Becoming a Bad Doctor*

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Abstract

We analyze a market with n rational firms (doctors) and a continuum of boundedly rational consumers (patients). Following Spiegler (2006a), we assume that patients are not familiar with the market and rely on anecdotes.
We analyze the price setting game played by doctors with given, different healing qualities. Doctors know their own quality, as well as the qualities of their competitors. In the unique equilibrium all doctors, no matter how bad, earn positive profits.
If doctors (even costlessly) choose their qualities, doctors mainly offer mediocre qualities in all SPNE. Welfare may strictly decrease in the number of doctors.

JEL Classification: D03, L13, L15

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1 Introduction

A consumer who turns to an unfamiliar market may rely on the advice of other consumers. Anticipating such search behavior, it may be beneficial for firms to commit on low qualities as a product differentiation strategy, even if raising quality is costless. With low qualities, every single firm is considered by less consumers. Yet the firm has to compete for potential consumers less fiercely, as these take only few other firms into consideration. This is the situation explored in this paper.

There are plenty of markets in which consumers are not fully aware of the different qualities of specialists and rely on anecdotal evidence. Whereas consumers do not know well how the market they are in exactly looks like, specialists know the market perfectly: They know how good they are, how good their competitors are, and how consumers use to search for them. Specialists act rationally, while consumers apply a rule of anecdotal reasoning.

Our running example is given by a patient thinking about consulting some doctor because a friend got cured at that doctor. Like in Spiegler's "Market for Quacks" (2006a) on which our model is based, patients are assumed to rely on the experiences of others in order to judge the qualities of different doctors. Each patient asks one former client of each doctor whether the doctor cured him or not. A positive report makes the patient believe that that doctor will cure him as well - whereas a negative report makes the patient shy away from that doctor. Among the recommended doctors, patients choose the cheapest one.

Another example is given by the market of piano-teaching: Parents, not knowing how to play the piano themselves, rely on recommendations to find a good piano teacher for their children. Further applications are the markets for financial advice, personal coaching, business consulting, special repair services, or spiritual guidance.

We define the quality of a doctor as the probability with which he can help a patient. If patients ask one client of each doctor while searching for a good treatment, the probability that a doctor is recommended to a patient is given by his success probability. We first assume that qualities are given exogenously and analyze the price setting behavior of the doctors. Later we allow doctors to choose their qualities. Thinking about choosing a quality as a process of specialization, it is natural to assume that doctors first choose their quality, then set their prices.

In the pricing game with exogenous, asymmetric qualities, we find that even the worst doctors survive in the market, and earn equilibrium payoffs much larger than their maxmin payoffs. The reason is that good doctors do not feel threatened enough by bad doctors to set low prices. Instead good doctors mix over rather high prices, leaving room for bad doctors to earn considerable payoffs.[1] No doctor has an incentive to reveal his true quality: If he would state his true quality, his position in competition with other recommended doctors would be weakened. For the analysis of this price-setting game - which differs from the one in Spiegler (2006a) since all doctors' qualities may be different - we can rely on the equilibrium analysis of Ireland (1993) and McAfee (1994) who independently solved the model under the interpretation of advertising to rational consumers. Uniqueness of this equilibrium was shown in Szech (2010).

We then analyze the full game in which doctors first choose their qualities. Extending the results of Ireland (1993) and McAfee (1994), we find that in all pure SPNE and in the symmetric mixed SPNE, doctors mainly offer low qualities. A low quality attracts fewer patients, but it softens price competition. The latter effect dominates, thus doctors do not set high qualities, even if setting higher qualities is costless. This extended model allows us to analyze how welfare (i.e. the overall proportion of patients cured[2]) is affected by the number of doctors in the market.

Heavily restricting the number of doctors improves welfare, i.e. the number of patients cured thanks to the doctors, since it increases the quality of treatments offered. If the maximal quality doctors can offer is high enough, having a monopolistic doctor maximizes welfare (and welfare strictly decreases in the number of doctors).[3]

The idea of a physician applying a bad cure on a patient instead of a good one may seem rather cynical. Yet note that a lot of treatments performed in health markets of generally high standard lack evidence that they help well. For example, vertebroplasty, an often performed spinal surgery to treat osteoporotic fractures, was questioned in its efficacy only recently.[4] Also arthroscopic surgery was recently

[1]This may explain why approved therapies do not crowd out the myriads of obscure nutritional supplements which were never clinically tested.

[2]In our model, all payments made are just transfers within society. Moreover, neither doctors nor patients face any direct costs. Thus, in our setting, maximizing social welfare is equivalent to maximizing the proportion of patients cured.

[3]It is even more beneficial for welfare not to limit entry but to exogenously prescribe a fixed price. Then doctors cannot take advantage of the weaker price competition induced by low qualities, and thus have no incentive to choose a lower than maximal quality.

[4]Compare Kallmes et al. (2009).

questioned by Kirkley et al. (2008), whose study raises serious doubt about this very common therapy to lower pain. Our model provides an explanation of why a variety of treatments of a disease survive, and why the best ones do not necessarily drive out inferior approaches.

1.1 Related Literature

Our model is based on Ran Spiegler's "Market for Quacks" (2006a). We extend his model by endogenizing quality: Instead of assuming fixed and symmetric qualities, we consider a pre-stage of quality-setting. This makes the model more suitable for the study of social welfare. The sampling rule patients apply to evaluate doctors is the S(1) rule which was introduced by Osborne and Rubinstein (1998). Spiegler and Rubinstein have utilized the S(1) rule to model consumer behavior in a variety of settings (see Spiegler (2006a, 2006b) and Rubinstein and Spiegler (2008)). We have used the S(1) procedure as well in a companion paper, Szech (2010).[5]

Besides S(1) there are other related approaches for modeling boundedly rational consumer behavior, such as Ellison and Fudenberg's (1995) "word-of-mouth learning" and Rabin's (2002) "law of small numbers". More broadly, our paper contributes to the literature on interactions between rational firms and boundedly rational consumers (see the survey by Ellison (2006)). To our knowledge, this paper is the first to extend a price competition game with boundedly rational consumers via introducing a preceding quality setting stage in order to study how bounded rationality affects welfare.

A vast experimental literature documents that anecdotes serve as convenient tools for transporting information and influencing people's behavior.[6] Especially patients, i.e. consumers in medical markets, have been shown to apply anecdotal reasoning.[7] Even if statistical information on different forms of therapy is available[8], patients

[5]In there, we consider a variant of Spiegler's (2006a) model in which the doctors' qualities are privately known random variables. We identify an equilibrium in monotone pricing strategies of that model and show that welfare goes to zero in the number of doctors. This happens because patients always attend the cheapest among the doctors who are recommended - in monotone strategies this is also the worst recommended doctor.

[6]Compare, e.g., Kahneman and Tversky (1973), Borgida and Nisbett (1977).

[7]See, e.g., Fagerlin et al. (2005) and the references therein and Enkin and Jadad (1998).

