE, i is pivotal for LEAVE reaching the κ-majority; if m_n^R others vote for LEAVE, he is pivotal for REMAIN reaching the κ-majority. Suppose that agent i believes that each other agent votes for LEAVE with probability $k \in (0,1)$. His Bayesian belief about the pivotality scenario is then $\mu = \mu_n(k)$ where

$$\mu_n(k) \equiv \frac{\binom{n-1}{m_n^L} k^{m_n^L}(1-k)^{n-1-m_n^L}}{\binom{n-1}{m_n^R} k^{m_n^R}(1-k)^{n-1-m_n^R} + \binom{n-1}{m_n^L} k^{m_n^L}(1-k)^{n-1-m_n^L}}. \tag{B.16}$$

For $k \in \{0,1\}$, we employ the consistency notion from Kreps and Wilson (1982): The belief μ_n is *consistent* with the probability $k_n \in [0,1]$ in the finite version of the model if there exists a sequence $(k_{n,\tau})_\tau$ in $(0,1)$ with $\lim_{\tau\to\infty} k_{n,\tau} = k_n$ such that $\lim_{\tau\to\infty} \mu_n(k_{n,\tau}) = \mu_n$. We say that the limit belief μ is *consistent* with the limit probability $k \in [0,1]$ if there exists a sequence of probabilities $(k_n)_n$ in $[0,1]$ with $\lim_{n\to\infty} k_n = k$ and a sequence of beliefs $(\mu_n)_n$ with $\lim_{n\to\infty} \mu_n = \mu$ such that, for all n, μ_n is consistent with k_n in the finite version of the model.

Lemma B.7. *For any $\kappa \in (\frac{1}{2}, 1)$, the limit belief $\mu \in [0,1]$ is consistent with the limit probability $k \in [0,1]$ if, and only if,*

$$\begin{cases} \mu = 1 & \text{if } k > \frac{1}{2} \\ \mu \in [0,1] & \text{if } k = \frac{1}{2} \\ \mu = 0 & \text{if } k < \frac{1}{2} \end{cases}.$$

Proof. For $k \in (0,1)$, we can rewrite (B.16) as

$$\begin{aligned} \mu_n(k) &= \frac{\frac{\binom{n-1}{m_n^L}}{\binom{n-1}{m_n^R}} \left(\frac{k}{1-k}\right)^{m_n^L - m_n^R}}{1 + \frac{\binom{n-1}{m_n^L}}{\binom{n-1}{m_n^R}} \left(\frac{k}{1-k}\right)^{m_n^L - m_n^R}} \\ &= \frac{\frac{(n-1-m_n^R)!}{m_n^L!} \frac{m_n^R!}{(n-1-m_n^L)!} \left(\left(\frac{k}{1-k}\right)^{(m_n^L - m_n^R)/n}\right)^n}{1 + \frac{(n-1-m_n^R)!}{m_n^L!} \frac{m_n^R!}{(n-1-m_n^L)!} \left(\left(\frac{k}{1-k}\right)^{(m_n^L - m_n^R)/n}\right)^n}. \end{aligned} \tag{B.17}$$

Step 1: Only belief $\mu_n = 0$ ($\mu_n = 1$) is consistent with probability $k = 0$ ($k = 1$) in the finite version of the model. Fix any n. Let $(k_{n,\tau})_\tau$ be any sequence in $(0,1)$ with $\lim_{\tau\to\infty} k_{n,\tau} = 0$ ($\lim_{\tau\to\infty} k_{n,\tau} = 1$). $\lim_{\tau\to\infty} \mu_n(k_{n,\tau}) = 0$ ($\lim_{\tau\to\infty} \mu_n(k_{n,\tau}) = 1$) immediately follows from (B.17).

