Individual versus Aggregate Income Elasticities for Heterogeneous Populations

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Abstract

The paper deals with different concepts of income elasticities of demand for a heterogenous population and the relationship between individual and aggregate elasticities is analyzed. In general, the aggregate elasticity is not equal to the mean of individual elasticities. The difference depends on the heterogeneity of the population and is quantified by a covariance term. Sign and magnitude of this term are determined by an empirical analysis based on the U.K. Family Expenditure Survey. It is shown that the relevant quantities can be identified from cross-section data and, without imposing restrictive structural assumptions, can be estimated by nonparametric techniques.

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1 Introduction

Long before the formal economic theory of consumer behavior (and the concept of a demand function) was developed, it was recognized that income is an important explanatory variable for consumer demand. We refer to Stigler (1954) for the early history of empirical studies. Certainly, there are other explanatory variables, such as prices and preferences. In order to derive a complete set of explanatory variables one needs a precise and complete description of the decision situation. Does the consumer face an a-temporal or inter-temporal decision with or without uncertainty?

These alternative decision problems are studied in detail in the microeconomic theory of consumer behavior (e.g., Deaton and Muellbauer, 1980, Romer, 2006; for a concise formulation see Section 5.2 of Hildenbrand and Kneip, 2005). For a given period (e.g., a specified year) this leads to a relation which generally can be written in the following form:

$$c^h \; = \; f(x^h, v^h),$$

where c^h denotes the expenditure in current prices on a certain category of consumption goods (such as food or services) of consumer h, x^h is disposable income, and v^h denotes the vector of all other explanatory variables. The nature of variables subsumed by v^h crucially depends on the decision situation. In any case v^h will contain prices and preference parameters. In an inter-temporal setting v^h will also incorporate suitably formalized future expectations.

In order to measure how sensitive consumer h reacts to an income change under the ceteris paribus condition that v^h remains constant, one considers the elasticity of consumption expenditure with respect to income, 'income elasticity' for short, defined by

$$\beta(x^h, v^h) := \frac{x^h}{c^h}\partial_x f(x^h, v^h) = \partial_y \log f(e^{y^h}, v^h),$$

where $y^h = \log x^h$. Thus, if the consumer's income increases by one percent then his consumption expenditure increases by β percent.

For economic policy analysis one needs mean (aggregate) consumption expenditure across a large and heterogeneous population H of households. Let $\nu_{x,v}$ denote the joint distribution of the explanatory variables x^h and v^h across the population H. Then mean consumption expenditure is equal to $C_{mean} := \int f(x,v)d\nu_{x,v} \equiv F(\nu_{x,v})$. Therefore the 'explanatory variable' for mean demand is the distribution $\nu_{x,v}$. The marginal ν_x is the income distribution, and $X_{mean} := \int x d\nu_x$ is mean income.

Consider a change in income on the micro-level, $x^h \rightarrow \tilde{x}^h$, under the above ceteris paribus condition. This leads to a new distribution $\tilde{\nu}_{x,v}$ and a changed mean income $\int x d\tilde{\nu}_x$. If, for a heterogeneous population, one wants to relate the resulting change in mean consumption expenditure to the change in mean income, then one has to specify, either on the micro-level how the change in mean income is allocated across the households in the population, or on the distributional level, how the changed distribution $\tilde{\nu}_{x,v}$ is generated from $\nu_{x,v}$.

Throughout this paper we consider a proportional change on the distributional level (a precise definition is given in Section 2). This corresponds to the concept of "mean scaled" income distributions as introduced by Lewbel (1990, 1992). Thus, relative income distributions remain unchanged and, hence, income inequality measures such as Gini or the coefficient of variation are unaffected.

In this setup the elasticity of mean consumption expenditure with respect to mean income, 'aggregate income elasticity' for short, is given by (see Section 2)

$$\beta_{agg} = \frac{X_{mean}}{C_{mean}} \partial_\mu \left(\int f(\frac{\mu}{X_{mean}} x, v) d\nu_{x,v} \right) \Big|_{\mu = X_{mean}} .$$

Hence, if, for example, the income of *each* consumer increases by one percent, then mean consumption expenditure increases by β_{agg} percent. We emphasize that β_{agg} will in general not be equal to the mean of individual elasticities. Indeed, a simple calculation given in Section 2 leads to

$$\beta_{agg} = \beta_{mean} + \frac{1}{C_{mean}} \mathrm{Cov}(f(x,v), \beta(x,v)),$$

where β_{mean} denotes the mean of individual elasticities, and $\mathrm{Cov}(f(x,v), \beta(x,v))$ is the covariance between individual consumption expenditures and individual elasticities with respect to the distribution $\nu_{x,v}$.

What can be said about the sign or the magnitude of the covariance term? Do households with large demand tend to have large or small elasticities? Under which circumstances can one expect the covariance term to be negligible? The latter is often implicitly assumed in applied work when the magnitude of the estimated aggregate elasticity is interpreted in terms of individual behavior.

Even in the case of a population which is homogeneous in demand behavior, without specifying the demand function, nothing definitive can be said about the sign of the covariance term. For example, even if the common demand function describes the demand for a necessity the sign of the covariance term can be positive or negative. Consequently, the above questions have to be answered by empirical studies.

Many contributions in the literature estimate elasticities based on cross-section and panel data. The standard approach relies on a parametric modeling of demand or consumption expenditure. In addition to specifying the functional relationship between consumption c^h, income x^h and prices p in the current period, possible dependencies between income level x^h and all remaining household specific explanatory variables in v^h have to be taken into account in parametric modeling. It is standard practice to stratify the population according to observable profiles a^h of household attributes (such as family size, age, etc.). One then assumes that for a given profile $a^h = a$ the corresponding subpopulation is homogeneous in the sense that remaining variation in consumption expenditure (e.g. due to heterogeneity in individual preferences) can be described by an additive error term ϵ^h. In our notation such an approach postulates a mapping $v^h \to (p, a^h, \epsilon^h)$ and a resulting parametric model may be written in the form $c^h = f(x^h, v^h) = g(p, x^h, a^h; \theta) + \epsilon^h$.[1] Here, g is a known model function, while θ is an unknown vector of coefficients which has to be estimated from the data. Based on estimates $\hat{\theta}$, one may then compute approximations of individual elasticities.

There is an extensive literature on estimating elasticities based on such parametric approaches. Some of these estimates correspond to β_{mean}, some to β_{agg}, while others are the elasticities of g evaluated at mean or median income. For example, Houthakker (1957) assumes a double logarithmic model. In this case the covariance term is zero and hence, β_{mean} equals β_{agg}. Banks et al. (1997) rely on a more general specification (QUAIDS) and their concept of income elasticity seems to correspond to β_{agg}, since they compute an expenditure weighted mean of individual elasticities. Blundell et al. (1993) also use a QUAIDS approach and determine budget elasticities computed at the average shares and household attributes. For further empirical studies, see, e.g., Jorgenson et al (1982) or Lewbel (1989). Recently nonparametric techniques have been used to estimate aggregate elasticities β_{agg} by Chakrabarty et al. (2006).

Another branch of literature deals with the estimation of income elasticities of consumption expenditure using time-series data. In such models aggregate consumption expenditure is assumed to be a function of aggregate current and past income and other explanatory variables. Hence, under the implicit assumption that changes over time in the income distribution are captured by changes in mean income one is able to estimate an aggregate income elasticity. Important contributions in this context are, for example, Davidson et al. (1978) and Campbell and Mankiw (1990).

In this paper we show that under general, qualitative conditions the various concepts of elasticities developed in the above sketched theoretical framework can be identified and estimated from cross-section data (Section 2). In Section 3 -5 we then present an empirical study based on the microdata from the British Family Expenditure Survey (FES) (1974-1993). We estimate β_{mean}, β_{agg}, as well as close approximations to individual elasticities and the covariance term. No functional form specification of the behavioral relations is required, and no restrictive distributional assumptions have to be made. We use recent identification results described in Hoderlein and Mammen (2007) as well as nonparametric techniques for estimating regression and quantile functions as proposed by Li and Racine (2004, 2006).

We emphasize that we analyze elasticities with respect to disposable income which can be considered as an exogeneous variable. Many estimates in the literature are elasticities with respect to total expenditure. This situation is also covered by our methodology provided that

[1] Models may also be formulated with respect to log consumption expenditure, $\log c^h = g(p, x^h, a^h; \theta) + \epsilon^h$, or budget shares $w^h = c^h/x^h$, $w^h = g(p, x^h, a^h; \theta) + \epsilon^h$.

total expenditure can be considered as an exogeneous variable. Otherwise, more involved nonparametric instrumental variable techniques may be applied. However, one can show that under additional assumptions our methodology offers an easy way to circumvent this problem.

The results of our empirical study are presented in Section 5. In particular, it turns out that the aggregate elasticity can be very different from the mean of individual elasticities. The magnitude of this difference varies from commodity to commodity. For expenditure on 'food' and 'services', as well as for 'total expenditure', aggregate income elasticity is significantly greater than the mean individual elasticities for almost all sample years. In the extreme, for expenditure on 'services' the difference can be as large as 30% of the aggregate elasticity. On the other hand, for the commodity groups 'clothing and footware' and 'fuel and light' aggregate and mean individual elasticities are quite close.

The outline of the paper is as follows: Section 2 provides precise descriptions of our theoretical setup and of corresponding identification results. Section 3 contains information about the FES data set used in our analysis, while nonparametric estimation procedures are discussed in Section 4. Empirical results and conclusions are presented in Section 5.

2 Individual and aggregate income elasticities

In this section we first define individual and aggregate elasticities for a population of households. The relation between these concepts is investigated. We then study the question in how far the quantities of interest can be identified and, therefore, are estimable from cross-section data. Our setup is based on general, qualitative conditions and avoids restrictive parametric model assumptions.

As explained in the introduction, a specific household with income x and a vector of further explanatory variables v determines his consumption $c \in \mathbb{R}$ of a specific commodity (e.g. food consumption) by

$$c = f(x, v),$$

The function f is assumed to be twice continuously differentiable in x. The individual income elasticity is then given by

$$\beta(x, v) := \frac{x}{c} \partial_x f(x, v) = \partial_y \log f(e^y, p), \tag{1}$$

where $y = \log x$.