[8]Note that even for standard Western medicine, e.g., for many surgical treatments, statistical evidence is often missing, compare Gattellari et al. (2001) and McCulloch et al. (2002). According to McCulloch et al. (2002), "treatments in general surgery are half as likely to be based on RCT [Randomised Control Trials] evidence as treatments in internal medicine" (p. 1448).

often prefer to rely on personal stories. Fagerlin et al. (2005) stress identification and emotional feelings as driving factors behind this.[9]

Fagerlin et al. (2005) also point out that anecdotes typically provide "a clear dichotomy − either an individual was cured or not" (p. 399). This makes anecdotal information easy-to-grasp (and thus compelling to rely on) for a lay person such as a patient. Indeed, most untrained people have difficulties to understand basic statistical concepts.[10] For example, subjects often oversee the importance of sample size. The $S(1)$ rule can be seen as a simple model of this behavioral bias.

Reinterpreting the model with rational patients, i.e., considering a model where each patient likes or dislikes (notices or does not notice) a firm's product with some probability,[11] one sees the close relation of our model to the advertising model of Butters (1977) and its variations by Ireland (1993) and McAfee (1994). A similar reinterpretation with rational agents is possible with the Spiegler model, which is then a special (yet in their paper uncovered) case of the Perloff and Salop (1985) model of product differentiation. Ireland (1993) considers a similar quality-setting stage as we do but, of course, the welfare implications are completely different. McAfee (1994) assumes costly quality-choice, which implies robustness of our results for this case. The quality-setting stage also parallels Shaked and Sutton (1982): They introduce a quality setting pre-stage to the Gabszewicz and Thisse (1979) pricing model, while we do the same with a Perloff-Salop-type pricing model. In Shaked and Sutton (1982), only a limited number of firms can earn positive profits. This is not true in our model. The reason lies in the different modeling of consumers' preferences: In Shaked and Sutton (1982), all consumers share the same ranking of products. In our paper, for each product there is a group of consumers who prefer it to all other products.

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[9] Compare also Jenni and Loewenstein (1997), Loewenstein et al. (2001) and Finucane et al. (2000).

[10] This has been shown in general studies, but also in medical contexts like cancer treatment or cancer screening. Compare among others Tversky and Kahneman (1971), Hamill et al. (1980), Garfield and Ahlgren (1988), Yamagishi (1997), Schwartz et al. (1997), Weinstein (1999), Lipkus et al. (2001), Weinstein et al. (2004).

[11] This reinterpretation of quality as "mass appeal" or "intensity of advertising" dramatically changes the welfare implications of our model, but not the equilibrium analysis.

1.2 Outline

The paper is structured as follows: Section 2 presents the model and describes the $S(1)$ procedure in detail. In Section 3, we present and discuss the equilibrium of the pricing stage. Equilibria of the quality setting stage are discussed in Section 4. Section 5 presents our results on social welfare. Section 6 discusses our results. Section 7 concludes. We delegate all proofs to the Appendix.

2 The Model

We consider a market with n doctors who are familiar with the market and act rationally. Patients are not familiar with the market and apply a simple sampling rule, called the **S(1)** rule, as described below. Patients form a continuum of mass one. We assume that the doctors know each other very well and hence know the qualities of each other when playing the pricing game. Qualities can be anything between zero and some upper bound $0 < \overline{\alpha} < 1$. A doctor's quality is the probability with which he can cure a patient. With the counter probability, the patient remains ill. Before we specify the patients' behavior, let us fix the **timing** of the model:

1. Doctors simultaneously set their qualities $\alpha_i \in [0, \overline{\alpha}]$.
2. Doctors observe each others' qualities.
3. Doctors simultaneously set their prices P_i.
4. Patients decide if they want to attend a doctor and if so, which one.

Patients are initially ill and have a utility of one from getting cured and a utility of zero from staying ill. They decide according to the behavioral S(1) rule as introduced by Osborne and Rubinstein (1998), and as utilized in Spiegler (2006a):

- Each patient samples each doctor once.
- With probability α_i, a patient receives a positive signal $S_i = 1$ on doctor i ("**a recommendation**").
- With probability $1 - \alpha_i$, a patient receives a negative signal $S_i = 0$ on doctor i ("**no recommendation**").
- A patient attends the doctor with the highest $S_i - P_i$ ...

- unless $\max_i S_i - P_i < 0$. Then the patient stays out of the market and expects a utility of 0 at a price of 0.

Note that the last two points implicitly contain a tie-breaking rule: If a patient has to choose between consulting a recommended doctor charging a price of one and staying at home, the patient opts for the doctor. It can be shown that in the pricing stage no equilibrium exists if we depart from this assumption. All other possible types of ties can be broken arbitrarily.

Note that patients rely far too much on the signal they get - they overinfer from their sample. The idea behind the S(1) rule is to capture a simple way of anecdotal reasoning: Each patient independently asks some "former" client of each doctor.[12] A client of doctor i got cured with probability α_i. Thus, with probability α_i, he recommends doctor i to the patient. The patient trusts in this report - he either thinks the doctor can cure him as well for sure or not at all. For evidence of such reasoning from the medical literature, see Fagerlin (2005).

Choosing a higher quality comes at no direct costs for the doctors. The motivation for this assumption is that we want to study how the patients' boundedly rational behavior induces doctors to set a low quality. Our model is to be understood as a benchmark case which ignores costs that give doctors another, separate reason for choosing a low quality.

3 The pricing stage

We search for SPNE using backwards induction, and thus we start with an analysis of the price setting game for given quality levels α_i. This game was solved independently by Ireland (1993) and McAfee (1994). Uniqueness of equilibrium was added by Szech (2010).

Proposition 1 (Ireland (1993), McAfee (1994), Szech (2010)) *Consider* $0 < \alpha_1 \leq \ldots \leq \alpha_n \leq \overline{\alpha}$. *Then the unique equilibrium of the pricing game is given as follows:*

Define a sequence of prices $p_0, \ldots, p_n$ *by*

$$p_i = \frac{(1-\alpha_{i+1}) \cdot \ldots \cdot (1-\alpha_{n-1})}{(1-\alpha_i)^{n-i-1}}$$

[12] Of course, we are not in a dynamic model here. This is only a motivating story.

for $1 \leq i \leq n-2$,

$$p_0 = \prod_{i=1}^{n-1}(1-\alpha_i) \quad \textit{and} \quad p_{n-1} = p_n = 1.$$

Each doctor i *mixes over the interval* $[p_0, p_i]$ *using the distribution function* G_i *defined by*

$$G_i(p) = \frac{1}{\alpha_i}\left(1 - \sqrt[n-j]{\frac{(1-\alpha_j)\cdot \ldots \cdot (1-\alpha_{n-1})}{p}}\right)$$

for $p \in [p_{j-1}, p_j] \subset [p_0, p_i]$ *with* $1 \leq j \leq n-1$. *On* $[0, p_0]$, *define* $G_i = 0$ *and on* $[p_i, 1]$, $G_i = 1$. G_n *places an atom of size* $1 - \frac{\alpha_{n-1}}{\alpha_n}$ *on* 1.[13]

Note that the upper boundaries p_i coincide if the corresponding α_i coincide. The distribution functions G_i are continuous except for G_n where doctor n puts an atom on $p = 1$ (if $\alpha_{n-1} \neq \alpha_n$). As seen in Figure 1, the doctors' price supports all start at the same lowest price p_0. Furthermore, we see that the higher the quality of a doctor, the larger the support of his pricing strategy.