Step 2: Implications of consistency of limit beliefs for probability sequences $(k_n)_n$ with $k_n =$

k for all n: $k \in (0, \frac{1}{2})$ implies $\lim_{n\to\infty} \mu_n(k) = 0$, $k = \frac{1}{2}$ implies $\lim_{n\to\infty} \mu_n(k) = \frac{1}{2}$, and $k \in (\frac{1}{2}, 1)$ implies $\lim_{n\to\infty} \mu_n(k) = 1$. Fix any $k \in (0,1)$. We note that $\lim_{n\to\infty} \frac{(n-1-m_n^R)!}{m_n^L!} = 1$, $\lim_{n\to\infty} \frac{m_n^R!}{(n-1-m_n^L)!} = 1$, and $\lim_{n\to\infty} \left(\frac{k}{1-k}\right)^{(m_n^L - m_n^R)/n} = \left(\frac{k}{1-k}\right)^{2\kappa-1}$. Since $k \in (0, \frac{1}{2})$ implies $\left(\frac{k}{1-k}\right)^{2\kappa-1} < 1$, we obtain from (B.17) that $\lim_{n\to\infty} \mu_n(k) = 0$. Analogously, since $k \in (\frac{1}{2}, 1)$ implies $\left(\frac{k}{1-k}\right)^{2\kappa-1} > 1$, we obtain from (B.17) that $\lim_{n\to\infty} \mu_n(k) = 1$. Finally, since $k = \frac{1}{2}$ implies $\left(\frac{k}{1-k}\right)^{m_n^L - m_n^R} = 1$, we obtain from (B.17) that $\lim_{n\to\infty} \mu_n(k) = \frac{1}{2}$.

Step 3: Implications of consistency of limit beliefs for probability sequences $(k_n)_n$.

Step 3.1: For any $k \in [0, \frac{1}{2})$ ($k \in (\frac{1}{2}, 1]$), only the limit belief $\mu = 0$ ($\mu = 1$) is consistent with the limit probability k. Let $(k_n)_n$ be a sequence in $[0,1]$ with $\lim_{n\to\infty} k_n = k$ and let $(\mu_n)_n$ be a sequence in $[0,1]$ with $\lim_{n\to\infty} \mu_n = \mu$ such that, for each n, belief μ_n is consistent with probability k_n in the finite version of the model. If follows from Steps 1 and 2 that $\lim_{n\to\infty} \mu_n(k_n) = 0$ if $k < \frac{1}{2}$ and that $\lim_{n\to\infty} \mu_n(k_n) = 1$ if $k > \frac{1}{2}$.

Step 3.2: Any limit belief $\mu \in (0,1)$ is consistent with the limit probability $k = \frac{1}{2}$. Since $\mu_n(k)$ is strictly increasing and continuous in k with $\lim_{k\to 0} \mu_n(k) = 0$ and $\lim_{k\to 1} \mu_n(k) = 1$, for any $\mu \in (0,1)$, there exists by the Intermediate Value Theorem a unique value $\mu_n^{-1}(\mu)$. Set $k_n = \mu_n^{-1}(\mu)$ and $\mu_n = \mu_n(k_n)$. By construction, $\lim_{n\to\infty} \mu_n = \mu$. Moreover, by Step 2, $\lim_{n\to\infty} k_n = \frac{1}{2}$.

Step 3.3: Any limit belief $\mu \in \{0,1\}$ is consistent with the limit probability $k = \frac{1}{2}$. Consider $k_n = \frac{1}{1+n^{1/(m_n^L - m_n^R)}}$ and $\mu_n = \mu_n(k_n)$. Then, since $\lim_{n\to\infty} n^{1/(m_n^L - m_n^R)} = 1$, $\lim_{n\to\infty} k_n = \frac{1}{2}$. Moreover,

$$\begin{aligned} \mu_n \le \frac{2}{n} \quad &\Leftrightarrow \quad \frac{n-2}{n} \frac{(n-1-m_n^R)!}{m_n^L!} \frac{m_n^R!}{(n-1-m_n^L)!} \left(\frac{k_n}{1-k_n}\right)^{m_n^L - m_n^R} \le \frac{2}{n} \\ &\Leftrightarrow \quad \frac{n-2}{n} \frac{(n-1-m_n^R)!}{m_n^L!} \frac{m_n^R!}{(n-1-m_n^L)!} \frac{1}{n} \le \frac{2}{n} \\ &\Leftrightarrow \quad \frac{n-2}{n} \frac{(n-1-m_n^R)!}{m_n^L!} \frac{m_n^R!}{(n-1-m_n^L)!} \le 2. \end{aligned} \tag{B.18}$$