In a large population heterogeneity in explanatory variables will generate a *joint distribution* of (x, v). Let C, X, V be corresponding generic random variables describing consumption expenditure, income and other explanatory variables of a randomly drawn household. We will use $\nu_{x,v}$ to denote the joint probability distribution of (X, V). $\nu_{x,v}$ then induces corresponding distributions of consumption $C = f(X,V)$ and individual elasticities $\beta(X,V) = \frac{X}{C}\partial_x f(X,V)$ in the population. The mean individual elasticity over the population is then given by

$$\beta_{mean} := \mathbb{E}(\beta(X,V)) = \mathbb{E}(\frac{X}{C}\partial_x f(X,V)) = \int \frac{x}{f(x,v)}\partial_x f(x,v) d\nu_{x,v} \tag{2}$$

Let $C_{mean} = \int f(x,v)\ d\nu_{x,v}$ denote mean consumption, while $X_{mean} = \int x\ d\nu_{x,v}$ denotes mean income. The idea motivating our definition of an aggregate income elasticity may be expressed as follows: quantify the proportional change of mean consumption in dependence of the proportion $\frac{\mu}{X_{mean}}$, when mean income is changed from X_{mean} to $\mu \neq X_{mean}$. For fixed distribution of (X,V), one considers the effect of a transformation $X \to (\frac{\mu}{X_{mean}}X)$. Obviously, $\mathbb{E}(\frac{\mu}{X_{mean}}X) = \mu$, and resulting mean consumption is given by $\mathbb{E}\left(f(\frac{\mu}{X_{mean}}X,V)\right)$. The aggregate income elasticity is then defined by

$$\begin{aligned}\beta_{agg} &= \frac{X_{mean}}{C_{mean}}\partial_\mu \mathbb{E}\left(f(\frac{\mu}{X_{mean}}X,V)\right)\Big|_{\mu=X_{mean}} = \frac{X_{mean}}{C_{mean}}\partial_\mu \int f(\frac{\mu}{\mu_x}x,v)d\nu_{x,v}\Big|_{\mu=X_{mean}} \\ &= \frac{1}{C_{mean}}\int x\partial_x f(x,v)d\nu_{x,v}\end{aligned} \tag{3}$$

It is now immediately seen that generally β_{agg} does not coincide with β_{mean}. Obviously,

$$\begin{aligned}\beta_{agg} &= \frac{1}{C_{mean}}\int f(x,v)\frac{x}{f(x,v)}\partial_x f(x,v)\ d\nu_{x,v} = \mathbb{E}(\frac{1}{C_{mean}}C\beta(X,V)) \\ &= \frac{1}{C_{mean}}C_{mean}\mathbb{E}(\beta(X,V)) + \frac{1}{C_{mean}}\text{Cov}(C,\beta(X,V)) = \beta_{mean} + \frac{1}{C_{mean}}\text{Cov}(C,\beta(X,V))\end{aligned} \tag{4}$$

Let us now consider the question which of the above quantities are identifiable from cross-section data. A basic problem is that individual preferences and, hence, the parameters v are not directly observable. However, all expenditure surveys provide information about important household attribute profiles a, as for example household size, employment status, age of household members, etc. Let us thus analyze the situation that there is an i.i.d. sample (C_i, X_i, A_i), $i = 1, \ldots, n$, containing information about consumption, income and household attributes of n randomly selected households. Following the above notation, the distribution of (C_i, X_i, A_i) corresponds to the distribution of generic variables (C, X, A). The introduction of attribute profiles is crucial, since generally A will be correlated with the unobservable random variable V. Let $\nu_{x,a}$ denote the joint distribution of (X, A), while $\nu_{v|x,a}$ stands for the conditional distribution of V given (X, A). Note that in our setup X denotes disposable income of a household (and not total expenditure). X (and $Y = \log X$) are thus assumed to be exogeneous variables. A standard assumption which implicitly or explicitly provides the very basis for almost all theoretical and applied work to be found in the literature is as follows:[2] If attribute profiles a provide sufficient information about the characteristics of an household, then for the subgroups of all household with the same attributes $A = a$ the variation in V only reflects variation in individual preferences which may be assumed to be independent of income. More formally, further analysis will rest upon the following assumption:

Assumption (*Conditional independence of X and V given A*):
For every income level x, $\nu_{v|x,a} = \nu_{v|a}$, where $\nu_{v|a}$ denotes the conditional distribution of V given $A = a$.

Let us now first study identification of β_{mean} and β_{agg}. Set $Y = \log X$,

$$\bar{c}(y,a) := \mathbb{E}(C \mid Y = y, A = a), \quad \bar{c}_{log}(y,a) := \mathbb{E}(\log C \mid Y = y, A = a),$$

and let $\nu_{y,a}$ be the joint distribution of (Y, A). Under the above assumption,

$$\begin{aligned} \beta_{mean} &= \mathbb{E}(\beta(X,V)) = \mathbb{E}(\frac{X}{C}\partial_x f(X,V)) = \int \left(\int \frac{x}{f(x,v)} \partial_x f(x,v) d\nu_{v|a} \right) d\nu_{x,a} \\ &= \int \partial_y \left(\int \log f(e^y, v) d\nu_{v|a} \right) d\nu_{y,a} = \int \partial_y \bar{c}_{log}(y,a) d\nu_{y,a} \end{aligned} \tag{5}$$

and

$$\begin{aligned} \beta_{agg} &= \frac{1}{C_{mean}} \int x \partial_x f(x,v) d\nu_{x,v} = \frac{1}{C_{mean}} \int \partial_y \left(\int f(e^y, v) d\nu_{v|a} \right) d\nu_{y,a} \\ &= \frac{1}{C_{mean}} \int \partial_y \bar{c}(y,a) d\nu_{y,a} \end{aligned} \tag{6}$$

The functions $\bar{c}(y,a)$ and $\bar{c}_{log}(y,a)$ are well identified regression functions. Nonparametric regression procedures can be used to determine estimates $\widehat{\bar{c}(y,a)}$ and $\widehat{\bar{c}_{log}(y,a)}$ by regressing C_i on (Y_i, A_i) and $\log C_i$ on (Y_i, A_i), respectively. By (5) and (6) the elasticities β_{mean} and β_{agg} then are (suitably scaled) **average derivatives** of $\bar{c}_{log}(y,a)$ and $\bar{c}(y,a)$, which may be estimated by $\frac{1}{n}\sum_{i=1}^n \partial_y \widehat{\bar{c}_{log}(Y_i, A_i)}$ and $\frac{1}{\bar{C}n}\sum_{i=1}^n \partial_y \widehat{\bar{c}(Y_i, A_i)}$, respectively, where $\bar{C} = \frac{1}{n}\sum_{i=1}^n C_i$. Details of our estimation procedures are given in Section 4.

Identification of individual elasticities is, of course, a much more difficult problem. Quite surprisingly, in a general setup it is possible to get some "close" approximations. Our identification strategy is based on the approach of Hoderlein and Mammen (2007).

[2] Parametric models of demand may be written in the form $C_i = g(p, X_i, A_i; \theta) + s(p, X_i, A_i; \theta) \cdot \epsilon_i$ (or $\log C_i = g(p, X_i, A_i; \theta) + s(p, X_i, A_i; \theta) \cdot \epsilon_i$) for some prespecified functions g and s, where $\epsilon_1, \epsilon_2, \ldots$ are i.i.d random errors ϵ_i with $\mathbb{E}(\epsilon_i) = 0$ and $Var(\epsilon_i) = 1$ which are assumed to be *independent* of X_i, A_i. The function s may be used to account for possible heteroscedasticity, while θ denotes some unknown vector of parameters that have to be estimated from the data. In such a setup the "error term" $\epsilon_i \equiv \epsilon(V_i)$ obviously captures remaining heterogeneity of C_i for given (X_i, A_i). Conditional independence of X_i and V_i given A_i is an immediate consequence.

For $0 \leq \tau \leq 1$, let $k(\tau; y, a)$ denote the conditional τ-quantile of $\log C$ given $Y = y$ and $A = a$. More formally, $P(\log C \leqslant k(\tau; y, a)|Y = y, A = a) = \tau$. We will assume that $k(\tau; y, a)$ is continuously differentiable with respect to y and that $k(\tau; y, a)$ is strictly increasing in τ for all (y, a). For any given (c, y, a) there then exists some $\tau_{c,y,a}$ such that $\log c = k(\tau_{c,y,a}; y, a)$ Under some mild regularity conditions on the distribution $\nu_{y,a}$, the results of Hoderlein and Mammen (2007) then imply that

$$\beta_{c,y,a} := \mathbb{E}\big[\partial_y \log f(e^y, V, p)|Y = y, A = a, C = c\big] = \partial_z k(\tau_{c,y,a}; z, a)\Big|_{z=y}. \tag{7}$$

We will refer to $\beta_{c,y,a}$ as a "local" elasticity. By definition,

$$\beta_{c,y,a} = \mathbb{E}\left(\beta(X, V)\ |Y = y, A = a, C = c\right),$$

and thus it is the conditional mean of $\beta(X, V)$ over a subpopulation of households with log income y, attributes a *and* consumption expenditure equal to c. Although households within such a subpopulation can still be heterogeneous in v, they show the same consumption behavior given y and a. Consequently, when using i.i.d. data providing information about consumption C_i , income X_i and household attributes A_i, $i = 1, \ldots, n$, approximating the individual elasticity $\beta(X_i, V_i)$ by the conditional mean $\beta_i := \beta_{C_i,Y_i,A_i} = \mathbb{E}\big[\partial_y \log f(e^{Y_i}, V, p)|Y = Y_i, A = A_i, C = C_i\big]$, $\quad i = 1, \ldots, n$ is the best we can do on the basis of the available information. Local and individual income elasticities will coincide if for given (y, a) there is a one-to-one relation between consumption c and (preference) parameters v.[3] Identification of demand function under such "monotonicity" constraints has been considered by Matzkin (2003).

By relying on nonparametric quantile estimation techniques, nonparametric estimates $\widehat{k(\tau; y, a)}$ of conditional quantile functions and their derivatives can be determined from cross-section data. For any observation (C_i, Y_i, A_i) the corresponding conditional quantile position of $\log C_i$ given (Y_i, A_i) can be computed in a straightforward way which then leads to estimates $\widehat{\beta}_i$ of local elasticities.

Local elasticities provide a mean to estimate $\text{Cov}(C, \beta(X, V))$. Obviously, $\mathbb{E}(\beta_i) = \mathbb{E}(\beta_{C,Y,A}) = E(\beta(X, V)) = \beta_{mean}$, and

$$\begin{aligned} \text{Cov}(C, \beta(X, V)) &= \mathbb{E}(C\beta(X, V)) - C_{mean}\mathbb{E}(\beta(X, V)) = \mathbb{E}\left[C\mathbb{E}(\beta(X, V)|Y, A, C)\right] - C_{mean}\beta_{mean} \\ &= \mathbb{E}\left[C\beta_{C,Y,A}\right] - C_{mean}\beta_{mean} = \text{Cov}(C, \beta_{C,Y,A}) = \text{Cov}(C_i, \beta_i) \end{aligned} \tag{8}$$

Remark: As mentioned in the introduction many contributions in the literature aim at estimating elasticities with respect to total expenditure on nondurables, "budget" for short,

[3] Such an assumption is made in any parametric model of demand. If for example, $C_i = g(p, X_i, A_i; \theta) + s(p, X_i, A_i; \theta) \cdot \epsilon_i$ for some prespecified g, s, then $\beta_i = \mathbb{E}\big[\partial_y \log f(e^{Y_i}, V)|Y = Y_i, A = A_i, C = C_i\big] = \partial_y \log(g(p, e^{Y_i}, A_i; \theta) + s(p, e^{Y_i}, A_i; \theta) \cdot \epsilon_i)$ equals the individual elasticity $\beta(X_i, V_i)$ (recall that $\epsilon_i \equiv \epsilon(V_i)$).

which is usually considered as an endogeneous variable. The behavioral relations imply that $C_{tot} = f_{tot}(X, V)$ and $C = f(X, V)$, where C_{tot} denotes budget and C refers to consumption expenditure on another commodity. Assuming monotonicity of f_{tot} in income, C can be rewritten as a function of budget, $C = \tilde{f}(C_{tot}, V) = \tilde{f}(f_{tot}(X, V), V)$. The elasticity with respect to budget is then given by $\tilde{\beta}(C_{tot}, V) = \frac{C_{tot}}{C} \partial_{c_{tot}} \tilde{f}(C_{tot}, V)$. Taking derivatives yields

$$\frac{C}{X}\beta(X, V) = \frac{C}{C_{tot}}\tilde{\beta}(C_{tot}, V) \cdot \frac{C_{tot}}{X}\beta_{tot}(X, V), \tag{9}$$

where β_{tot} denotes the elasticity of C_{tot} with respect to income. Under the additional assumption that local elasticities are equal to individual elasticities, we can infer from (9) that the mean elasticity $\beta_{mean,tot} := \mathbb{E}(\tilde{\beta}(C_{tot}, V))$ with respect to budget corresponds to

$$\beta_{mean,tot} = \mathbb{E}\left(\frac{\beta(X, V)}{\beta_{tot}(X, V)}\right) = \mathbb{E}\left(\frac{\beta_{C,Y,A}}{\beta_{tot;C_{tot},Y,A}}\right), \tag{10}$$

where $\beta_{tot;C_{tot},Y,A}$ denotes local elasticities of C_{tot} with respect to income. This elasticity may then be estimated by $\widehat{\beta}_{mean,tot} = \frac{1}{n}\sum_i \frac{\hat{\beta}_i}{\hat{\beta}_{tot,i}}$. Similar arguments may then be used to show that the corresponding aggregate elasticity $\beta_{agg,tot}$ can also be determined from local elasticities.