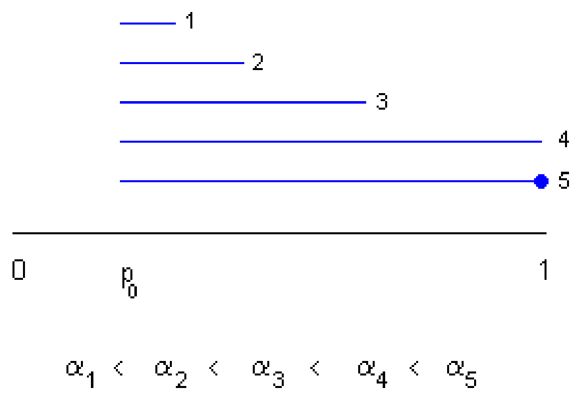

Figure 1: Supports of the doctors' pricing strategies for $n = 5$

The proposition immediately yields the following expression for equilibrium payoffs:

[13]Thus, this result still holds in the (excluded) case where exactly one doctor has a quality of 1. If there are more than one doctor with a quality of 1, payoffs will be zero due to Bertrand competition. Then, many equilibria are possible as long as two doctors of quality 1 set prices of 0.

Corollary 1 *The equilibrium payoff of doctor i is given by*

$$\pi_i = \alpha_i \prod_{j \neq j^*} (1 - \alpha_j) \tag{1}$$

where $j^* \in argmax_j \ \alpha_j$.

The intuition for this is as follows: Consider doctor j^* who offers the highest quality.[14] This doctor can earn a strictly positive payoff independently of his competitors' strategies as there will be patients who only receive a positive report on him, but not on the other doctors. (The fraction of these patients is $\alpha_{j^*} \prod_{j \neq j^*}(1 - \alpha_j)$). Our doctor will thus set a strictly positive lowest price in any equilibrium. Consider one equilibrium and assume our doctor yields there a payoff of $\alpha_{j^*}C$ and sets a lowest price of p_L. By charging an only slightly lower price than this, every other doctor i can earn, at least, a payoff arbitrarily close to $\alpha_i C$. Reasoning vice versa we see that the payoffs indeed have to equal $\alpha_i C$ and cannot be higher. To determine C, one has to argue that there is one doctor i who has a price of 1 in the support of his equilibrium price setting strategy. This doctor only earns a profit if he is the only recommended doctor, which happens with a probability of $\alpha_i \prod_{j \neq i}(1 - \alpha_j)$. Finally, we find that this doctor has to be the best doctor, as otherwise C would be so low that the best doctor would prefer to deviate. Thus we can specify $C = \prod_{j \neq j^*}(1 - \alpha_j)$.

We hence see that the quality of the best doctor does not appear in the payoff formulae of the other doctors. Hence it does not matter for the competitors' profits whether the best doctor is as good as the second best doctor or far better than him. For the second best doctor, this is not true: His quality always affects the competitors' profits.[15]

Note that in our equilibrium weak doctors typically earn much more than their maxmin payoffs. As an example, consider the case $n = 2, \alpha_1 = 0.9, \alpha_2 = 0.3$. Then

$$\pi_1 = \text{maxmin}_1 = 0.9(1 - 0.3) = 0.63,$$

[14] If there is more than one, we (arbitrarily) determine as best doctor one of the doctors with highest quality.

[15] Thus, if more than one quack in Spiegler's market would be changed into some expert, the payoffs of the remaining quacks would be diminished. This result complements Proposition 2 of Spiegler (2006a).

whereas

$$\pi_2 = 0.3(1 - 0.3) = 0.21 > \text{maxmin}_2 = 0.3(1 - 0.9) = 0.03.$$

A good doctor can yield a high payoff from being a monopolist. Thus when facing a weak competitor he is "unwilling" to play too low prices as the probability that he is the only recommended doctor remains high. Hence there is room left for the weak doctor to earn a considerable payoff.

The following proposition characterizes the equilibrium further. It shows that while better doctors hold larger market shares in equilibrium, differences in market shares are smaller than differences in qualities. Hence patients attend bad doctors over-proportionally often.

Proposition 2 *Consider two doctors i and j with qualities $\alpha_i < \alpha_j$, $i, j \in \{1, \ldots, n\}$. Denote by m_i and m_j the doctors' market shares, i.e. the proportions of patients attending doctors i and j. Then*

$$\frac{\alpha_i}{\alpha_j} < \frac{m_i}{m_j} < 1.$$

Note that α_i/α_j is also the ratio between the equilibrium payoffs of doctors i and j. Thus the corollary implies that better doctors yield higher payoffs partly by attracting more patients and partly by charging higher prices.

4 The quality setting stage

As the equilibrium of the price setting game is unique, the doctors' payoffs in every SPNE of the complete game just depend on the qualities chosen by the doctors in the first stage. From now on, we can thus consider the game as a one-stage quality setting game and search for its Nash equilibria. The results of this section extend those of Ireland (1993) and McAfee (1994) by including an upper quality limit $\overline{\alpha}$ and by giving a more complete treatment of mixed equilibria. For the qualitatively similar case where costs are involved, see McAfee (1994).

For intuition, consider first the two doctors case with $\overline{\alpha} > \frac{1}{2}$. By Corollary 1, the payoff of doctor 1, given quality choices α_1 and α_2, is

$$\Pi_1(\alpha_1, \alpha_2) = \begin{cases} \alpha_1(1-\alpha_2) & \text{if } \alpha_1 \geq \alpha_2 \\ \alpha_1(1-\alpha_1) & \text{if } \alpha_1 \leq \alpha_2. \end{cases}$$

It is thus not surprising that the best response curve of doctor 1 contains only $\frac{1}{2}$ and $\overline{\alpha}$:

$$BR_1(\alpha_2) = \begin{cases} \overline{\alpha} & \text{if } \alpha_2 \leq 1 - \frac{1}{4\overline{\alpha}} \\ \frac{1}{2} & \text{if } \alpha_2 \geq 1 - \frac{1}{4\overline{\alpha}}. \end{cases}$$

The two equilibria in pure quality setting strategies are, accordingly: $(\overline{\alpha}, \frac{1}{2})$ and $(\frac{1}{2}, \overline{\alpha})$.

With more than two doctors, it remains true that a doctor's best response to any pure strategies of his opponents is either $\overline{\alpha}$ or $\frac{1}{2}$. Furthermore, if $\overline{\alpha} > \frac{1}{2}$, given that one of his opponents plays $\overline{\alpha}$, doctor i prefers playing $\frac{1}{2}$ to $\overline{\alpha}$. Intuitively, doctor i chooses a low quality to soften price competition against the strong competitor. We can now characterize the SPNE as follows:

Proposition 3 (***i***) *If $\overline{\alpha} \in (\frac{1}{2}, 1)$, all SPNE in pure quality setting strategies are of the following form: One doctor i sets $\alpha_i = \overline{\alpha}$, all other doctors j set $\alpha_j = \frac{1}{2}$. Pricing strategies are as given in Proposition 1. We call these equilibria the* ***$\frac{1}{2}$-$\overline{\alpha}$-equilibria****.*
(***ii***) *If $\overline{\alpha} < \frac{1}{2}$, the unique SPNE is given by all doctors i setting $\alpha_i = \overline{\alpha}$ and choosing prices as in Proposition 1.*

An immediate corollary of Proposition 3 is the following:

Corollary 2 *With $\overline{\alpha} > \frac{1}{2}$, there is no SPNE where all doctors offer the highest quality.*

There is a positive effect of raising α above $\frac{1}{2}$: With a higher quality, a doctor gets recommended to more patients. Yet, if strong competitors are present, this effect on payoff is dominated by the negative effect of strengthening competition. This already suggests the following conclusion which is made precise in Section 5: Having many doctors in the market harms welfare since the average quality offered by the doctors decreases with competition.