The first equivalence follows from plugging the definition of $\mu_n(k_n)$ in (B.17) and from simplifying; the second equivalence follows from using the definition of k_n and simplifying. Since the left-hand side of (B.18) converges to 1 as $n \to \infty$, we have shown that the limit belief $\mu = 0$ is consistent with the limit probability $k = \frac{1}{2}$. A similar construction can be used to show that the limit belief $\mu = 1$ is consistent with the limit probability $k = \frac{1}{2}$. $\square$

Proof of Proposition 5

We first formally introduce the continuation values implied by a Markov strategy and the meaning of optimality under pivotality considerations: If all agents vote according to the Markov strategy $(\lambda(0, p_t), \lambda(1, p_t))$, the decision is

$$d(p_t) = \begin{cases} L & \text{if } l_t \geq \kappa \\ R & \text{if } l_t \in (1-\kappa, \kappa) \\ R\infty & \text{if } l_t \leq 1-\kappa \end{cases} \text{ with } l_t = p_t \lambda(1, p_t) + (1-p_t)\lambda(0, p_t). \tag{B.19}$$

Consider the system of linear equations

$$\begin{cases} V_L = \mathbb{E}_{p_t}[\mathbb{E}_{\pi_t^i|p_t}[\pi_t^i + \delta V_L]] \\ V_R = \mathbb{E}_{p_t}[\mathbb{E}_{\pi_t^i|p_t}[\mathbf{1}_{d(p_t)=L}(\pi_t^i + \delta V_L) + \mathbf{1}_{d(p_t)=R}(\frac{1}{2} + \delta V_R) + \mathbf{1}_{d(p_t)=R\infty}(\frac{1}{2} + \delta V_{R\infty})]] \\ V_{R\infty} = \frac{1}{2} + \delta V_{R\infty} \end{cases} \tag{B.20}$$

that is implied by policy (B.19). It has a unique solution.[39] δV_d describes the continuation value from decision $d \in \{R\infty, R, L\}$, common to all agents.

$(\lambda(0, p_t), \lambda(1, p_t))$ is optimal under pivotal voting if and only if, for any $\pi_t^i \in \{0, 1\}$ and any $p_t \in [0, 1]$,

$$\begin{cases} \lambda(\pi_t^i, p_t) = 1 & \text{if } \mu(p_t)(\pi_t^i + \delta V_L) + (1-\mu(p_t))(\frac{1}{2} + \delta V_R) \\ & \quad > \mu(p_t)(\frac{1}{2} + \delta V_R) + (1-\mu(p_t))(\frac{1}{2} + \delta V_{R\infty}) \\ \lambda(\pi_t^i, p_t) \in [0,1] & \text{if } \mu(p_t)(\pi_t^i + \delta V_L) + (1-\mu(p_t))(\frac{1}{2} + \delta V_R) \\ & \quad = \mu(p_t)(\frac{1}{2} + \delta V_R) + (1-\mu(p_t))(\frac{1}{2} + \delta V_{R\infty}) \\ \lambda(\pi_t^i, p_t) = 0 & \text{if } \mu(p_t)(\pi_t^i + \delta V_L) + (1-\mu(p_t))(\frac{1}{2} + \delta V_R) \\ & \quad < \mu(p_t)(\frac{1}{2} + \delta V_R) + (1-\mu(p_t))(\frac{1}{2} + \delta V_{R\infty}) \end{cases}. \tag{B.21}$$