3 Data Description

Our empirical analysis bases on the British Family Expenditure Survey, which contains cross-section data on consumption expenditure, income and socioeconomic characteristics of British households. FES was launched in the late 50s but due to changes in survey design and the following inconsistency in variable definitions we restrict our analysis to the period 1974-1993.

Annually, FES asks approximately 7000 households to keep a detailed account of their expenditures on a variety of commodity groups for 14 consecutive days. Depending on how necessary the good is, one might expect different demand behavior for different categories of goods. Therefore we perform our analysis for the major four commodity groups: 'food', 'fuel and light', 'services', and 'clothing and footware,' as well as for total (nondurable) expenditure. As far as the income variable is concerned, it is the natural logarithm of the disposable non-property income, which is obtained by deducting investment income and all taxes from total income.[4]

[4]Following HBAI standards, household incomes are obtained by extracting relevant items from the elementary database. The task of elaborating the database and specifying consistent variables has mainly been accomplished by Jürgen Arns and described in Arns (2006) and Arns and Bhattacharya (2005). His careful work is gratefully acknowledged.

A correct specification of the econometric model and the need for stratification of the population on attributes induced by the theoretical model motivates the inclusion of further explanatory variables for household demand in the empirical analysis. The following variables[5] have been chosen in our analysis: number of adults, children and persons working in the household, and age as well as employment status of the household's head.

4 Estimation procedures

In this section we this section we give a detailed description of our procedure for estimating the quantities β_{agg}, β_{mean}, β_i, and $\mathrm{Cov}(C, \beta(X, V))$ from cross-section data. It is assumed that for a given time period of interest there are observations (C_i, X_i, A_i) of consumption, income and attributes, $i = 1, \ldots, n$, for an i.i.d. sample of n households.

4.1 Estimation of β_{agg} and β_{mean}

By definition in (6) and (5) β_{agg} and β_{mean} are average derivatives with respect to the regression functions $\bar{c}_(y, a) = \mathbb{E}(C|Y = y, A = a)$ and $\bar{c}_{\log}(y, a) = \mathbb{E}(\log C_i|Y = y, A = a)$, where $Y = \log X$. In other words, in order to estimate β_{agg} we have to regress C on (Y, A), while for approximating β_{mean} one has to regress $\log C$ on (Y, A).

In principle, estimation could be based on valid parametric models for $\bar{c}(y, a)$ and $\bar{c}_{\log}(y, a)$, respectively. A straightforward approach would be to use a model for $\bar{c}(y, a)$, which is quadratic in log income and age, linear in the number of adults and children and dummies for employment status and allows for interaction between log income and age. Such a model could be then estimated by the least squares method and the derivative $\partial_y \bar{c}(y, a)$ could be computed as a function of estimated parameters. However, as shown by Chakrabarty et al. (2006) for the FES data, such a model suffers from misspecification according to the Ramsey (1969) RESET test for all commodity groups but for 'fuel and light' and 'clothing and footware' for some years.

In this paper we therefore adopt a nonparametric approach for estimating $\bar{c}$ and $\bar{c}_{\log}$. We rely on the methodology developed in Li and Racine (2004) which is well-adapted to the fact that some regressors are continuous (log income and age), while others are categorical (number of adults and children, employment status).

More precisely, the vector A_i of household attributes is split into the continuous attribute 'age of household head', denoted by age_i, and a vector $\tilde{A}_i$ of six discrete variables: three

[5] For a more detailed exposition of the elementary data set and variable definitions we refer to the FES Handbook by Kemsley et al. (1980).

dummies for the employment status of the household's head (unemployed/unoccupied, self-employed, and retired), number of persons in the household, number of children, and number of persons working). Among the discrete variables we distinguish unordered and ordered ones. We treat the three employment status dummies as unordered and the remaining discrete regressors as ordered variables. Following Li and Racine (2004), estimates of the regression functions are obtained by local linear weighted regressions.

Let $Z_i = (Y_i, \text{age}_i, \tilde{A}_i^T)^T$, $i = 1, \ldots, n$ denote the individual vectors of all explanatory variables. For a given point $z = (y, \text{age}, \tilde{a}^T)^T$ estimates of $\bar{c}(y, a)$ and $\partial_y \bar{c}(y, a)$ are then determined by $\widehat{\bar{c}(y, a)} := \hat{\zeta}_0$ and $\partial_y \widehat{\bar{c}(y, a)} := \hat{\zeta}_1$, respectively, where $\hat{\zeta}_0$, $\hat{\zeta}_1$, $\hat{\zeta}_2$ minimize

$$\sum_{i=1}^{n} \left[C_i - \zeta_0 - \zeta_1(Y_i - y) - \zeta_2(\text{age}_i - \text{age})\right]^2 W_{i,\mathbf{h}}(z),$$

over all possible values $\zeta_0, \zeta_1, \zeta_2$. Hereby, $W_{i,\mathbf{h}}$ is a kernel weight for household i at point z, which depends on the bandwidth vector $\mathbf{h}$. These weights are computed as a product of univariate kernel functions, where the functional forms of the kernels are chosen according to the nature of the respective variables: Epanechnikov kernels $\kappa(\cdot)$ for continuous, Aitchison and Aitken (1976) kernel $l^u(\cdot)$ for unordered categorical, and Wang and Van Ryzin (1981) kernel $l^o(\cdot)$ for ordered discrete variables.[6] Consistency and asymptotic normality of this estimators follow from the results of Li and Racine (2004).

Similarly, estimates $\widehat{\bar{c}_{\log}(y, a)} := \hat{\zeta}_0^*$ and $\partial_y \widehat{\bar{c}_{\log}(y, a)} := \hat{\zeta}_1^*$ are calculated from the minimizers $\hat{\zeta}_0^*$, $\hat{\zeta}_1^*$, $\hat{\zeta}_2^*$ of

$$\sum_{i=1}^{n} \left[\log C_i - \zeta_0^* - \zeta_1^*(Y_i - y) - \zeta_2^*(\text{age}_i - \text{age})\right]^2 W_{i,\mathbf{h}}(z).$$

By (6) and (5) this then leads to the estimates

$$\hat{\beta}_{agg} = \frac{1}{\bar{C}n} \sum_{i=1}^{n} \partial_y \widehat{\bar{c}(Y_i, A_i)}, \quad \hat{\beta}_{mean} = \frac{1}{n} \sum_{i=1}^{n} \partial_y \widehat{\bar{c}_{log}(Y_i, A_i)}.$$

The optimal smoothing parameters for estimating $\bar{c}$ and $\bar{c}_{log}$, which are denoted by $\mathbf{h}_{CV}$ and $\mathbf{h}_{CV}^*$, respectively, are chosen by a least-squares cross-validation algorithm as described

[6] More precisely,

$$W_{i,\mathbf{h}}(z) = \kappa\Big(\frac{Y_i - y}{\mathbf{h}_1}\Big)\kappa\Big(\frac{age_i - age}{\mathbf{h}_2}\Big) \prod_{s=1}^{3} l^u(\tilde{A}_{is}, \tilde{a}_s, \mathbf{h}_s) \prod_{s=4}^{6} l^o(\tilde{A}_{is}, \tilde{a}_s, \mathbf{h}_s),$$

where κ, l^u, and l^o are continuous, unordered discrete, and ordered discrete kernels, respectively. They are defined by $\kappa(u) = \begin{cases} \frac{3}{4\sqrt{5}}(1 - \frac{1}{5}u^2), & \text{if } u^2 < 5 \\ 0, & \text{else} \end{cases}$, $l^u(\tilde{A}_{is}, \tilde{a}_s, \mathbf{h}_s) = \begin{cases} 1 - \mathbf{h}_s, & \text{if } \tilde{A}_{is} = \tilde{a}_{as} \\ \mathbf{h}_s/(o_s - 1), & \text{else} \end{cases}$,
and $l^o(\tilde{A}_{is}, \tilde{a}_s, \mathbf{h}_s) = \begin{cases} 1 - \mathbf{h}_s, & \text{if } \tilde{A}_{is} = \tilde{a}_s \\ \frac{1}{2}(1 - \mathbf{h}_s)\mathbf{h}_s^{|\tilde{A}_{is} - \tilde{a}_s|}, & \text{else} \end{cases}$, where o_s is the number of possible outcomes of $\tilde{A}_{is}$.

in Racine and Li (2004).[7] However, recall that we are interested in estimating corresponding average derivative of the regression function and not the regression function itself. Averaging reduces variability of the estimate but not bias. In a similar context, Härdle and Stoker (1989) show that by applying 'undersmoothing' bandwidths parametric rates of convergence can be achieved for average derivative estimators. We therefore determine $\hat{\beta}_{agg}$ and $\hat{\beta}_{mean}$ by using bandwidths $0.8\mathbf{h}_{CV,1}$ and $0.8\mathbf{h}^*_{CV,1}$ for log income, respectively.[8] Additionally, for the sake of stability of results, while computing the average derivative and the covariance term we neglect the highest and the lowest 0.5% of the values of the point derivatives.

Separately for each period, standard errors of $\hat{\beta}_{agg}$ and $\hat{\beta}_{mean}$ can be obtained by bootstrap. For i.i.d bootstrap resamples $(\log C_1^*, Z_1^*), \ldots, (\log C_n^*, Z_n^*)$ the distributions of $\hat{\beta}_{agg} - \beta_{agg}$, $\hat{\beta}_{mean} - \beta_{mean}$ are approximated by the conditional distribution of $\hat{\beta}^*_{agg} - \hat{\beta}_{agg}$, $\hat{\beta}^*_{mean} - \hat{\beta}_{mean}$ given (C_i, Y_i, A_i), $i = 1, \ldots, n$. Theoretical support for the use of such a naive bootstrap in the context of average derivative estimation can be found in Härdle and Hart (1991).