To round out the game-theoretic analysis we close this section with a discussion of mixed equilibria, demonstrating that these cannot be expected to yield qualitatively different welfare results. Since there is a unique pure equilibrium for $\overline{\alpha} \leq \frac{1}{2}$, we consider $\overline{\alpha} \in (\frac{1}{2}, 1)$ for the remainder of this section. While showing existence of a symmetric mixed strategy equilibrium based on abstract existence results is straightforward, we cannot provide such equilibria explicitly for $n > 2$. Nevertheless, in the following proposition we show that symmetric mixed equilibria are characterized by rather low qualities as well.

Proposition 4 *For $\overline{\alpha} \in (\frac{1}{2}, 1)$, the quality setting game has a symmetric mixed strategy equilibrium. Fix such an equilibrium given by a distribution function F on $[0, \overline{\alpha}]$. Then supp $F \subseteq [\frac{1}{2}, \overline{\alpha}]$ and $\{\frac{1}{2}, \overline{\alpha}\} \subseteq$ supp F. Denote by μ a doctor's expected equilibrium quality, i.e.*

$$\mu = \int_{\frac{1}{2}}^{\overline{\alpha}} \alpha \, dF(\alpha).$$

Then

$$\mu \leq 1 - \frac{1}{4\overline{\alpha}} \in \left(\frac{1}{2}, \frac{3}{4}\right).$$

Furthermore, equilibrium payoffs π are bounded from below by

$$\pi \geq \frac{1}{4} \left(\frac{1}{4\overline{\alpha}}\right)^{n-2}.$$

Note that our bound on the expected quality μ has the appeal that it does not depend on n. Recall from Proposition 2 that market shares are more evenly distributed than qualities. Thus, although for large n with high probability there is a considerable number of high quality doctors, many patients end up at low quality doctors. Hence also in the mixed strategy equilibrium, competition does not force doctors to offer high qualities and welfare remains significantly bounded away from $\overline{\alpha}$.

For the two doctor case we can state a symmetric equilibrium of the quality setting game explicitly: It is easy to check that F given by

$$F(\alpha) = \frac{4\alpha^2 - 1}{4\alpha^2} \text{ for } \alpha \in [\frac{1}{2}, \overline{\alpha})$$

and $F(\overline{\alpha}) = 1$ is such an equilibrium.

Finally note that there are also asymmetric mixed strategy equilibria.[16] For instance, with two doctors, an equilibrium is given by one doctor setting a quality of $1 - \frac{1}{4\overline{\alpha}}$

[16]Hence note that Ireland (1993)'s informal claim that all mixed equilibria are symmetric in this game is not correct.

and the other doctor setting a quality of $\overline{\alpha}$ with probability $\frac{1}{3-\frac{1}{\overline{\alpha}}}$ and of $\frac{1}{2}$ with the counter probability.

5 Welfare

In this section we explicitly discuss the interplay between competition and welfare. We start with the case $\overline{\alpha} > \frac{1}{2}$ and determine the market size which maximizes welfare in the $\frac{1}{2}$-$\overline{\alpha}$-equilibria. We find that a single doctor is optimal for high maximal qualities, namely $\overline{\alpha} \geq \frac{3}{4}$. For smaller $\overline{\alpha}$, having more than one, but still only few doctors in the market is best.

Proposition 5 *As n increases, welfare in the $\frac{1}{2}$-$\overline{\alpha}$-equilibria converges to $\frac{1}{2}$ from above. For $\frac{3}{4} < \overline{\alpha} < 1$ welfare strictly decreases in $n \geq 1$. For $\frac{1}{2} < \overline{\alpha} \leq \frac{3}{4}$ welfare increases up to some finite optimal market size n^* and decreases from there on. Furthermore, for $\frac{1}{2} < \overline{\alpha}$ the optimal market size is bounded from above by*

$$n^* \leq 10.4 + 2.3 \ln \left(1.7 + \frac{0.35}{\overline{\alpha} - \frac{1}{2}} \right).$$

To see why for lower maximal qualities a market with a small number of doctors generates higher welfare than a monopolistic doctor, consider as an example $\overline{\alpha} = 0.6$. If there is only one doctor, he offers the best possible quality of 0.6 and sets a price of 1. Thus 60% of all patients receive a positive report and attend the doctor. Of this fraction of patients, 60% get cured. Thus, in monopoly, 36% of all patients get cured. With more doctors, more patients receive at least one positive report and attend a doctor at all. In our example, with two doctors, we have $1-(1-0.6)\cdot(1-\frac{1}{2}) = 80\%$ of all patients receiving at least one positive report and thus attending a doctor at all. Formula (4) of the Appendix shows that in this case 44.25% of patients get cured, and that the optimal number of doctors for $\overline{\alpha} = 0.6$ is 7. This is the positive welfare effect of a larger number of doctors. Yet as the number of doctors increases, more and more doctors offer only low quality treatments ($\alpha = \frac{1}{2}$). This is the downside of a high number of doctors, which dominates if n gets larger: The positive effect of increasing the market size vanishes exponentially in n while the market share of the good doctor decreases much more slowly (roughly like $\frac{1}{n}$). Thus for n sufficiently large the proportion of patients cured is always above $\frac{1}{2}$.

The upper bound on the optimal market size demonstrates that the optimal market size goes to infinity only very slowly as $\overline{\alpha}$ approaches $\frac{1}{2}$. For instance, for $\overline{\alpha} = 0.50001$ we obtain $n^* \leq 34$:[17] The presence of a doctor with a slightly better quality drastically reduces the optimal market size, as infinitely many doctors would maximize welfare if $\overline{\alpha} = \frac{1}{2}$. The proposition is summed up in Figure 2 which depicts the proportion of patients cured as a function of market size for different values of $\overline{\alpha}$.

Let us now turn to the case where the best possible method of healing only leads to a recovery probability of $\overline{\alpha} \leq \frac{1}{2}$. Then, as noted in Proposition 3, the unique SPNE is that all doctors offer the best possible treatment with healing probability $\overline{\alpha}$. But what would happen if there was a new technology that could lead to a higher $\overline{\alpha}$? Corollary 3 tells us that a rise of $\overline{\alpha}$ in between $[\frac{1}{n}, \frac{1}{2}]$ would not be welcomed by the doctors:

Corollary 3 *The equilibrium profit of each doctor*

$$\overline{\alpha}(1 - \overline{\alpha})^{n-1}$$

is strictly decreasing in $\overline{\alpha}$ *on* $[\frac{1}{n}, \frac{1}{2}]$.