[39] The system can be rewritten as

$$\begin{pmatrix} 1-\delta & 0 & 0 \\ -\delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=L}] & 1-\delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}] & -\delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R\infty}] \\ 0 & 0 & 1-\delta \end{pmatrix} \begin{pmatrix} V_L \\ V_R \\ V_{R\infty} \end{pmatrix} = \begin{pmatrix} \mathbb{E}_{p_t}[p_t] \\ \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=L} p_t + \mathbf{1}_{d(p_t)\in\{R,R\infty\}} \frac{1}{2}] \\ \frac{1}{2} \end{pmatrix}.$$

Since $\delta \in (0, 1)$ implies that the matrix has full rank, a unique solution exists.

Next, we argue that, in any REMAIN-friendly environment, $\delta^{TM} \in (\delta^*, 1)$. $\delta^{TM} < 1$ is obvious. Since

$$\delta^* < \delta^{TM} \Leftrightarrow \frac{1}{2(1-\bar{p})} < \frac{1}{1+F(\frac{1}{2})(1-2\bar{p})} \Leftrightarrow F(\tfrac{1}{2})(1-2\bar{p}) < 1-2\bar{p},$$

$\bar{p} < \frac{1}{2}$ implies $\delta^* < \delta^{TM}$.

Suppose that the environment is REMAIN-friendly and that $\delta > \delta^{TM}$. We argue in two steps.

Step 1: The optimal policy is implementable. Since $\delta > \delta^{TM}$ implies $\delta > \delta^*$, it is optimal to never LEAVE (by Lemma 1). Thus, if the Markov strategy $(\lambda(0, p_t), \lambda(1, p_t)) = (0, 0)$ is part of an equilibrium, the optimal policy is implementable. This Markov strategy implies $V_R = V_{R\infty}$ (by (B.20)) and $\mu(p_t) = 0$ for all p_t (by (11)). It follows that, for any p_t, an agent believes to be pivotal between REMAIN for now and REMAIN FOREVER; that is, he compares $\frac{1}{2} + V_R$ with $\frac{1}{2} + V_{R\infty}$. Because $V_R = V_{R\infty}$, he is indifferent and $\lambda(\pi_t^i, p_t) = 0$ is optimal for any π_t^i and any p_t under pivotality considerations for the continuation values and the belief system that it induces.

Step 2: No other policy is implementable. Assume to the contrary that there exists an equilibrium that implements a non-optimal policy. Since the social planner strives to maximize V_R, $V_R < V_R^* = \frac{1}{1-\delta}\frac{1}{2}$ must be true in such an equilibrium. Since it is optimal to never LEAVE, any postponement of LEAVE is socially beneficial. This implies that $V_R \geq V_L$. It follows that

$$0 + \delta V_L < \tfrac{1}{2} + \delta V_R \text{ and } \tfrac{1}{2} + \delta V_R < \tfrac{1}{2} + \delta V_{R\infty}.$$

Hence, no matter in which event a LEAVE-loser believes to be pivotal, he has by (B.21) a strict incentive to vote for REMAIN; that is, only $\lambda(0, p_t) = 0$ is optimal under pivotality considerations.

$\lambda(0, p_t) = 0$ implies $l_t = p_t \lambda(1, p_t) \leq p_t$. For given p_t, we distinguish now three cases and argue that, in each case, the decision is not LEAVE. This yields a contradiction to a non-optimal policy being implemented:

Case i: $\lambda(1, p_t) < \frac{1}{2p_t}$. Then, $l_t < \frac{1}{2}$ and, by (11), $\mu(p_t) = 0$. As this means that LEAVE-winners compare $\frac{1}{2} + \delta V_R$ with $\frac{1}{2} + \delta V_{R\infty}$, only $\lambda(1, p_t) = 0$ is optimal under pivotal voting by our observation in the first paragraph of Step 2 and by (B.21). Hence, $d(p_t) = R\infty$.

