

Part A

1. Show: The core of the marriage market equals the set of stable matchings.

For this exercise, you can use the following definition of the core:

Definition 1. For a marriage market, a matching μ' **dominates** another matching μ if and only if there exists a coalition A contained in $M \cup W$, such that, for all men m and women w in A ,

$$\begin{aligned}\mu'(m) &\in A \\ \mu'(w) &\in A \\ \mu'(m) &\underset{m}{>} \mu(m) \\ \mu'(w) &\underset{w}{>} \mu(w).\end{aligned}$$

Definition 2. The **core** of a marriage market is the set of undominated matchings.

2. There are $i = 1, \dots, 8$ sellers each of one horse and $j = 1, \dots, 10$ potential buyers each of one horse in a horse market. All agents' utility gains in the horse market can be identified with their monetary gains, and the horses are homogeneous goods. The reserve price c_i of the sellers and the maximal willingness-to-pay h_j of the buyers are known to be given as in the following tables:

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
10	11	15	17	20	21.5	25	26

and

h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
30	28	26	24	22	21	20	18	17	15

- (a) Identify the gains from trade each coalition can achieve.
- (b) Show that the existence of two buyer-seller pairs who trade at different prices contradicts the requirements of the core.
- (c) Determine the core.
- (d) Show that each allocation in the core can be supported by a Walrasian price. (Note that this is the converse of the standard result whereby each Walrasian allocation is in the core!)