Hence, in our model, doctors would try to block or delay the approval of promising new drugs or treatments.[18]

[17] The exact value (which can be found numerically) is $n^* = 24$.

[18] Note, however, that doctors would, of course, welcome innovations that allowed themselves but not the other doctors to set a higher quality.

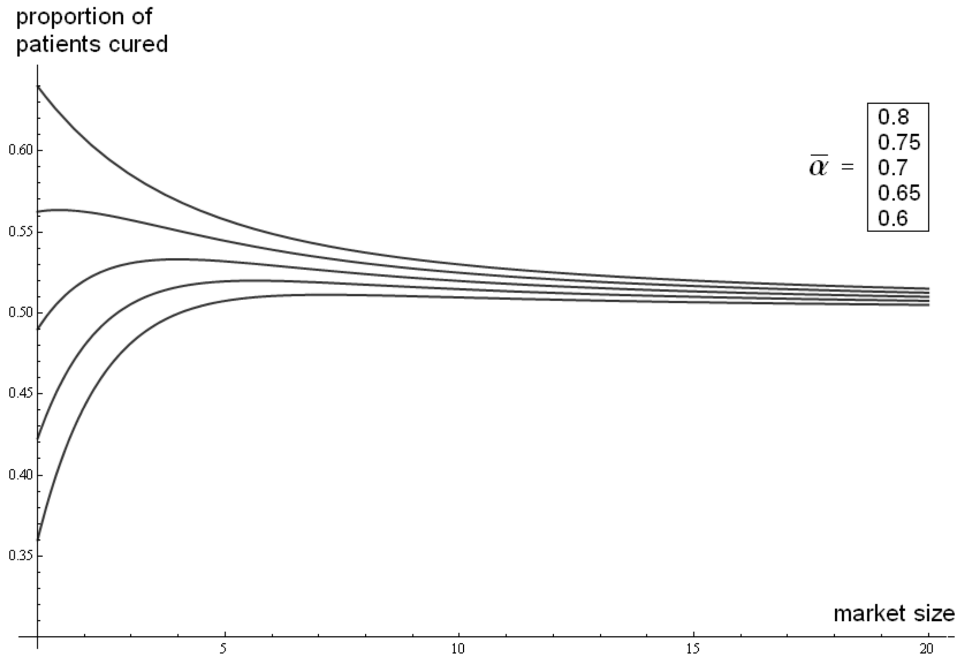

Figure 2: The effect of market size on welfare

For $\overline{\alpha} \leq \frac{1}{2}$, the proportion of patients cured increases in market size. Note, however, that a restriction to a finite market size does not do much harm unless $\overline{\alpha}$ is small since the proportion of patients cured, which is given by

$$\overline{\alpha}\left[1-(1-\overline{\alpha})^{n}\right],$$

converges to $\overline{\alpha}$ exponentially fast.

6 Discussion

6.1 Capacity Constraints

Throughout we assumed that a doctor's capacity is not constrained. While capacity constraints would make having some more doctors in the market beneficial for welfare, our general line of reasoning remains valid: If there are many doctors in the market, they will soften competition vis quality dispersion and hence reduce welfare.

6.2 Comparison with a Model of Incomplete Information

It is natural to ask whether our results could be reproduced in a model with incompletely informed but rational patients. Despite some similarities, this is generally not the case. Consider first a price setting game in which patients believe in some prior distribution of doctors' qualities. Patients receive either a good or a bad report on each doctor and update their prior in a Bayesian way. Thus there are two possible expectations α^l and α^h a patient can have about a doctor's quality. This does lead to a mixed strategy equilibrium like in our model. Yet, apart from this, there are considerable differences. For instance, in the incomplete information model, if we change the patients' prior beliefs to more optimistic ones (such that α^l and α^h are replaced by $L\alpha^l$ and $L\alpha^h$, $L > 1$) the patients' willingness to pay increases, and doctors then charge higher prices in equilibrium. This is in contrast to the $S(1)$ model, in which increasing qualities leads to lower equilibrium prices. For a more detailed discussion, see Spiegler (2006a).

A two stage model with strategical patients differs even more drastically from our model. In such a model, in equilibrium patients know the doctors' strategies (even though they may not observe qualities perfectly). It is thus an SPNE for doctors to set the highest possible quality and charge a price of zero and for patients to choose an arbitrary doctor among those offering the lowest price.

Having results such as Milgrom and Roberts (1986) in mind one might expect that if doctors had a chance to disclose their qualities they would do so in equilibrium and most of our results would break down. Yet this is not the case. In our model, doctors have no incentive to disclose their qualities: A doctor who is assumed to have a quality of 1 by a fraction α_i of patients is in a better position when competing with other doctors than a doctor who is known to have quality α_i by all patients. Unlike rational agents, our boundedly rational patients do not draw any conclusions from a doctor's decision not to disclose his quality. For a detailed exposition of this point, see Proposition 3 in Spiegler (2006a).[19] For further discussion of the relation to standard models see also Spiegler (2010).

[19] Spiegler shows that any strategy involving disclosure is weakly dominated by some strategy involving no disclosure. The equilibrium we have identified in Theorem 2 persists because if some doctor could profitably deviate to a strategy involving disclosure there would also exist a strategy involving no disclosure he could deviate to. But then our equilibrium would not have been an equilibrium in the original game. Furthermore, in the working paper version Spiegler (2003), Spiegler shows that no Nash equilibrium involves disclosure. Hence the equilibrium from our Theorem 2 is still the unique equilibrium of the game where disclosure is allowed.

6.3 Reversed Timing

What happens if we reverse the timing, such that doctors first choose their prices and then choose their qualities? For a given vector of (positive) prices, it is a unique best response for doctors to set their qualities to $\overline{\alpha}$. Thus the unique SPNE of the game with reversed timing is the following: Doctors mix over prices according to the distribution functions from Proposition 1 for the case of $\alpha_i = \overline{\alpha}$ and set their qualities to $\overline{\alpha}$.

The fact that doctors respond to a fixed vector of prices by setting their qualities to the maximal value also has a policy implication: If a policy maker could prescribe a fixed price for the doctors' services, the problem with the low qualities would vanish. Doctors would set their qualities as high as possible.

If we consider the simultaneous choice of qualities and prices we thus get - in the case $\overline{\alpha} > \frac{1}{2}$ - two types of Nash equilibria with pure quality setting strategies. Among these, from the view of the doctors, the $\frac{1}{2}$-$\overline{\alpha}$-equilibria strictly Pareto-dominate the $\overline{\alpha}$-equilibria.

7 Conclusion

We have seen that if consumers are unfamiliar with the market and rely on anecdotes, all firms, no matter how bad, yield positive profits. Low quality firms typically earn much more than their maxmin payoffs. If firms can choose, they mostly opt for low qualities: A lower quality makes firms attract less consumers. Yet it also softens price competition, and thus allows them to set higher prices in equilibrium. The latter, positive effect on payoffs dominates.

Having many firms in the market does not help to cure the problem. Indeed, welfare falls for larger numbers of firms, as the average quality offered decreases. Depending on the maximally possible quality, a monopoly or oligopoly of firms is best for welfare.

Fixing prices exogenously would destroy the incentives of the firms to choose low qualities. Then, all firms would offer the best possible quality. An increase in qualities offered could also be achieved by making the market more transparent to consumers.