Case ii: $\lambda(1, p_t) > \frac{1}{2p_t}$. Then, $l_t > \frac{1}{2}$ and only $\mu(p_t) = 1$ is by (11) consistent with the Markov strategy. By (B.21), $\lambda(1, p_t) > 0$ requires $1 + \delta V_L \geq \frac{1}{2} + \delta V_R$. Because, for any $p_t < \frac{1}{2}$, Case i applies, we obtain the following lower bound on V_R:

$$V_R \geq F(\tfrac{1}{2}) V_{R\infty} + (1 - F(\tfrac{1}{2})) V_L.$$

Necessary for $1 + \delta V_L \geq \frac{1}{2} + \delta V_R$ is thus

$$\begin{aligned} 1 + \delta V_L \geq \tfrac{1}{2} + \delta(F(\tfrac{1}{2}) V_{R\infty} + (1 - F(\tfrac{1}{2})) V_L) \quad &\Leftrightarrow \quad \tfrac{1}{2} \geq \delta F(\tfrac{1}{2})(V_{R\infty} - V_L) \\ &\Leftrightarrow \quad \tfrac{1}{2} \geq \delta F(\tfrac{1}{2}) \tfrac{1}{1-\delta}(\tfrac{1}{2} - \bar{p}) \\ &\Leftrightarrow \quad \delta \leq \delta^{TM}. \end{aligned}$$

As this violates our assumption that $\delta > \delta^{TM}$, Case ii cannot occur.

Case iii: $\lambda(1, p_t) = \frac{1}{2p_t}$. Then, $l_t = \frac{1}{2}$ such that $d(p_t) = R$.

Proof of Proposition 6

Suppose that $\delta < \delta^*$. By Lemma 1, a cutoff policy $p^* \in (\frac{1}{2}, 1)$ is optimal. Consider two-sided p^*-majority voting. We will argue that the Markov strategy

$$(\lambda(0, p_t), \lambda(1, p_t)) = \begin{cases} (\frac{1-2p_t}{2-2p_t}, 1) & p_t \leq \frac{1}{2} \\ (0, 1) & p_t > \frac{1}{2} \end{cases} \tag{B.22}$$

together with some belief system constitutes an equilibrium. If each agent votes according to this strategy, the mass of LEAVE-votes is $l_t = \max\{\frac{1}{2}, p_t\}$. It follows that, on the equilibrium path, the decision is never $R\infty$ and that LEAVE is chosen in the first period such that $p_t \geq p^*$. That is, the optimal policy is implemented.

It remains to argue that there exists a belief system which is consistent with the Markov strategy, and for which the Markov strategy is optimal. Consider a period t such that $d_{t-1} = R$. We distinguish two cases:

Case 1: $p_t > \frac{1}{2}$. Then, $l_t = p_t$ and, by (11), only $\mu(p_t) = 1$ is consistent with the Markov strategy (B.22). As this means than an agent believes to be pivotal for LEAVE reaching the p^*-majority, he faces the same trade-off as under normal majority voting; i.e., he compares $p_t+\delta V_L$ with $\frac{1}{2}+\delta V_R$. Since the Markov strategy implies that future decisions are taken according to the optimal policy, we obtain that $\delta(V_R - V_L) = \delta(V_R^* - V_L^*) = \Delta^*$. Since $\Delta^* \in (0, \frac{1}{2})$ for $p^* \in (\frac{1}{2}, 1)$, myopic voting is indeed optimal.