4.2 Estimation of local elasticities and Cov$(C, \beta(X, V))/C_{mean}$

As already explained in Section 2, the strategy for estimating the individual values of the local elasticities β_i, $i = 1, \ldots, n$, stems from Hoderlein and Mammen (2007). We apply a two-step procedure. In the first step, we determine estimates $\hat{\tau}_i$ of the quantiles $\tau_i := tau_{C_i,Y_i,A_i}$ with $\log C_i = k(\tau_{C_i,Y_i,A_i}; Y_i, A_i)$, i.e., of the quantile positions of $\log C_i$ in the distribution of log expenditure across the subpopulation with log income and attributes equal to (Y_i, A_i). In the second step, one estimates the partial derivative of $k(\tau_{C_i,Y_i,A_i}; y, A_i)$ at $y = Y_i$. As we do not want to impose any restrictive assumptions on the shape of the conditional quantile function $k(\cdot)$, our approach again relies on nonparametric procedures. As in the case of estimation of $\bar{c}(y, a)$ and $\bar{c}_{\log}(y, a)$ described above, we have to account for the presence of both discrete and continuous variables. We therefore apply a general method for quantile estimation which has been developed in a recent work by Li and Racine (2006).

Consistent estimators of τ_i, $i = 1, \ldots, n$, are then given by

$$\hat{\tau}_i = \frac{\sum_{j=1}^n G\big(\frac{\log C_j - \log C_i}{\mathbf{h}_0}\big) W_{j,\mathbf{h}}(Z_i)}{\sum_{j=1}^n W_{j,\mathbf{h}}(Z_i)},$$

where G is the cummulative univariate continuous kernel function, i.e., $G(t) = \int_{-\infty}^t \kappa(u)du$, $\mathbf{h}_0$ is the bandwidth parameter for C, and $W_{j,\mathbf{h}}(Z_i)$ is the kernel weight for the household j

[7] Numerical search for optimal smoothing parameters was performed using the N library made available by Jeff Racine. The estimation procedure itself was programmed in MATLAB and the corresponding routines are available from authors upon request.

[8] The estimates of $\hat{\beta}_{agg}$ and $\hat{\beta}_{mean}$ obtained with this bandwidth vector were very similar to those obtained when using factors 0.7 or 0.9. This may indicate that we are close to the optimal bandwidth (for the average derivative estimator).

at Z_i, which has already been defined above. Bandwidth parameters for the estimation of τ_i were chosen through a numerical search algorithm presented for conditional density estimation in Hall et al. (2004) method and properly adjusted for the estimation of conditional cummulative distribution functions as advocated by Li and Racine (2006).

In the second step, we perform a local linear[9] quantile regression at the quantile $\hat{\tau}_i$. More precisely, for each $i = 1, \ldots, n$ we calculate the values $\hat{\eta}_{0,i}, \hat{\eta}_{1,i}, \hat{\eta}_{2,i}$ minimizing

$$\sum_{j=1}^{n} \rho_{\hat{\tau}_i}\big[\log C_j - \eta_{0,i} - \eta_{1,i}(Y_j - Y_i) - \eta_{2,i}(\text{age}_j - \text{age}_i)\big] W_{j,\mathbf{h}}(Z_i), \tag{11}$$

with respect to all $\eta_{0,i}$, $\eta_{1,i}$, and $\eta_{2,i}$, where $\rho_\tau(u) = u[\tau - I(u \leq 0)]$ is a 'check function' typical for quantile regression problems.[10] In the above regression $\hat{\eta}_{1,i}$ estimates the partial derivative of $k(\tau_{C_i,Y_i,A_i}; y, A_i)$ at $y = Y_i$, and therefore $\hat{\beta}_i := \hat{\eta}_{1,i}$. By (8) these estimates $\hat{\beta}_i$ of local elasticities can then be used to estimate $\text{Cov}(C, \beta(X, V))$:

$$\widehat{\text{Cov}(C, \beta(X, V))} = \frac{1}{n}\sum_{i=1}^{n}(C_i - \bar{C})(\hat{\beta}_i - \bar{\beta}),$$

where $\bar{\beta} = \frac{1}{n}\sum_{i=1}^{n}\hat{\beta}_i$. Whereas estimation of average derivatives (as in the case of β_{agg} or β_{mean}) relies on a smaller bandwidth for log income than the optimal one for estimating the regression function, point derivatives of quantiles should be estimated using a *larger* bandwidth. Since the direct data-driven bandwidth selection methods in this situation are still an open question, we proceed as follows. First, we multiply the cross-validated bandwidth for log income by a factor 1.5 which results in $\mathbf{h}^*_d$ with $\mathbf{h}^*_{d,1} = 1.5\mathbf{h}^*_{CV,1}$ and $\mathbf{h}^*_{d,s} = \mathbf{h}^*_{CV,s}$, for $s > 1$.[11] Then, as advocated by Yu and Jones (1998), in order to obtain a suitable bandwidth for quantile derivative estimation we adjust smoothing parameters for log income $\mathbf{h}^*_{d,1}$ and age $\mathbf{h}^*_{d,2}$ in dependence of $\hat{\tau}_i$ by multiplying them by a factor $\left[\frac{\hat{\tau}_i(1-\hat{\tau}_i)}{\phi[\Phi^{-1}(\hat{\tau}_i)]^2}\right]^{1/6}$. Here ϕ and Φ are the pdf and the cdf of the standard normal distribution. For discrete variables we use the same bandwidths as in the mean regression case.[12]

[9] From the theoretical point of view local quadratic smoother outperforms the linear one in estimating the derivative of $k(\tau; y, a)$. However, in our application local quadratic regression (even for large bandwidths) leads to more instable estimates. In particular, for food expenditure we then obtain an unplausibly high percentage of negative elasticities.

[10] It is important to note that the presence of this function is the only difference between a typical (mean) regression and a quantile regression.

[11] Our estimates of β and $\text{Cov}(C, \beta(X, V))$ are stable with respect to changes in this multiplier between 1 and 2.

[12] Numerical search for optimal bandwidths in the first step was carried out by the N library made available by Jeff Racine. Estimators for both estimation steps were programmed in MATLAB. The solution to (11) was found by the interior point (Frisch-Newton) algorithm implemented in the RQ.m routine and described by Portnoy and Koenker (1997). Program codes for these routines are available from authors upon request.

4.3 Inference about Cov$(C, \beta(X, V))/C_{mean}$

Having estimated the aggregate elasticity, the mean of individual elasticities, and the covariance term, it is of interest to assess whether the difference $\beta_{agg} - \beta_{mean}$, or equivalently, the covariance term $\text{Cov}(C, \beta(X, V))/C_{mean}$ is significantly different from zero. We propose two different tests for equality of β_{agg} and β_{mean}.

In the first test, for each year of the sample we test the null hypothesis $H_0 : \text{Cov}(C, \beta(X, V))/C_{mean} = 0$. As there does not exist a closed form for the asymptotic standard error of the covariance term, in order to analyze its significance, the test is based on bootstrap confidence intervals. Bootstrap resamples $(\log C_1^*, Z_1^*), \ldots, (\log C_n^*, Z_n^*)$ are generated by drawing independently, with replacements n observations from the original sample $(\log C_1, Z_1), \ldots, (\log C_n, Z_n)$. For each bootstrap sample corresponding estimates $\text{Cov}(\widehat{C, \beta(X}, V))^*$ and $\bar{C}^* = \frac{1}{n}\sum_{i=1}^n C_i^*$ are determined. We then approximate the distribution of $(\text{Cov}(\widehat{C, \beta(X}, V))/\bar{C} - \text{Cov}(C, \beta(X, V))/C_{mean})$ by the bootstrap distribution $(\text{Cov}(\widehat{C, \beta(X}, V))^*/\bar{C}^* - \text{Cov}(\widehat{C, \beta(X}, V))/\bar{C})$ and obtain 95% confidence intervals for $\text{Cov}(C, \beta(X, V))/C_{mean}$, which are computed as

$$\text{C.I.} = [\text{Cov}(\widehat{C, \beta(X}, V))/\bar{C} - t^*_{0.975},\ \text{Cov}(\widehat{C, \beta(X}, V))/\bar{C} - t^*_{0.025}],$$

where t^*_α denotes the α quantile of the distribution of $(\text{Cov}(\widehat{C, \beta(X}, V))^*/\bar{C}^* - \text{Cov}(\widehat{C, \beta(X}, V))/\bar{C})$. As shown by Koenker (1994), this type of bootstrap performs very well in quantile regression problems under heteroscedasticity which is present in our data.

In the second test, we consider the significance of the average difference between β_{agg} and β_{mean} over the sample period (1974-1993). We obtain two series $\{\hat{\beta}_{agg,t}\}$ and $\{\hat{\beta}_{mean,t}\}$ for $t = 74, \ldots, 93$ and test their equality by means of the Wilcoxon (1945) test for matched pairs. Additionally, we perform the Wilcoxon signed-rank test for zero mean of $\text{Cov}(C, \beta(X, V))/C_{mean}$ based on observations $\text{Cov}(\widehat{C, \beta(X}, V))_t/\bar{C}_t$, $t = 74, \ldots, 93$. All empirical results are given in the next section.

5 Estimation Results and Conclusions

Tables 1-5 report our estimates of the aggregate elasticity $\hat{\beta}_{agg}$ (first column) and the mean individual elasticity $\hat{\beta}_{mean}$ (second column) for each commodity group. The third and the fourth column provide corresponding estimates $\bar{\beta} = \frac{1}{n}\sum_i \hat{\beta}_i$ of mean local elasticities and of $\text{Cov}(\widehat{C, \beta(X}, V))/\bar{C}$, respectively. In the parentheses next to the estimates we report their bootstrapped standard errors. Furthermore, in Figures 1-5 we plot the time-series of estimates $\{\hat{\beta}_{agg,t}\}$, $\{\hat{\beta}_{mean,t}\}$, as well as $\text{Cov}(\widehat{C, \beta(X}, V))_t/\bar{C}_t$, $t = 74, \ldots, 93$, with corresponding 95% confidence intervals.

Our estimation results lead to the following conclusions:

1) There are large differences in the magnitude of the elasticities among different commodity groups. In particular, an increase in aggregate income of 1% drives up aggregate expenditure for 'food' or 'fuel and light' by approximately 0.2%, whereas for expenditure on 'services' this increase is of roughly 1%. Total expenditure for all nondurable goods rises by about 0.5%.

2) From Figures 6-8, where we present kernel density estimates of the distribution of local elasticities $\hat{\beta}_i$ for different commodity groups in 1993 we see that these distributions are unimodal and exhibit a significant spread. This last feature indicates a substantial degree of heterogeneity in demand behavior across the population. Furthermore, according to the Jarque-Bera test, these distributions are very far from being normal for all years and for all commodity groups.[13]

3) The estimates of elasticities seem to be fairly stable over time. During the period 1974-1993, one can observe no pronounced trend in the estimates of both the aggregate elasticity and the mean individual elasticity.

4) The estimates of $\mathbb{E}(\beta(X,V)) = \beta_{mean}$ obtained by $\hat{\beta}_{mean}$ and by the average $\bar{\beta} = \frac{1}{n}\sum_i \hat{\beta}_i$ of local elasticities are of very similar magnitude, which could serve as a support for the reliability and robustness of these estimates. Further, for most commodity groups and sample years we can recover the relationship from the proposition saying that $\beta_{agg} = \beta_{mean} + \text{Cov}(C, \beta(X,V))/C_{mean}$, which provides further evidence for the appropriateness of our crucial assumption and our estimation strategy.