In the German health market, legislation recently curbed price competition among health insurers to a minimum. The idea was to shift consumers' attention from price differences to quality differences.[20] While there may be even better means to strengthen quality-competition, our hunch is that the reasoning behind this policy was guided by observations that fit well to the theoretical framework and predictions of our model.

[20]Compare the following statement by the German Federal Ministry of Health (2009), translated: "The uniform insurance fee ends the unfair competition for the cheapest fee. Instead it opens a fair competition for the best service and additional benefits to the insured." Clearly, such "fairness" considerations would not make sense if patients were assumed to be rational.

A Proofs

Proof of Proposition 2

Note that we can write

$$m_i = \int_{p_0}^{p_i} \alpha_i \prod_{k \neq i} (1 - \alpha_k G_k(p)) dG_i(p).$$

Note furthermore that the G_k were chosen such that for $p \in [p_0, p_i]$

$$\prod_{k=1}^{n-1} (1 - \alpha_k) = p \prod_{k \neq i} (1 - \alpha_k G_k(p)).$$

Thus

$$m_i = \alpha_i \prod_{k=1}^{n-1} (1 - \alpha_k) \int_{p_0}^{p_i} \frac{1}{p} dG_i(p).$$

Define $b_i = \int_{p_0}^{p_i} \frac{1}{p} dG_i(p)$. Recall that because of the shape of the G_k and because of $\alpha_i < \alpha_j$ we can see G_j as a mixture of G_i and a probability distribution over $[p_i, p_j]$. Since $\frac{1}{p}$ is decreasing, this implies $b_i > b_j$ and thus

$$\frac{m_i}{m_j} = \frac{\alpha_i b_i}{\alpha_j b_j} > \frac{\alpha_i}{\alpha_j}.$$

It remains to be shown that $m_i < m_j$. For this denote by $g_i(p)$ and $g_j(p)$ the densities of $G_i(p)$ and $G_j(p)$.[21] Note that for $p \in [p_0, p_i]$ we have $\alpha_i G_i(p) = \alpha_j G_j(p)$ and thus $\alpha_i g_i(p) = \alpha_j g_j(p)$. Thus we can deduce:

$$\begin{aligned} m_i &= \prod_{k=1}^{n-1} (1 - \alpha_k) \int_{p_0}^{p_i} \frac{1}{p} \alpha_i g_i(p) dp = \prod_{k=1}^{n-1} (1 - \alpha_k) \int_{p_0}^{p_i} \frac{1}{p} \alpha_j g_j(p) dp \\ &< \prod_{k=1}^{n-1} (1 - \alpha_k) \int_{p_0}^{p_j} \frac{1}{p} \alpha_j g_j(p) dp = m_j. \end{aligned}$$

□

Proof of Proposition 4

[21]Note that these densities are well-defined but exhibit jumps at the p_k. Without loss of generality we can choose the densities to be left-continuous. In this proof we chose to ignore the atom of G_n since it only complicates notation without adding further insight.

Note that payoffs in the quality setting game are continuous, bounded and symmetric. Thus, for instance, the main result of Becker and Damianov (2006) shows existence of a symmetric equilibrium. Recall that a doctor's best response to any set of pure strategies by his opponents is playing either $\frac{1}{2}$ or $\overline{\alpha}$. Thus $(\frac{1}{2}, \ldots, \frac{1}{2})$ and $(\overline{\alpha}, \ldots, \overline{\alpha})$ are the only candidates for symmetric pure equilibria. Since neither of these is an equilibrium, we can conclude the existence of a symmetric equilibrium in (non-degenerate) mixed strategies. Fix such an equilibrium and denote the equilibrium strategy by a distribution function F over $[0, \overline{\alpha}]$. Define $S = \text{supp } F$. Since qualities in $[0, \frac{1}{2})$ are strictly dominated by playing $\frac{1}{2}$ we can conclude $S \subseteq [\frac{1}{2}, \overline{\alpha}]$. Define $\underline{s} = \inf S$ and $\overline{s} = \sup S$. We must have $\overline{s} = \overline{\alpha}$ because otherwise a doctor could profitably deviate by playing $\overline{\alpha}$ instead of qualities near $\overline{s}$: For a doctor who is the (weakly) best doctor with high probability it is optimal to choose $\overline{\alpha}$. Analogously we must have $\underline{s} = \frac{1}{2}$ because otherwise a doctor would want to move probability mass from near $\underline{s}$ down to $\frac{1}{2}$.

What remains to be shown is the lower bound on expected equilibrium qualities and payoffs. Without loss of generality consider the payoff of doctor 1. For doctor 1, $\alpha_2, \ldots, \alpha_n$ are independent random variables with distribution function F. The main idea is to compare payoffs from playing $\frac{1}{2}$ and $\overline{\alpha}$. Denote these payoffs by $\pi_{\frac{1}{2}}$ and $\pi_{\overline{\alpha}}$. Note that

$$\pi_{\overline{\alpha}} = \overline{\alpha} E[\prod_{j=2}^{n} (1 - \alpha_j)] = \overline{\alpha}(1 - \mu)^{n-1}$$

since a doctor playing $\overline{\alpha}$ is the (weakly) best doctor with certainty. Denote by $\alpha_{j:n-1}$ the jth largest of $\alpha_2, \ldots, \alpha_n$. We can then write doctor 1's payoff from setting a quality of $\frac{1}{2}$ as

$$\pi_{\frac{1}{2}} = \frac{1}{4} E\left[\prod_{j=2}^{n-1} (1 - \alpha_{j:n-1})\right] \geq \frac{1}{4} E\left[\prod_{j=2}^{n-1} (1 - \alpha_j)\right] = \frac{1}{4}(1 - \mu)^{n-2}$$

where in the middle step we used that we can bound the product over the $n - 2$ smallest α_j by a product over an arbitrary collection of $n - 2$ of the α_j.[22] In equilibrium we must have that $\pi_{\frac{1}{2}} = \pi_{\overline{\alpha}}$. We can thus conclude that

$$\overline{\alpha}(1 - \mu)^{n-1} \geq \frac{1}{4}(1 - \mu)^{n-2}.$$

[22]Note that how much we are giving away in this bound depends on n: Thus while our upper bound is independent of n, the same is not true for μ itself.