Case 2: $p_t \leq \frac{1}{2}$. Then, $l_t = \frac{1}{2}$ and, by (11), any $\mu(p_t) \in [0,1]$ is consistent with the Markov strategy (B.22). By (B.21), it suffices to argue that there exists some $\mu(p_t) \in [0,1]$ such that a LEAVE-loser is indifferent between voting for LEAVE and for REMAIN for the continuation values the Markov strategy induces:

$$\begin{aligned} & \mu(p_t)(0+\delta V_L) + (1-\mu(p_t))(\tfrac{1}{2}+\delta V_R) \\ = \; & \mu(p_t)(\tfrac{1}{2}+\delta V_R) + (1-\mu(p_t))(\tfrac{1}{2}+\delta V_{R\infty}). \end{aligned}$$

Since the Markov strategy (B.22) implies that future decisions are taken according to the optimal policy, $V_R \geq V_L$ and $V_R \geq V_{R\infty}$ must be true because REMAIN has an option value both relative to LEAVE and relative to REMAIN FOREVER. It follows that

$$0+\delta V_L < \tfrac{1}{2}+\delta V_R \text{ and } \tfrac{1}{2}+\delta V_R \geq \tfrac{1}{2}+\delta V_{R\infty}.$$

Hence, there exists some $\mu(p_t) \in [0,1]$ such that the equality holds.

Proofs for Extension I

Proof of Lemma 4

By subtracting the first equation in (A.2) from the second equation, multiplying both sides of the resulting equation by δ, and using the definition of $\Delta(\pi_{t-1}^i, p_{t-1})$, we obtain

$$\begin{aligned} \Delta(\pi_{t-1}^i, p_{t-1}) &= \delta \mathbb{E}_{p_t}[\mathbb{E}_{\pi_t^i|\pi_{t-1}^i, p_{t-1}, p_t}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - \pi_t^i + \Delta(\pi_t^i, p_t))] \\ &= \delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}\mathbb{E}_{\pi_t^i|\pi_{t-1}^i, p_{t-1}, p_t}[(\tfrac{1}{2} - \pi_t^i + \Delta(\pi_t^i, p_t))]] \\ &= \delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - q_{\pi_{t-1}^i}(p_{t-1}, p_t) \\ &\qquad + q_{\pi_{t-1}^i}(p_{t-1}, p_t)\Delta(1, p_t) + (1 - q_{\pi_{t-1}^i}(p_{t-1}, p_t))\Delta(0, p_t))]. \quad \text{(B.23)} \end{aligned}$$

The future advantage of REMAIN induced by the same policy in the original model is determined by the equation

$$\Delta = \delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - p_t + \Delta)]. \tag{B.24}$$

Define

$$\begin{aligned} \bar{\Delta}(p_{t-1}) &\equiv p_{t-1}\Delta(1,p_{t-1}) + (1-p_{t-1})\Delta(0,p_{t-1}) \\ &= \delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - p_t + p_t\Delta(1,p_t) + (1-p_t)\Delta(0,p_t))] \\ &= \delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - p_t + \bar{\Delta}(p_t))]. \end{aligned} \tag{B.25}$$

The equality in the second line follows from using the definition of $\Delta(\pi^i_{t-1}, p_{t-1})$ in (B.23) and from Assumption 2. Since the right-hand side does not depend on p_{t-1}, $\bar{\Delta}(p_{t-1})$ must be constant, say $\bar{\Delta}$. Since $\bar{\Delta}(p_{t-1}) = \bar{\Delta}$ implies that $\bar{\Delta}$ is characterized through the same equation as Δ, i.e., through (B.24), we have shown that $\bar{\Delta}(p_{t-1}) = \Delta$.

It follows from (B.25) with $\bar{\Delta}(p_{t-1}) = \Delta$ that, for each p_{t-1}, either $\Delta(0,p_{t-1}) \leq \Delta \leq \Delta(1,p_{t-1})$ or $\Delta(0,p_{t-1}) \geq \Delta \geq \Delta(1,p_{t-1})$. We conclude the proof of this Lemma by showing that $\Delta(0,p_{t-1}) \geq \Delta(1,p_{t-1})$ for any p_{t-1}.

