

Exercises:

1. There is a set of n men, $M = \{m_1, \dots, m_n\}$, and a set of p women, $W = \{w_1, \dots, w_p\}$. If man m_i and woman w_j are paired, they create a monetary value of $v(m_i, w_j)$, single individuals do not create value. Utility is given by the monetary value an agent realizes.

- (a) Suppose that utility is transferable, $n = p = 2$ and match values are given as follows:

	w_1	w_2
m_1	10	18
m_2	1	10

Compute the core of the game and draw the set of payoffs for men that are part of core allocations.

- (b) Now one additional man arrives, corresponding match values are given as follows:

	w_1	w_2
m_1	10	18
m_2	1	10
m_3	3	5

Determine the payoff vector in the core that men prefer the least. Compare to the payoff vector men prefer the least in part (a).

- (c) Consider the general setting with n men and p women and arbitrary match values. Suppose that utility is not transferable across agents and each man that is matched receives a share $s \in (0, 1)$ of the match value, each woman receives a share $1 - s$. Assume that match values are such that individuals have strict preferences. Show that there is a unique stable matching.
2. Consider the Derman-Lieberman-Ross model from the lecture. Suppose there are 2 objects with quality $0 < q_2 \leq q_1$. Suppose there are 3 agents and agents' types are distributed iid uniformly on the unit interval.
- (a) Calculate the cutoffs that implement the dynamically efficient policy.
- (b) Compute expected welfare.
- (c) Calculate the payments that implement this policy.
3. Consider Albright's model. Suppose the seller has a single object for sale. Buyers arrive according to a Poisson process with arrival rate $\lambda = 3$. Valuations are iid uniformly on the unit interval. There is a fixed deadline $T = 1$ after which no object can be sold.
- (a) Suppose the seller uses cutoff $\frac{1}{2}$ throughout. Compute his expected revenue given this policy.
- (b) Suppose the seller uses cutoff $\frac{1}{2}$ until period $\frac{T}{2}$ and cutoff $\frac{1}{4}$ thereafter. Compute his expected revenue given this policy.
4. Consider the example about learning from the lecture: There is one object for sale and 2 potential buyers arriving sequentially. Buyers' valuations are drawn uniformly from $[0, 1]$ (with probability 0.5) or from $[1, 2]$ (with probability 0.5). Buyers must be served upon arrival and the object cannot be reallocated. Buyers are privately informed about their valuations. Delayed payments are not possible.
- (a) Compute the second-best optimal policy that maximizes welfare among all implementable deterministic policies.
- (b) Compute the second-best optimal policy that maximizes revenue among all implementable deterministic policies.