5) The perhaps most interesting empirical result is that aggregate elasticity can be very different from the mean of individual elasticities. The magnitude of this difference varies from commodity to commodity. For expenditure on food and services, as well as for total expenditure, aggregate income elasticity is greater than the mean individual elasticity for all sample years. In the extreme case of expenditure on services, the difference can be as large as 30% of the aggregate elasticity. On the other hand, for the commodity groups 'clothing and footware' and 'fuel and light' aggregate and mean individual elasticities are quite close.

As mentioned in the last section, in order to assess whether the discrepancy between β_{agg} and β_{mean} is statistically significant we expose this difference to several tests. The p-values from the Wilcoxon test for matched pairs of the hypothesis that the average (over the period

[13] Note that for a small group of households the elasticity is estimated to be negative. The size of this group varies by commodity group and year and is of magnitude of one to three percent of the sample. The explanation for the occurence of negative elasticities is the methodical artefact of the nonparametric smoother applied in our paper.

1974-1993) difference between β_{agg} and β_{mean} is zero are given in the last line of the table. The last line of the first and the second column report p-values based on the comparison of $\hat{\beta}_{agg}$ with $\hat{\beta}_{mean}$ and $\bar{\beta}$, respectively. Asterisks in Tables 1-5 denote the significance at the 95% level. According to these p-values, the aggregate elasticity is significantly greater than the mean of individual elasticities for 'food', 'services', and total expenditure. For 'clothing and footware' and 'fuel and light' the difference between β_{agg} and β_{mean} is not significant.

Similarly, according to the Wilcoxon signed-rank test of the hypothesis $\mathrm{Cov}(C, \beta(X, V))/C_{mean} = 0$, we reject it in favor of $\mathrm{Cov}(C, \beta(X, V))/C_{mean} > 0$ for 'food', 'services', and total expenditure. For 'fuel and light' and 'clothing and footware' the covariance term is not significantly different from zero.

The discussion above regards the average difference between the aggregate elasticity and the mean individual elasticity over the sample period of 20 years. We perform a bootstrap to assess the statistical significance of this difference for each year of the sample. The figures in the last column of the Tables 1-5 are the 95% bootstrap confidence interval for $\mathrm{Cov}(\widehat{C, \beta(X}, V))/\bar{C}$, which we can use to test $H_0 : \mathrm{Cov}(C, \beta(X, V))/C_{mean} = 0$. The main result of this test is that for expenditure on 'food', 'services', and 'total expenditure' the covariance term is significantly positive for almost all sample years. For the remaining commodity groups 'clothing and footware' and 'fuel and light' the covariance term is not significant.

It is important to note that estimation results in Tables 1-5 are elasticities with respect to income. However, under additional assumptions described in Section 2 our methodology allows estimation of elasticities with respect to budget. For the sake of completeness and comparability with other studies we present estimates of the mean of individual budget elasticities $\beta_{mean,tot}$ for several commodity groups in Table 6. It is not surprising that these estimates are substantially greater than the corresponding mean of income elasticities as the latter do not take the savings behavior into account. Indeed, $\hat{\beta}_{mean,tot}$ is roughly twice as large as the corresponding $\hat{\beta}_{mean}$, which seems intuitive as $\hat{\beta}_{mean}$ for total expenditure is approximately equal to 0.5.

To sum up, we found strong empirical evidence for aggregate elasticity to be greater than the mean of individual elasticities for commodity groups 'food', 'services' and total expenditure. In contrast, for commodity groups 'fuel and light' and 'clothing and services' the aggregate elasticity seems neither to overestimate, nor to underestimate the average individual elasticity.

The above result has extensive implications for both policy makers and applied researchers. As for the former, the knowledge of the relationship between the aggregate elasticity and the distribution of individual elasticities is crucial for correct evaluation of economic reforms. For instance, if one wants to assess possible changes in demand due to an income tax reform, one should take heterogeneity in income elasticities into account. For the latter, it is important to know that one must not interpret the aggregate elasticity in terms of mean individual elasticities, since the difference between them can be of magnitude of even 30% of the aggregate elasticity.

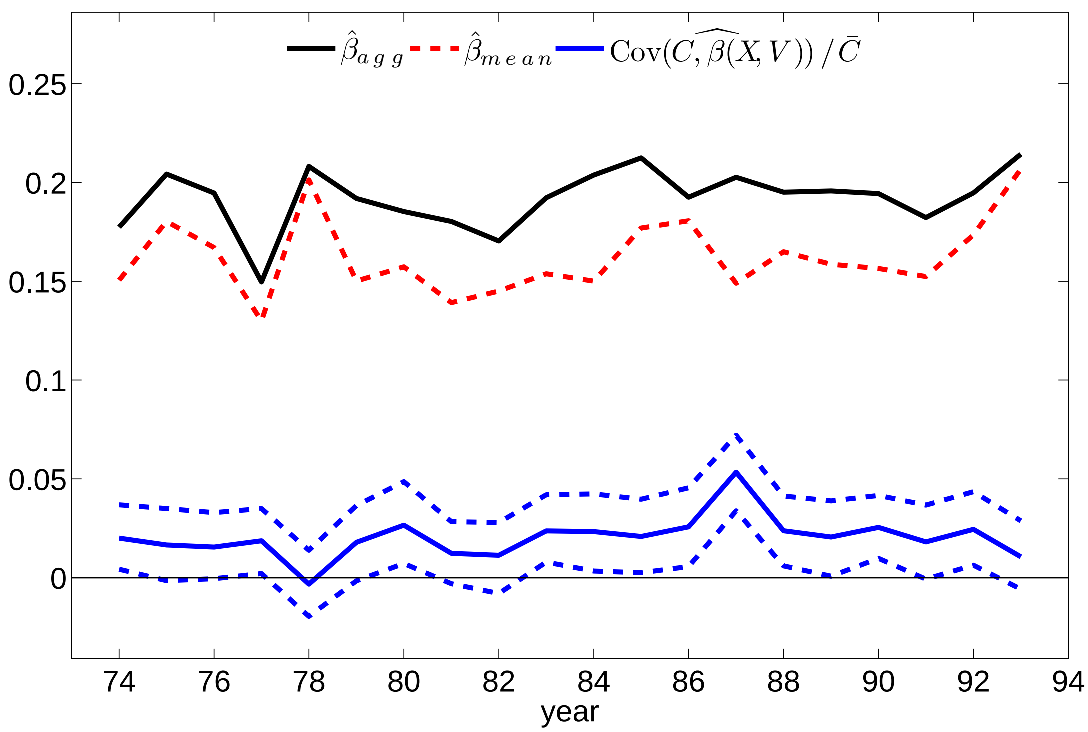

Figure 1: Estimates for ‘food expenditure’

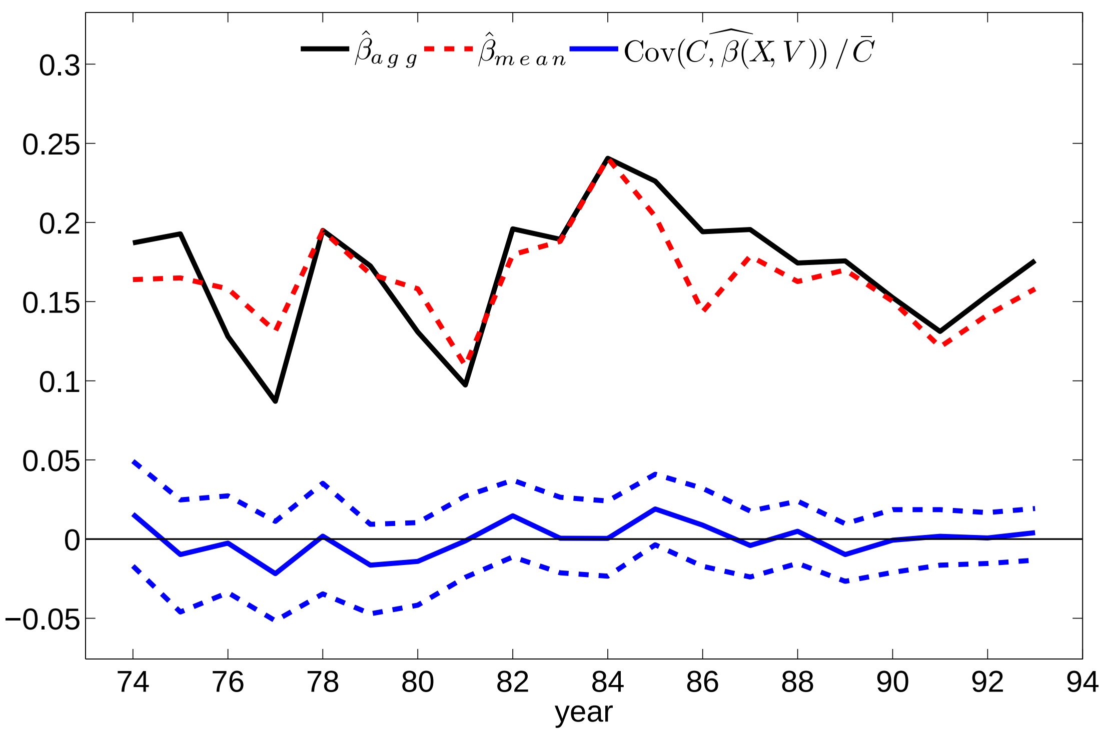

Figure 2: Estimates for 'fuel and light expenditure'

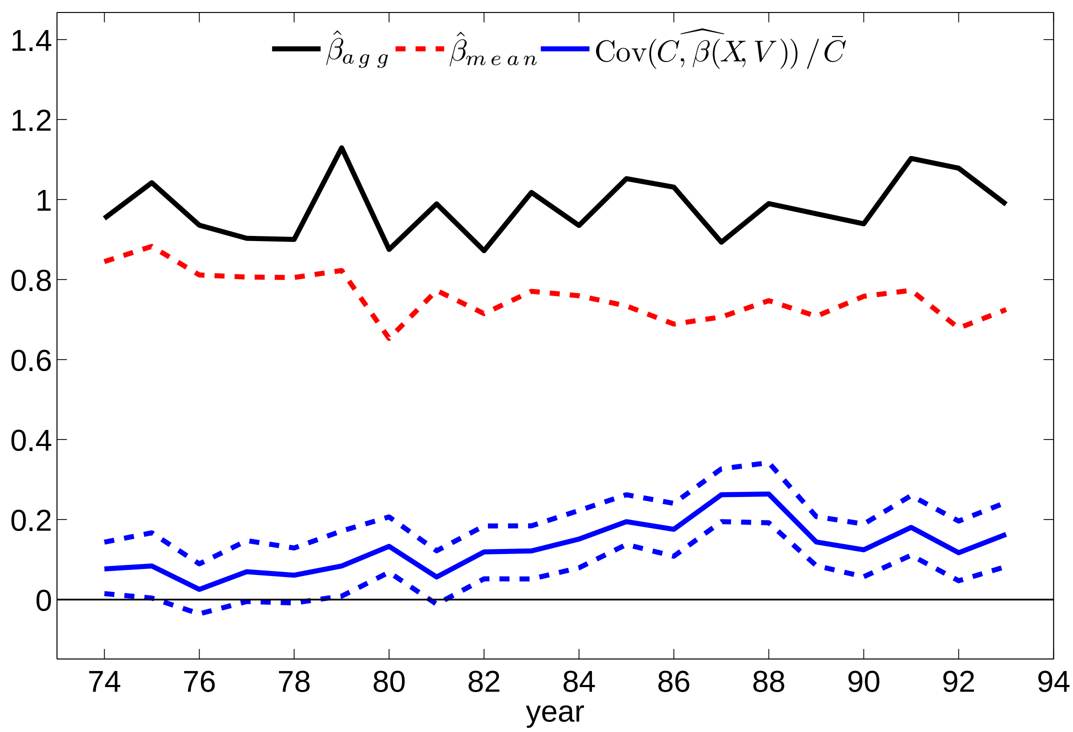

Figure 3: Estimates for 'services expenditure'