We have

$$\begin{aligned} & \Delta(0,p_{t-1}) - \Delta(1,p_{t-1}) \\ \overset{(B.23)}{=} \; & \delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - q_0(p_{t-1},p_t) + q_0(p_{t-1},p_t)\Delta(1,p_t) + (1-q_0(p_{t-1},p_t))\Delta(0,p_t))] \\ & -\delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}(\tfrac{1}{2} - q_1(p_{t-1},p_t) + q_1(p_{t-1},p_t)\Delta(1,p_t) + (1-q_1(p_{t-1},p_t))\Delta(0,p_t))] \\ = \; & \delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}(q_1(p_{t-1},p_t) - q_0(p_{t-1},p_t))] \\ & +\delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}(q_1(p_{t-1},p_t) - q_0(p_{t-1},p_t))(\Delta(0,p_t) - \Delta(1,p_t))]. \end{aligned} \tag{B.26}$$

Since Assumption 1 implies $\mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}(q_1(p_{t-1},p_t) - q_0(p_{t-1},p_t))] \geq 0$, we obtain

$$\begin{aligned} & \Delta(0,p_{t-1}) - \Delta(1,p_{t-1}) \\ \geq \; & \delta \mathbb{E}_{p_t}[\mathbf{1}_{d(p_t)=R}(q_1(p_{t-1},p_t) - q_0(p_{t-1},p_t))(\Delta(0,p_t) - \Delta(1,p_t))]. \end{aligned} \tag{B.27}$$

Substituting $t+1$ for t in (B.27), we obtain a lower bound on $\Delta(0,p_t) - \Delta(1,p_t)$ that can be used in (B.27) to obtain an even smaller lower bound on $\Delta(0, p_{t-1}) - \Delta(1, p_{t-1})$. We obtain

$$\begin{aligned} & \Delta(0, p_{t-1}) - \Delta(1, p_{t-1}) \\ \geq \;& \delta^2 \mathbb{E}_{p_t}[\mathbb{E}_{p_{t+1}}[\left(\prod_{t'=t}^{t+1} \mathbf{1}_{d(p_{t'})=R}(q_1(p_{t'-1}, p_{t'}) - q_0(p_{t'-1}, p_{t'}))\right)(\Delta(0, p_{t+1}) - \Delta(1, p_{t+1}))]]. \end{aligned}$$

By repeatedly applying this logic k times, the lower bound becomes

$$\begin{aligned} & \Delta(0, p_{t-1}) - \Delta(1, p_{t-1}) \\ \geq \;& \delta^{k+1} \mathbb{E}_{p_t}[\cdots \mathbb{E}_{p_{t+k}}[\left(\prod_{t'=t}^{t+k} \mathbf{1}_{d(p_{t'})=R}(q_1(p_{t'-1}, p_{t'}) - q_0(p_{t'-1}, p_{t'}))\right)(\Delta(0, p_{t+k}) - \Delta(1, p_{t+k}))] \cdots]. \end{aligned}$$

Since, in each single period, the stage payoff from LEAVE is by at most $\frac{1}{2}$ different from the stage payoff from REMAIN, $\Delta(0, p_t) - \Delta(1, p_t)$ is bounded for given δ. Specifically, we have $|\Delta(0, p_t) - \Delta(1, p_t)| \leq \frac{1}{1-\delta}$. This implies that the lower bound on $\Delta(0, p_{t-1}) - \Delta(1, p_{t-1})$ converges to 0 as $k \to \infty$. Hence, $\Delta(0, p_{t-1}) - \Delta(1, p_{t-1})$ cannot be negative.

