Micro II , SS 2014

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Quasi-Linear Utility

- $x = (k, t_1, .., t_I)$, where: $k \in K$ (physical outcomes, "projects"), $t_i \in \mathbb{R}$ (money)
- $u_i(x, \theta_i) = v_i(k, \theta_i) + t_i$
- $f(\theta) = f(\theta_1, ..\theta_I) = (k(\theta), t_1(\theta), .., t_I(\theta))$

Definition

Efficient SCF $f^*(\theta) = (k^*(\theta), t_1^*(\theta), .., t_I^*(\theta))$:

1. Value maximization: $\forall\theta$, $k^*(\theta) \in \arg\max_k \sum_i v_i(k, \theta_i)$
2. Budget Balance: $\forall\theta$, $\sum_i t_i^*(\theta) = 0$

Example: Allocation of indivisble good

- Indivisible good owned by seller
- I buyers
- $k = (y_1, .., y_I)$ where $y_i \in \{0, 1\}$ and $\sum y_i \leq 1$
- $v_i(k, \theta_i) = v_i((y_1, .., y_I), \theta_i) = y_i \theta_i$
- Efficient allocation: $y_i = 1$ if $\theta_i \in \arg\max_j \theta_j$; all monetary transfers from buyers go to seller

The Vickrey-Clarke-Groves (VCG) Mechanism

- Direct Revelation Mechanism
- $k(\theta) = k^*(\theta)$ (value maximization)
- $t_i^*(\theta) = \sum_{j \neq i} v_j(k^*(\theta), \theta_j) + h_i(\theta_{-i})$, where h_i is arbitrary

Theorem

The VCG mechanism truthfully implements the value maximizing SCF in dominant strategies

The Pivot Mechanism

Problem

VCG mechanism requires huge transfers to the agents.

Solution

Appropriate definition of the h_i functions

- Denote by $k^*_{-i}(\theta_{-i})$ the value maximizing project in the absence of i
- Define

$$\begin{aligned} t_i^*(\theta) &= \sum_{j \neq i} v_j(k^*(\theta), \theta_j) + h_i(\theta_{-i}) \\ &= \sum_{j \neq i} v_j(k^*(\theta), \theta_j) - \sum_{j \neq i} v_j(k^*_{-i}(\theta_{-i}), \theta_j) \end{aligned}$$

- Exercise: Prove that : $\forall \theta,\ \sum_i t_i^*(\theta) \leq 0$

Example: Allocation of Indivisble Object

- Efficient allocation: $y_i = 1$ if $\theta_i \in \arg\max_j \theta_j$
- In VCG mechanism:

$$t_i^*(\theta) = \left\{ \begin{array}{l} 0 + h_i(\theta_{-i}), \text{ if } i = \arg\max \theta_j \\ \arg\max \theta_j + h_i(\theta_{-i}), \text{ otherwise} \end{array} \right\}$$

- In pivot mechanism:

$$t_i^*(\theta) = \left\{ \begin{array}{l} -\arg\max_{j \neq i} \theta_j, \text{ if } i = \arg\max \theta_j \\ 0, \text{ otherwise} \end{array} \right\}$$

- Second price auction !

The Green-Laffont Theorem for Bilateral Bargaining I

- Agent 1 is seller, owns indivisble object , value for object θ_1
- Agent 2 is buyer, value for object θ_2
- Values are distributed independently on interval $[0, 1]$ according to densities ϕ_1, ϕ_2.
- VCG Mechanism:

$$k^*(\theta) = k^*(\theta_1, \theta_2) = \left\{ \begin{array}{l} 1, \text{ if } \theta_1 \geq \theta_2 \\ 2, \text{ otherwise} \end{array} \right\}$$

$$t_1^*(\theta) = \left\{ \begin{array}{l} 0 + h_1(\theta_2), \text{ if } \theta_1 \geq \theta_2 \\ \theta_2 + h_1(\theta_2), \text{ otherwise} \end{array} \right\}$$

$$t_2^*(\theta) = \left\{ \begin{array}{l} \theta_1 + h_2(\theta_1), \text{ if } \theta_1 \geq \theta_2 \\ 0 + h_2(\theta_1), \text{ otherwise} \end{array} \right\}$$

The Green-Laffont Theorem for Bilateral Bargaining II

- Budget Balance:

$$
\begin{aligned}
t_1^*(\theta) + t_2^*(\theta) &= 0 \Rightarrow \\
\int_0^1 \int_0^1 [t_1^*(\theta) + t_2^*(\theta)]\phi_1(\theta_1)\phi_2(\theta_2)d\theta_1 d\theta_2 &= 0 \Rightarrow \\
H_1 + H_2 + \int_0^1 \int_0^1 \max[\theta_1, \theta_2]\phi_1(\theta_1)\phi_2(\theta_2)d\theta_1 d\theta_2 &= 0
\end{aligned}
$$

where $H_i = E_{\theta_{-i}} h_i$. Noting that $\theta_1 < \max[\theta_1, \theta_2]$ a.e., this yields:

$$H_1 + H_2 < -E\theta_1$$

With positive probability

The Green-Laffont Theorem for Bilateral Bargaining III

- Participation Constraints:

$$\begin{aligned} \text{Highest Seller Type} &: \quad 1 + H_1 \geq 1 \Rightarrow H_1 \geq 0 \\ \text{Lowest Buyer Type} &: \quad E\theta_1 + H_2 \geq 0 \Rightarrow H_2 \geq -E\theta_1 \end{aligned}$$

- This yields:

$$H_1 + H_2 \geq -E\theta_1$$

a contradiction !

Equivalent Mechanisms

Definition

1. Two mechanisms $\mathbf{M}$ and $\widetilde{\mathbf{M}}$ are *P-equivalent* if, for each i, k and x_i, it holds that $Q_i^k(x_i) = \widetilde{Q}_i^k(x_i)$, where Q_i^k and $\widetilde{Q}_i^k$ are the conditional expected probabilities associated with $\mathbf{M}$ and $\widetilde{\mathbf{M}}$, respectively.
2. Two mechanisms $\mathbf{M}$ and $\widetilde{\mathbf{M}}$ are U-equivalent if they provide the same interim utilities for each agent i and each type x_i of agent i.

- For each agent i, interim utility is obtained (up to a constant) by integrating the function $\sum_{k=1}^{K} a_i^k Q_i^k(x_i)$ with respect to x_i - this is the **Payoff Equivalence Theorem**. Thus P-equivalence implies U-equivalence.