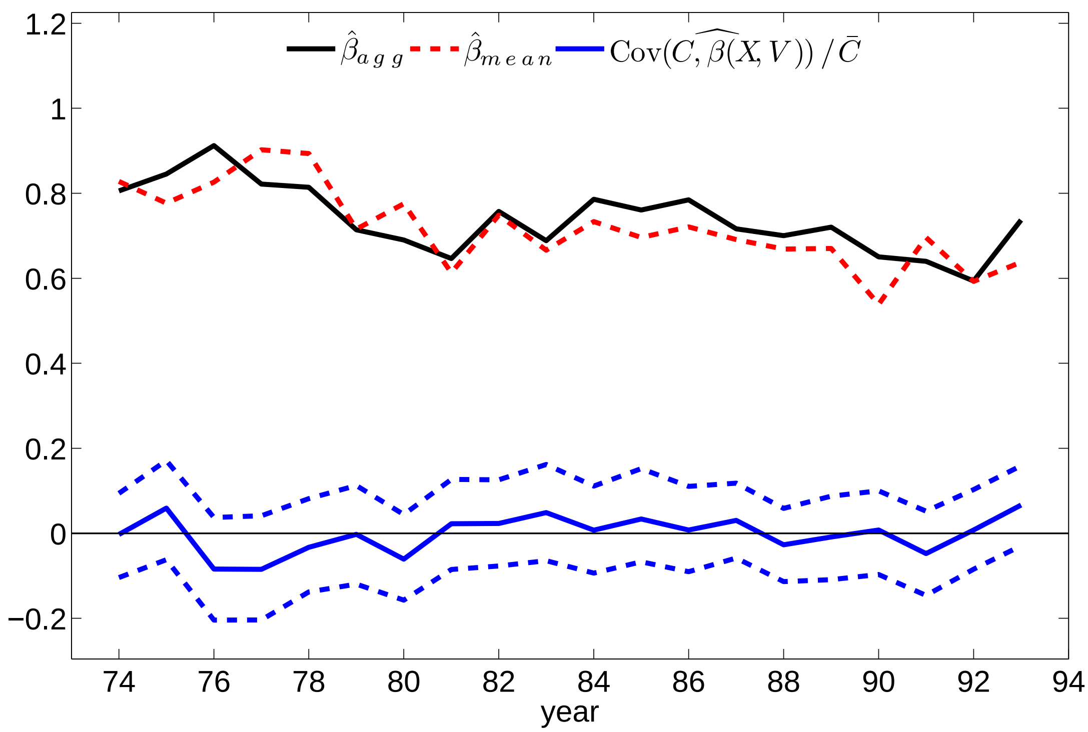

Figure 4: Estimates for 'clothing and footware expenditure'

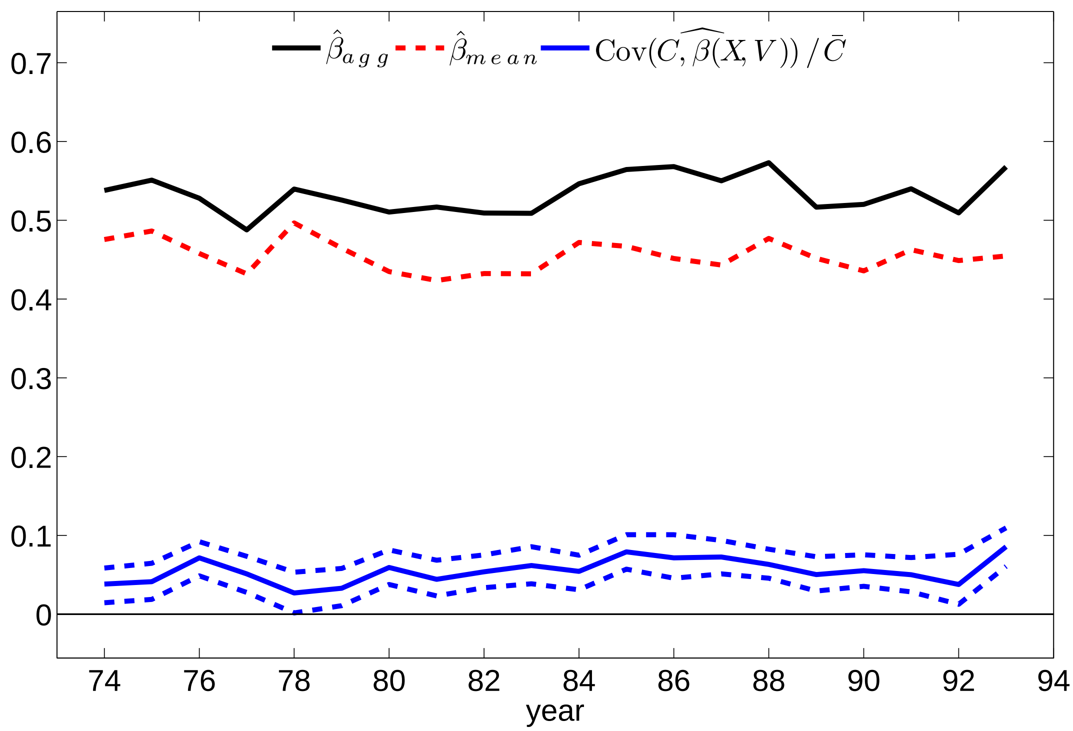

Figure 5: Estimates for ‘total (nondurable) expenditure’

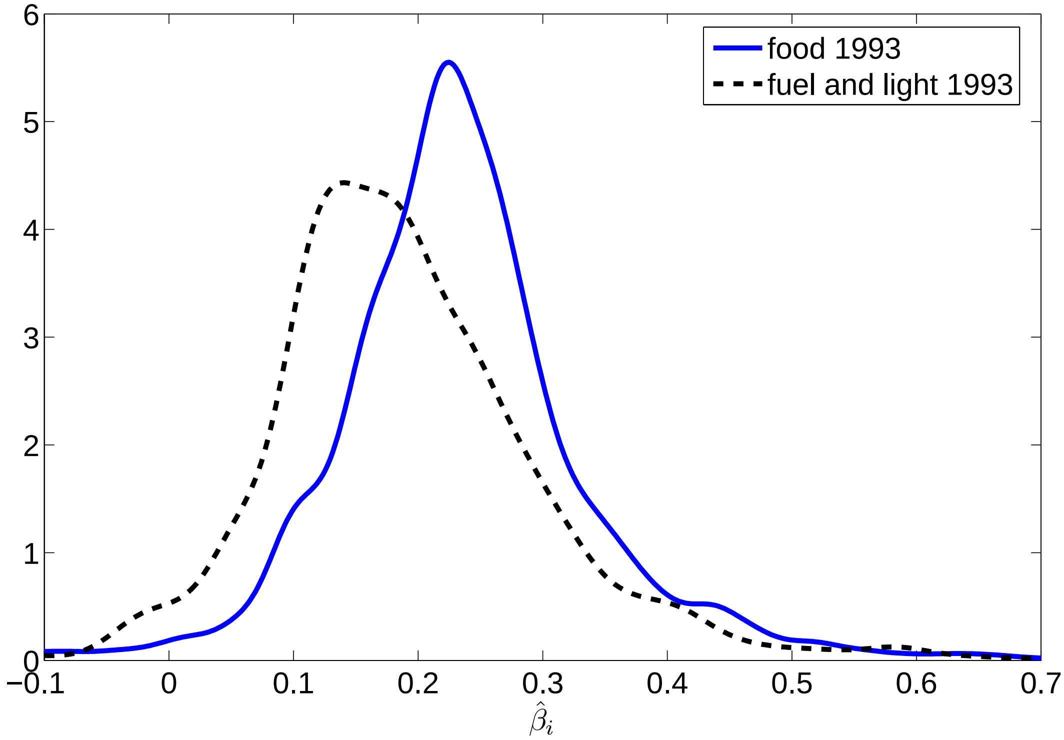

Figure 6: Distribution of $\hat{\beta}_i$ for expenditures on 'food' and 'fuel and light' in 1993.

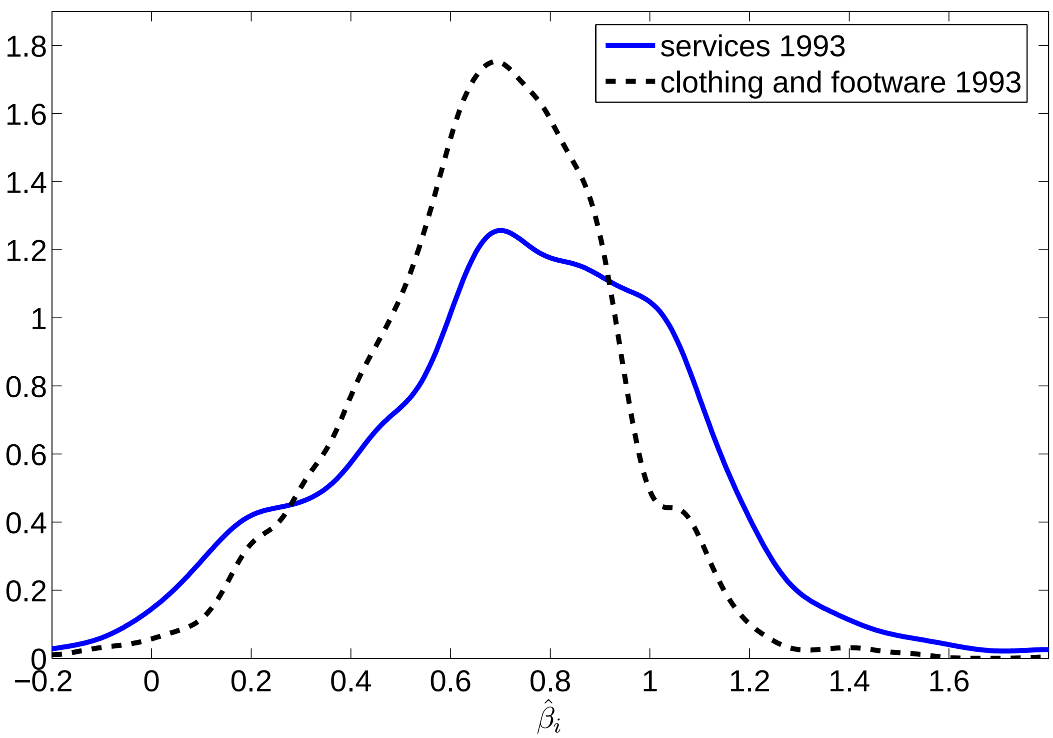

Figure 7: Distribution of $\hat{\beta}_i$ for expenditures on ‘services’ and ‘clothing and footware’ in 1993.