Solving for μ we get the bound

$$\mu \leq 1 - \frac{1}{4\overline{\alpha}}.$$

Plugging our upper bound on μ into the expression for $\pi_{\overline{\alpha}}$ we finally get the desired lower bound on equilibrium payoffs. $\square$

Proof of Proposition 5

The quantity of interest in this proof is $w(\overline{\alpha}, n)$, the proportion of patients healed in the $\frac{1}{2}$-$\overline{\alpha}$-equilibria with n doctors where $n \geq 1$ and $\frac{1}{2} < \overline{\alpha} \leq 1$. Clearly, $w(\overline{\alpha}, n)$ can be written as

$$w(\overline{\alpha}, n) = p_g\overline{\alpha} + p_b\frac{1}{2} + p_0 0 \tag{2}$$

where p_g, p_b and p_0 denote the fractions of patients consulting the good doctor (i.e. the doctor offering $\overline{\alpha}$), the fraction consulting the other doctors and the fraction who stays at home, respectively. Note that p_g, p_b and p_0 are unique since the price setting game has a unique equilibrium by Proposition 1 and that they can be calculated from the equilibrium strategies given there: The good doctor mixes over $[2^{-(n-1)}, 1]$ using the distribution function

$$F_g(p) = \frac{1}{\overline{\alpha}}\left(1 - \sqrt[n-1]{\frac{1}{2^{n-1}p}}\right)$$

and puts an atom of size $1 - \frac{1}{2\overline{\alpha}}$ on 1. The remaining doctors mix over the same interval with

$$F_b(p) = 2\left(1 - \sqrt[n-1]{\frac{1}{2^{n-1}p}}\right).$$

Note that the pricing strategy of the good doctor can be interpreted in the following way: With probability $1 - \frac{1}{2\overline{\alpha}}$ he sets a price of 1 and with probability $\frac{1}{2\overline{\alpha}}$ he uses exactly the same pricing strategy as the other doctors. In the first case the good doctor only gets patients if no other doctor is recommended (which happens with probability $2^{-(n-1)}$). In the second case, the good doctor has exactly the same chances to acquire a patient as the other recommended doctors. Let the random variable r_n denote the number of bad doctors who are recommended to a patient. Then the market share of the good doctor can be written as

$$p_g = \overline{\alpha}\left[\frac{1}{2\overline{\alpha}}E\left[\frac{1}{1+r_n}\right] + (1 - \frac{1}{2\overline{\alpha}})2^{-(n-1)}\right] \tag{3}$$

where the leading factor $\overline{\alpha}$ results from the fact that the doctor is only competing for the patients to which he is recommended. Note that r_n is distributed binomially with parameters $n-1$ and $\frac{1}{2}$ and thus

$$\begin{aligned} E\left[\frac{1}{1+r_n}\right] &= \sum_{k=0}^{n-1}\binom{n-1}{k}2^{-(n-1)}\frac{1}{1+k} = \frac{1}{n}\sum_{k=0}^{n-1}\binom{n}{k+1}2^{-(n-1)} \\ &= \frac{2}{n}\sum_{k=1}^{n}\binom{n}{k}2^{-n} = \frac{2}{n}\left(-2^{-n}+\sum_{k=0}^{n}\binom{n}{k}2^{-n}\right) \\ &= \frac{2}{n}(1-2^{-n}) \end{aligned}$$

where in the first step we have used that $n\binom{n-1}{k} = (k+1)\binom{n}{k+1}$ and in the final step we used that the sum equals 1 since it simply adds up all probabilities of a Binomial $(n, \frac{1}{2})$ distribution. Putting this into (3) and rearranging terms gives us

$$p_g = \frac{1}{n}(1-2^{-n}) + (\overline{\alpha} - \frac{1}{2})2^{-(n-1)}.$$

Clearly $p_0 = (1-\overline{\alpha})2^{-(n-1)}$. Thus we see that - as claimed in the main text - as n gets large p_g goes to zero like $\frac{1}{n}$ and thus much slower than p_0 which decreases exponentially. Inserting these expressions for p_g and p_0 and $p_b = 1 - p_g - p_0$ into (2) and rearranging we obtain

$$w(\overline{\alpha}, n) = \frac{1}{2} + (\overline{\alpha} - \frac{1}{2})\frac{1}{n}(1-2^{-n}) + (2\overline{\alpha}^2 - \overline{\alpha} - \frac{1}{2})2^{-n}. \tag{4}$$

The remainder of the proof consists of an analysis of the function $w(\overline{\alpha}, n)$.[23] We will proceed in the following way: First, we will show that for every α as n gets large w approaches $\frac{1}{2}$ from above. Then we will show that $w(\overline{\alpha}, x)$, where $x \in \mathbb{R}_{\geq 1}$, is monotonically decreasing in x for $\alpha \geq \frac{1}{4}(1+\sqrt{5}) \approx 0.809$. For $\frac{1}{2} < \overline{\alpha} < \frac{1}{4}(1+\sqrt{5})$ we will show that w increases in x up to some value $x^* \geq 1$ and decreases from there on. From this we can conclude that $w(\overline{\alpha}, 1) \geq w(\overline{\alpha}, 2)$ is a sufficient condition for $n^* = 1$ being a maximizer of $w(\overline{\alpha}, n)$. Then we will verify that $w(\overline{\alpha}, 1) \geq w(\overline{\alpha}, 2)$ if and only if $\overline{\alpha} \geq \frac{3}{4}$. From these results we can conclude that for $\overline{\alpha} > \frac{3}{4}$ $w(\overline{\alpha}, n)$ is maximized at $n^* = 1$ while for $\overline{\alpha} \geq \frac{3}{4}$ there is a finite $n^* > 1$ which maximizes $w(\overline{\alpha}, n)$. Furthermore it follows that there are at most two maximizers and if there

[23]While this rather technical analysis is of course necessary to complete the proof, the impatient reader is invited to skip it. Ultimately the calculations only verify that Figure 2 delivers the complete picture.

are two, these must be subsequent integers. In the final part of the proof we will show an upper bound on n^* in terms of $\overline{\alpha}$.

It is immediate that $w(\overline{\alpha}, n)$ converges to $\frac{1}{2}$ as n goes to infinity. To show that this convergence is from above we have to show for every fixed $\overline{\alpha}$ that $w(\overline{\alpha}, n) > \frac{1}{2}$ for sufficiently large n. Here and in the following we will substitute n by the real-valued parameter x and view w as the weighted sum of two functions f and g which do not depend on $\overline{\alpha}$:

$$w(\overline{\alpha}, x) = \frac{1}{2} + (\overline{\alpha} - \frac{1}{2})f(x) + (2\overline{\alpha}^2 - \overline{\alpha} - \frac{1}{2})g(x)$$

where

$$f(x) = \frac{1}{x}(1 - 2^{-x}) \text{ and } g(x) = 2^{-x}.$$

Note that the coefficient of f is always positive while the coefficient of g is zero at $\overline{\alpha} = \frac{1}{4}(1 + \sqrt{5})$ and strictly increasing over $[\frac{1}{2}, 1]$. Note furthermore that g and f are strictly decreasing and positive. For g this is clear, for f note that

$$f'(x) = -[2^x - (1 + x\ln(2))]x^{-2}2^{-x}.$$

The factors outside the brackets are clearly positive. The term in squared brackets is positive for all $x \geq 1$ since it is the difference between the convex function 2^x and its first order Taylor approximation in 0. Thus $f'(x) < 0$ for all $x \geq 1$.

Note that in the case $\overline{\alpha} \geq \frac{1}{4}(1 + \sqrt{5})$ where the coefficients of f and g are both non-negative it is immediate that w is strictly decreasing in n, always greater than $\frac{1}{2}$ and maximized by $n^* = 1$. The case where the coefficient of g is negative requires more work however. The key observation driving the argumentation that follows now is that f decreases much slower than g and thus the term with the positive coefficient will eventually dominate - even if for $\overline{\alpha}$ near $\frac{1}{2}$ this coefficient is very small.