Proofs for Extension II

Derivation of the $(1-\kappa)$-quantile Agent's Stage Payoff, $\pi^{(1-\kappa)}(p_t)$

Define

$$H(\pi|p_t) \equiv \mathbb{P}_{\pi_t^i|p_t}\{\pi_t^i \leq \pi\} = \begin{cases} (1-\alpha) - p_t + 2\alpha G(\pi) & \text{if } \pi \in [0, 1) \\ 1 & \text{if } \pi = 1 \end{cases}.$$

We have then $\pi^{(1-\kappa)}(p_t) = 1$ if $\lim_{\pi \to 1} H(\pi|p_t) < 1 - \kappa$ and $\pi^{(1-\kappa)}(p_t) = 0$ if $H(0|p_t) > 1 - \kappa$. Otherwise, $\pi^{(1-\kappa)}(p_t)$ is the unique solution π to the equation $H(\pi|p_t) = 1 - \kappa$. We get

$$\pi^{(1-\kappa)}(p_t) = \begin{cases} 1 & \text{if } p_t > \kappa + \alpha \\ G^{-1}(\frac{p_t + \alpha - \kappa}{2\alpha}) & \text{if } \kappa - \alpha \leq p_t \leq \kappa + \alpha \\ 0 & \text{if } p_t < \kappa - \alpha \end{cases}. \tag{B.28}$$

Auxiliary Result for the Proof of Lemma 5

Lemma B.8. *(a) If g is quasi-convex, then $\pi \leq G^{-1}(\pi)$ for all $\pi \in [\frac{1}{2}, 1]$.*

(b) If g is strictly quasi-concave, then $g(\frac{1}{2}) > 1$.

Proof. (a) Quasi-convexity of g and symmetry of g around $\frac{1}{2}$ imply that g is weakly increasing on $[\frac{1}{2}, 1]$ which in turn implies that G is weakly convex on $[\frac{1}{2}, 1]$. It follows from this together with $G(\frac{1}{2}) = \frac{1}{2}$ and $G(1) = 1$ that, for all $\pi \in [\frac{1}{2}, 1]$, $G(\pi) \leq \pi$ and thus, $\pi \leq G^{-1}(\pi)$.

(b) Strict quasi-concavity of g and symmetry of g around $\frac{1}{2}$ imply that g is non-constant and attains its maximum at $\frac{1}{2}$. As the maximum of a non-constant density with support $[0, 1]$ must be larger than 1, we obtain $g(\frac{1}{2}) > 1$. □

Proof of Lemma 5

(a) Suppose that g is quasi-convex. If $p_t \in (\frac{1}{2} + \alpha, 1 - \alpha]$, (A.3) holds because $\pi^{(1/2)}(p_t) = 1$. It remains to consider the case with $p_t \in (\frac{1}{2}, \min\{\frac{1}{2} + \alpha, 1 - \alpha\}]$. We need to show that $G^{-1}(\frac{p_t+\alpha-1/2}{2\alpha}) > p_t$. By Lemma B.8 (a), $\frac{p_t+\alpha-1/2}{2\alpha} > p_t$ is sufficient for this. Since this inequality simplifies to $p_t > \frac{1}{2}$, the desired result follows.

(b) Suppose that g is strictly quasi-concave. (B.28) and symmetry of g around $\frac{1}{2}$ together imply $G^{-1}(\frac{1}{2}) = \frac{1}{2}$ such that $\pi^{(1/2)}(\frac{1}{2}) = \frac{1}{2}$. It suffices thus to show that there exists $\hat{\alpha} \in (0, \frac{1}{2})$ such that for all $\alpha \in (\hat{\alpha}, \frac{1}{2})$, $\frac{\mathrm{d}}{\mathrm{d}p_t}\pi^{(1/2)}(\frac{1}{2}) < 1$. We have

$$\frac{\mathrm{d}}{\mathrm{d}p_t}\pi^{(1/2)}(\tfrac{1}{2}) = \frac{1}{2\alpha}\frac{1}{g(G^{-1}(\frac{1/2+\alpha-1/2}{2\alpha}))} = \frac{1}{2\alpha}\frac{1}{g(\frac{1}{2})}.$$

By Lemma B.8 (b), $g(\frac{1}{2}) > 1$ and the result follows.

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