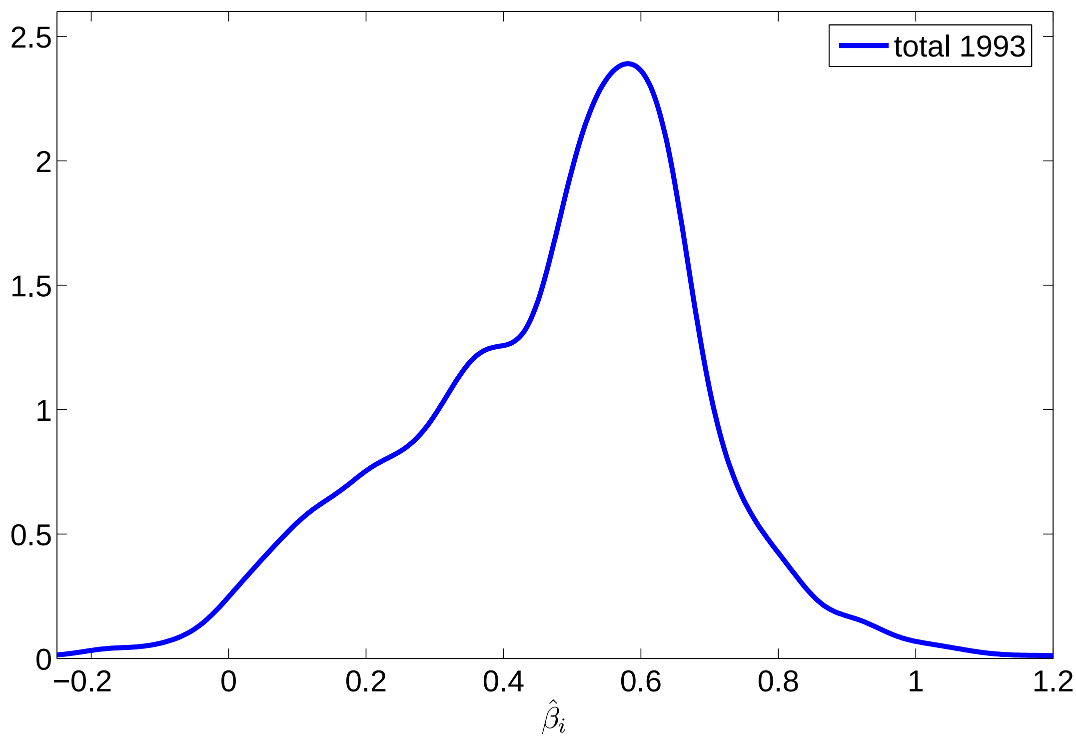

Figure 8: Distribution of $\hat{\beta}_i$ for expenditures on 'total (nondurable) expenditure' in 1993.

year	$\hat{\beta}_{agg}$	$\hat{\beta}_{mean}$	$\bar{\beta}$	$\widehat{\text{Cov}(C, \beta(X,V))}/\bar{C}$	C.I.
1974	0.177 (0.017)	0.151 (0.017)	0.148 (0.020)	0.020 (0.008)	*[0.004,0.037]
1975	0.204 (0.020)	0.180 (0.019)	0.173 (0.021)	0.017 (0.009)	[-0.002,0.035]
1976	0.195 (0.017)	0.167 (0.018)	0.161 (0.020)	0.015 (0.009)	[-0.001,0.033]
1977	0.150 (0.019)	0.130 (0.019)	0.132 (0.022)	0.019 (0.009)	*[0.002,0.035]
1978	0.208 (0.018)	0.201 (0.018)	0.194 (0.023)	-0.003 (0.009)	[-0.020,0.014]
1979	0.192 (0.021)	0.150 (0.020)	0.156 (0.022)	0.018 (0.010)	[-0.002,0.037]
1980	0.185 (0.019)	0.157 (0.021)	0.161 (0.022)	0.027 (0.010)	*[0.007,0.049]
1981	0.180 (0.017)	0.139 (0.017)	0.140 (0.020)	0.012 (0.008)	[-0.003,0.028]
1982	0.170 (0.017)	0.145 (0.018)	0.141 (0.024)	0.011 (0.009)	[-0.008,0.028]
1983	0.192 (0.017)	0.154 (0.018)	0.160 (0.021)	0.024 (0.009)	*[0.008,0.042]
1984	0.204 (0.016)	0.150 (0.019)	0.146 (0.023)	0.023 (0.010)	*[0.003,0.042]
1985	0.212 (0.017)	0.177 (0.019)	0.179 (0.021)	0.021 (0.009)	*[0.003,0.040]
1986	0.193 (0.020)	0.181 (0.020)	0.188 (0.021)	0.026 (0.010)	*[0.006,0.045]
1987	0.203 (0.018)	0.149 (0.019)	0.132 (0.020)	0.053 (0.010)	*[0.034,0.072]
1988	0.195 (0.015)	0.165 (0.018)	0.165 (0.023)	0.024 (0.009)	*[0.006,0.041]
1989	0.196 (0.018)	0.159 (0.019)	0.161 (0.021)	0.021 (0.010)	*[0.001,0.039]
1990	0.194 (0.014)	0.156 (0.017)	0.161 (0.019)	0.025 (0.009)	*[0.010,0.042]
1991	0.182 (0.016)	0.152 (0.017)	0.161 (0.021)	0.018 (0.010)	[-0.001,0.037]
1992	0.195 (0.015)	0.173 (0.016)	0.165 (0.019)	0.024 (0.009)	*[0.006,0.043]
1993	0.214 (0.017)	0.207 (0.017)	0.201 (0.019)	0.011 (0.009)	[-0.006,0.029]
MEAN	0.192	0.162	0.161	0.020	
p-value		0.000*	0.000*	0.000*	

- C.I. denotes the confidence interval for $\text{Cov}(C, \beta(X,V))/C_{mean}$
- Last line of the table contains p-values for the hypotheses: (on average over 20 years) $\beta_{agg} - \beta_{mean} = 0$, $\beta_{agg} - \beta = 0$, and $\text{Cov}(C, \beta(X,V))/C_{mean} = 0$, respectively.
- Asterisks denote rejection of equality of the aggregate elasticity and the average individual elasticity at the 5% level.

Table 1: Income elasticities of demand for 'food expenditure.'

year	$\hat{\beta}_{agg}$	$\hat{\beta}_{mean}$	$\bar{\beta}$	$\text{Cov}(\widehat{C, \beta(X}, V))/\bar{C}$	C.I.
1974	0.187 (0.043)	0.164 (0.031)	0.165 (0.036)	0.016 (0.017)	[-0.017,0.049]
1975	0.193 (0.048)	0.165 (0.033)	0.177 (0.033)	-0.010 (0.019)	[-0.046,0.025]
1976	0.128 (0.035)	0.158 (0.026)	0.158 (0.032)	-0.003 (0.015)	[-0.034,0.027]
1977	0.087 (0.033)	0.131 (0.028)	0.145 (0.030)	-0.022 (0.015)	[-0.052,0.011]
1978	0.195 (0.035)	0.195 (0.034)	0.205 (0.039)	0.002 (0.017)	[-0.035,0.035]
1979	0.173 (0.032)	0.167 (0.028)	0.163 (0.033)	-0.016 (0.014)	[-0.047,0.009]
1980	0.131 (0.041)	0.158 (0.030)	0.167 (0.031)	-0.014 (0.014)	[-0.042,0.010]
1981	0.097 (0.030)	0.109 (0.023)	0.114 (0.029)	-0.001 (0.013)	[-0.024,0.027]
1982	0.196 (0.032)	0.180 (0.027)	0.174 (0.030)	0.015 (0.013)	[-0.011,0.037]
1983	0.189 (0.027)	0.188 (0.025)	0.195 (0.027)	0.001 (0.012)	[-0.021,0.026]
1984	0.241 (0.030)	0.241 (0.023)	0.239 (0.028)	0.000 (0.012)	[-0.023,0.024]
1985	0.226 (0.028)	0.204 (0.024)	0.194 (0.027)	0.019 (0.011)	[-0.004,0.041]
1986	0.194 (0.031)	0.144 (0.025)	0.138 (0.028)	0.009 (0.012)	[-0.017,0.032]
1987	0.196 (0.023)	0.179 (0.022)	0.184 (0.022)	-0.004 (0.010)	[-0.024,0.018]
1988	0.174 (0.025)	0.163 (0.021)	0.172 (0.022)	0.005 (0.010)	[-0.015,0.024]
1989	0.176 (0.022)	0.170 (0.019)	0.173 (0.021)	-0.010 (0.009)	[-0.027,0.010]
1990	0.153 (0.024)	0.150 (0.019)	0.156 (0.022)	-0.001 (0.010)	[-0.021,0.019]
1991	0.131 (0.020)	0.121 (0.017)	0.118 (0.022)	0.002 (0.009)	[-0.016,0.019]
1992	0.154 (0.023)	0.142 (0.023)	0.154 (0.021)	0.001 (0.009)	[-0.015,0.017]
1993	0.176 (0.020)	0.158 (0.020)	0.154 (0.021)	0.004 (0.009)	[-0.013,0.019]
MEAN	0.170	0.164	0.167	0.000	
p-value		0.117	0.478	0.941	

- C.I. denotes the confidence interval for $\text{Cov}(C, \beta(X, V))/C_{mean}$
- Last line of the table contains p-values for the hypotheses: (on average over 20 years) $\beta_{agg} - \beta_{mean} = 0$, $\beta_{agg} - \beta = 0$, and $\text{Cov}(C, \beta(X, V))/C_{mean} = 0$, respectively.
- Asterisks denote rejection of equality of the aggregate elasticity and the average individual elasticity at the 5% level.