Note first that we can rewrite w to

$$w(\overline{\alpha}, x) = \frac{1}{2} + g(x)\left[(\overline{\alpha} - \frac{1}{2})\frac{f(x)}{g(x)} + (2\overline{\alpha}^2 - \overline{\alpha} - \frac{1}{2})\right].$$

In order to show that this is greater than $\frac{1}{2}$ for x sufficiently large (and finite) it is sufficient to show that f/g tends to infinity as x gets large because this guarantees that the first (positive) summand in the squared brackets will eventually dominate the second one making the term in the squared brackets positive. Now f/g is easily calculated to be

$$\frac{f(x)}{g(x)} = \frac{2^x - 1}{x}$$

which obviously tends to infinity as x gets large.

Next we show for fixed $\overline{\alpha}$ that $w(\overline{\alpha}, x)$ has exactly one local maximum in x. This is equivalent to showing that (depending on $\overline{\alpha}$) the x-derivative of $w(\overline{\alpha}, x)$ is either negative for all x or changes signs exactly once on $[1, \infty)$ from positive to negative. Note that we can write this derivative as

$$\frac{\partial w}{\partial x}(\overline{\alpha}, x) = g'(x)\left[(\overline{\alpha} - \frac{1}{2})\frac{f'(x)}{g'(x)} + (2\overline{\alpha}^2 - \overline{\alpha} - \frac{1}{2})\right]. \tag{5}$$

Recalling that $g' < 0$ it is clear that it is sufficient to prove that f'/g' is monotonically increasing and tends to infinity. It is easily calculated that

$$\frac{f'(x)}{g'(x)} = \frac{-1 + 2^x - x\ln(2)}{x^2\ln(2)} \tag{6}$$

which clearly tends to infinity as x gets large. In order to show monotonicity consider the derivative of f'/g' which can be written as

$$\frac{d}{dx}\frac{f'(x)}{g'(x)} = \frac{2^x(x\ln(2) - 2) - (-x\ln(2) - 2)}{x^3\ln(2)}.$$

To determine the sign of this expression we can concentrate on the numerator. Note that the numerator is exactly the difference between the function $2^x(x\ln(2) - 2)$ and its first order Taylor approximation around 0. Since $2^x(x\ln(2) - 2)$ is strictly convex on $(0, \infty)$ (its second derivative is $2^x x\ln(2)$) this difference is positive for $x > 0$. Thus f'/g' is monotonically increasing for $x \geq 0$ as desired. Thus we have shown for every $\overline{\alpha} > \frac{1}{2}$ that $w(\overline{\alpha}, x)$ has a unique maximizer $1 \leq x^* < \infty$.

Now we will consider $w(\overline{\alpha}, n)$ with an integer parameter n again. From the previous analysis it is clear that $w(\overline{\alpha}, n)$ is globally maximized by $n = 1$ if and only if

$$w(\overline{\alpha}, 1) - w(\overline{\alpha}, 2) \geq 0$$

(and $n = 1$ is the unique maximizer if the inequality holds strictly). Now we have

$$w(\overline{\alpha}, 1) - w(\overline{\alpha}, 2) = \frac{1}{2}\overline{\alpha}^2 + \frac{1}{8}\overline{\alpha} + \frac{3}{16}.$$

Since this is a quadratic polynomial, it is easily seen that it is increasing over $[\frac{1}{2}, 1]$ and zero for $\overline{\alpha} = \frac{3}{4}$. Thus $n^* = 1$ for $\overline{\alpha} \geq \frac{3}{4}$ and $n^* > 1$ for $\overline{\alpha} < \frac{3}{4}$. That for fixed $\overline{\alpha}$ there are at most two integer maximizers of $w(\overline{\alpha}, n)$ and that, if there are two, those must be subsequent integers follows trivially from the fact that $w(\overline{\alpha}, x)$ has a unique real-valued local (and thus global) maximizer.

In the final part of the proof we show an upper bound on n^* in terms of $\overline{\alpha}$. We will again consider the function $w(\overline{\alpha}, x)$ with real valued argument $x \geq 1$. Note that since $w(\overline{\alpha}, x)$ has a unique local maximum for fixed $\overline{\alpha}$, any point x where the x-derivative of $w(\overline{\alpha}, x)$ is negative must lie to the right of the maximizer x^*. We will construct a function $B(\overline{\alpha})$ with the property that

$$x > B(\overline{\alpha}) \Rightarrow \frac{\partial}{\partial x} w(\overline{\alpha}, x) < 0.$$

This implies $x^* \leq B(\overline{\alpha})$ and thus $n^* \leq B(\overline{\alpha}) + 1$.

Note that from (5) and (6) $\frac{\partial}{\partial x} w(\overline{\alpha}, x) < 0$ is equivalent to

$$\frac{-1 + 2^x - x\ln(2)}{x^2 \ln(2)} > \frac{\overline{\alpha} + \frac{1}{2} - 2\overline{\alpha}^2}{\overline{\alpha} - \frac{1}{2}}.$$

We will now try to find a sufficient condition for this which is of the desired form. Note that the numerator of the right hand side only fluctuates between $\frac{1}{2}$ and $-\frac{1}{2}$ and thus a sufficient condition is

$$\frac{2^x}{x^2 \ln(2)} > \frac{1 + x\ln(2)}{x^2 \ln(2)} + \frac{1}{2(\overline{\alpha} - \frac{1}{2})}.$$

Now note that the first term on the right hand side is at most $(1 + \ln(2))/\ln(2)$ and thus a sufficient condition is

$$\frac{2^x}{x^2} > 1 + \ln(2) + \frac{\ln(2)}{2(\overline{\alpha} - \frac{1}{2})}.$$

Taking logarithms on both sides yields

$$x\ln(2)-2\ln(x)>\ln\left(1+\ln(2)+\frac{\ln(2)}{2(\overline{\alpha}-\frac{1}{2})}\right).$$

Since $\ln(x)$ is concave we can bound it from above by its first order Taylor approximation in 8 which is $\ln(8)+(1/8)x$.[24] Thus a sufficient condition is

$$x>\frac{2\ln(8)}{\ln(2)-\frac{1}{4}}+\frac{1}{\ln(2)-\frac{1}{4}}\ln\left(1+\ln(2)+\frac{\ln(2)}{2(\overline{\alpha}-\frac{1}{2})}\right).$$

Since this bound is not very sharp anyway we can afford to improve readability by bounding the logarithms by real numbers in a way that the condition remains sufficient and get

$$x>9.4+2.3\ln\left(1.7+\frac{0.35}{\overline{\alpha}-\frac{1}{2}}\right)=:B(\overline{\alpha}).$$

As argued above this implies

$$n^*\leq 10.4+2.3\ln\left(1.7+\frac{0.35}{\overline{\alpha}-\frac{1}{2}}\right).$$

□

Proof of Corollary 3

The corollary follows immediately from the fact that

$$\frac{\partial}{\partial\overline{\alpha}}\,\overline{\alpha}(1-\overline{\alpha})^{n-1}=(1-n\overline{\alpha})(1-\overline{\alpha})^{n-2}$$

is negative for $\overline{\alpha}\in(\frac{1}{n},1)$. □

[24] The choice of 8 as the expansion point is essentially arbitrary, it only matters that the value is large enough so that the left hand side of the equation remains increasing in x.

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