Table 2: Income elasticities of demand for 'fuel and light.'

year	$\hat{\beta}_{agg}$	$\hat{\beta}_{mean}$	$\bar{\beta}$	$\widehat{\text{Cov}(C, \beta(X,V))}/\bar{C}$	C.I.
1974	0.953 (0.059)	0.845 (0.038)	0.814 (0.046)	0.077 (0.034)	*[0.015,0.143]
1975	1.042 (0.070)	0.883 (0.039)	0.855 (0.047)	0.084 (0.040)	*[0.004,0.167]
1976	0.936 (0.058)	0.811 (0.043)	0.804 (0.043)	0.025 (0.030)	[-0.036,0.089]
1977	0.903 (0.063)	0.806 (0.039)	0.799 (0.045)	0.070 (0.039)	[-0.005,0.147]
1978	0.900 (0.057)	0.805 (0.039)	0.780 (0.052)	0.061 (0.038)	[-0.009,0.129]
1979	1.129 (0.087)	0.823 (0.040)	0.775 (0.048)	0.084 (0.041)	*[0.008,0.171]
1980	0.875 (0.063)	0.653 (0.040)	0.622 (0.049)	0.133 (0.033)	*[0.068,0.207]
1981	0.989 (0.058)	0.773 (0.033)	0.745 (0.043)	0.056 (0.033)	[-0.011,0.122]
1982	0.872 (0.085)	0.715 (0.037)	0.697 (0.047)	0.119 (0.034)	*[0.052,0.184]
1983	1.018 (0.056)	0.771 (0.037)	0.735 (0.043)	0.122 (0.033)	*[0.051,0.184]
1984	0.935 (0.056)	0.759 (0.041)	0.750 (0.048)	0.151 (0.036)	*[0.079,0.223]
1985	1.052 (0.055)	0.734 (0.037)	0.706 (0.044)	0.195 (0.032)	*[0.138,0.262]
1986	1.031 (0.081)	0.689 (0.040)	0.671 (0.047)	0.176 (0.034)	*[0.108,0.240]
1987	0.894 (0.051)	0.707 (0.033)	0.692 (0.047)	0.262 (0.034)	*[0.195,0.327]
1988	0.990 (0.057)	0.747 (0.035)	0.731 (0.037)	0.264 (0.040)	*[0.192,0.342]
1989	0.964 (0.072)	0.708 (0.030)	0.690 (0.039)	0.144 (0.035)	*[0.084,0.207]
1990	0.939 (0.046)	0.758 (0.033)	0.746 (0.041)	0.124 (0.031)	*[0.057,0.189]
1991	1.103 (0.069)	0.773 (0.033)	0.754 (0.041)	0.181 (0.038)	*[0.111,0.260]
1992	1.079 (0.062)	0.679 (0.030)	0.653 (0.041)	0.117 (0.038)	*[0.047,0.196]
1993	0.988 (0.069)	0.725 (0.034)	0.716 (0.037)	0.163 (0.039)	*[0.083,0.243]
MEAN	0.980	0.758	0.737	0.130	
p-value		0.000*	0.000*	0.000*	

- C.I. denotes the confidence interval for $\text{Cov}(C, \beta(X,V))/C_{mean}$
- Last line of the table contains p-values for the hypotheses: (on average over 20 years) $\beta_{agg} - \beta_{mean} = 0$, $\beta_{agg} - \beta = 0$, and $\text{Cov}(C, \beta(X,V))/C_{mean} = 0$, respectively.
- Asterisks denote rejection of equality of the aggregate elasticity and the average individual elasticity at the 5% level.

Table 3: Income elasticities of demand for ‘services.’

year	$\hat{\beta}_{agg}$	$\hat{\beta}_{mean}$	$\bar{\beta}$	$\mathrm{Cov}(\widehat{C,\beta(X,V)})/\bar{C}$	C.I.
1974	0.805 (0.065)	0.828 (0.073)	0.810 (0.070)	-0.003 (0.052)	[-0.104,0.094]
1975	0.845 (0.069)	0.777 (0.074)	0.766 (0.072)	0.059 (0.062)	[-0.062,0.171]
1976	0.912 (0.099)	0.826 (0.072)	0.821 (0.081)	-0.084 (0.064)	[-0.204,0.037]
1977	0.822 (0.068)	0.902 (0.074)	0.883 (0.078)	-0.085 (0.061)	[-0.204,0.041]
1978	0.814 (0.065)	0.893 (0.070)	0.864 (0.075)	-0.033 (0.058)	[-0.138,0.082]
1979	0.714 (0.070)	0.716 (0.076)	0.725 (0.079)	-0.003 (0.062)	[-0.120,0.112]
1980	0.690 (0.061)	0.776 (0.070)	0.753 (0.073)	-0.061 (0.050)	[-0.157,0.044]
1981	0.646 (0.060)	0.613 (0.067)	0.607 (0.081)	0.022 (0.053)	[-0.085,0.127]
1982	0.757 (0.060)	0.749 (0.067)	0.732 (0.074)	0.023 (0.053)	[-0.077,0.126]
1983	0.688 (0.060)	0.666 (0.071)	0.651 (0.072)	0.049 (0.056)	[-0.064,0.162]
1984	0.786 (0.066)	0.733 (0.070)	0.733 (0.076)	0.008 (0.053)	[-0.094,0.111]
1985	0.760 (0.060)	0.696 (0.068)	0.701 (0.073)	0.034 (0.057)	[-0.067,0.152]
1986	0.785 (0.068)	0.721 (0.066)	0.720 (0.068)	0.008 (0.052)	[-0.090,0.110]
1987	0.716 (0.051)	0.691 (0.058)	0.691 (0.060)	0.030 (0.045)	[-0.058,0.118]
1988	0.700 (0.053)	0.669 (0.057)	0.670 (0.064)	-0.027 (0.046)	[-0.114,0.058]
1989	0.720 (0.054)	0.670 (0.059)	0.679 (0.064)	-0.009 (0.049)	[-0.109,0.088]
1990	0.651 (0.054)	0.538 (0.055)	0.545 (0.064)	0.008 (0.048)	[-0.097,0.100]
1991	0.640 (0.052)	0.696 (0.056)	0.707 (0.065)	-0.048 (0.050)	[-0.146,0.052]
1992	0.594 (0.052)	0.593 (0.054)	0.597 (0.064)	0.008 (0.048)	[-0.084,0.103]
1993	0.737 (0.057)	0.638 (0.056)	0.648 (0.059)	0.066 (0.048)	[-0.029,0.159]
MEAN	0.739	0.720	0.715	-0.002	
p-value		0.145	0.086	0.911	

- C.I. denotes the confidence interval for $\mathrm{Cov}(C,\beta(X,V))/C_{mean}$
- Last line of the table contains p-values for the hypotheses: (on average over 20 years) $\beta_{agg}-\beta_{mean}=0$, $\beta_{agg}-\beta=0$, and $\mathrm{Cov}(C,\beta(X,V))/C_{mean}=0$, respectively.
- Asterisks denote rejection of equality of the aggregate elasticity and the average individual elasticity at the 5% level.

Table 4: Income elasticities of demand for 'clothing and footware.'

year	$\hat{\beta}_{agg}$	$\hat{\beta}_{mean}$	$\bar{\beta}$	$\widehat{\text{Cov}(C, \beta(X,V))/\bar{C}}$	C.I.
1974	0.538 (0.023)	0.476 (0.019)	0.474 (0.023)	0.038 (0.011)	*[0.014,0.058]
1975	0.551 (0.021)	0.486 (0.017)	0.482 (0.022)	0.041 (0.012)	*[0.019,0.064]
1976	0.528 (0.025)	0.458 (0.019)	0.453 (0.023)	0.072 (0.011)	*[0.049,0.092]
1977	0.488 (0.022)	0.432 (0.019)	0.441 (0.021)	0.051 (0.011)	*[0.028,0.073]
1978	0.540 (0.023)	0.497 (0.019)	0.497 (0.024)	0.027 (0.013)	*[0.002,0.053]
1979	0.526 (0.023)	0.465 (0.019)	0.464 (0.023)	0.033 (0.012)	*[0.011,0.058]
1980	0.510 (0.021)	0.435 (0.018)	0.434 (0.023)	0.059 (0.011)	*[0.038,0.081]
1981	0.517 (0.021)	0.423 (0.018)	0.423 (0.019)	0.044 (0.011)	*[0.023,0.068]
1982	0.509 (0.028)	0.432 (0.018)	0.434 (0.021)	0.054 (0.011)	*[0.033,0.075]
1983	0.509 (0.023)	0.432 (0.019)	0.435 (0.024)	0.062 (0.011)	*[0.038,0.086]
1984	0.546 (0.026)	0.472 (0.019)	0.470 (0.025)	0.054 (0.011)	*[0.031,0.075]
1985	0.564 (0.020)	0.467 (0.018)	0.456 (0.021)	0.079 (0.011)	*[0.057,0.101]
1986	0.568 (0.025)	0.451 (0.018)	0.438 (0.024)	0.071 (0.014)	*[0.046,0.101]
1987	0.550 (0.019)	0.443 (0.016)	0.441 (0.019)	0.072 (0.012)	*[0.051,0.094]
1988	0.573 (0.020)	0.477 (0.016)	0.465 (0.021)	0.063 (0.010)	*[0.046,0.082]
1989	0.517 (0.022)	0.452 (0.017)	0.454 (0.021)	0.050 (0.011)	*[0.029,0.073]
1990	0.520 (0.017)	0.436 (0.015)	0.435 (0.019)	0.055 (0.010)	*[0.035,0.075]
1991	0.540 (0.018)	0.463 (0.015)	0.453 (0.020)	0.050 (0.011)	*[0.028,0.072]
1992	0.509 (0.034)	0.449 (0.015)	0.449 (0.020)	0.038 (0.016)	*[0.013,0.076]
1993	0.568 (0.024)	0.455 (0.018)	0.459 (0.020)	0.086 (0.013)	*[0.061,0.109]
MEAN	0.534	0.455	0.453	0.055	
p-value		0.000*	0.000*	0.000*	

- C.I. denotes the confidence interval for $\text{Cov}(C, \beta(X,V))/C_{mean}$
- Last line of the table contains p-values for the hypotheses: (on average over 20 years) $\beta_{agg} - \beta_{mean} = 0$, $\beta_{agg} - \beta = 0$, and $\text{Cov}(C, \beta(X,V))/C_{mean} = 0$, respectively.
- Asterisks denote rejection of equality of the aggregate elasticity and the average individual elasticity at the 5% level.

Table 5: Income elasticities of demand for 'total (nondurable) expenditure.'

year	food	fuel	services	clothing
1974	0.328 (0.038)	0.346 (0.062)	1.863 (0.095)	1.742 (0.111)
1975	0.379 (0.042)	0.331 (0.061)	1.9431 (0.098)	1.656 (0.099)
1976	0.384 (0.039)	0.340 (0.072)	2.0446 (0.110)	1.979 (0.118)
1977	0.304 (0.032)	0.430 (0.070)	2.0289 (0.126)	2.052 (0.125)
1978	0.393 (0.045)	0.390 (0.057)	1.922 (0.127)	1.812 (0.126)
1979	0.287 (0.046)	0.435 (0.056)	1.7942 (0.110)	1.784 (0.098)
1980	0.366 (0.040)	0.369 (0.053)	1.681 (0.122)	1.827 (0.123)
1981	0.359 (0.053)	0.291 (0.052)	2.1376 (0.112)	1.686 (0.140)
1982	0.279 (0.047)	0.390 (0.063)	1.7013 (0.086)	1.773 (0.143)
1983	0.329 (0.049)	0.404 (0.066)	2.0172 (0.114)	1.594 (0.139)
1984	0.313 (0.042)	0.490 (0.063)	1.7483 (0.113)	1.818 (0.145)
1985	0.357 (0.048)	0.382 (0.045)	1.6626 (0.115)	1.645 (0.123)
1986	0.463 (0.048)	0.376 (0.075)	1.9541 (0.086)	1.957 (0.135)
1987	0.312 (0.061)	0.487 (0.073)	1.6134 (0.130)	1.593 (0.112)
1988	0.261 (0.042)	0.346 (0.046)	1.796 (0.109)	1.726 (0.121)
1989	0.390 (0.045)	0.363 (0.056)	1.556 (0.104)	1.659 (0.103)
1990	0.382 (0.048)	0.329 (0.050)	1.8716 (0.095)	1.452 (0.096)
1991	0.388 (0.030)	0.260 (0.046)	1.7051 (0.086)	1.744 (0.093)
1992	0.387 (0.044)	0.311 (0.051)	1.6847 (0.081)	1.514 (0.104)
1993	0.308 (0.041)	0.283 (0.038)	1.6436 (0.092)	1.613 (0.094)
MEAN	0.348	0.368	1.818	1.731

Table 6: Estimates of $\beta_{mean,tot}$ for various commodity groups.

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