Microeconomics II (PhD)
Tutorial 6, June 11th, 2018

martin.pollrich@uni-bonn.de

Hand in written solutions *before* the tutorial on june 11th.
You may work in groups of at most two students.

Exercises:

1. Reconsider the auction framework from the lecture. Assume bidders' valuations are iid and uniform on $[0, 1]$. The seller's value for the object is zero.

   (a) Assume there are $n \geq 1$ bidders. Argue that a second-price auction with optimally chosen reserve price maximizes revenue. How does the reserve price change if the number of bidders increases?

   (b) Compute the revenue in a second-price auction without reserve price when there are n bidders.

   (c) Compare the revenues obtained in an optimal mechanism with n bidders with the revenue obtained in a second-price auction with $n+1$ bidders. Interpret your result.

2. *Interdependent value auction*

   Suppose there is one object for sale and N potential buyers. Each agent privately observes a signal X_i, which is independently and identically distributed on $[0, \overline{X}]$ with cdf F and density f. Denote by G the cdf of the first-order statistic of $N-1$ of these random variables.

   Buyers have quasi-linear utilities: in case of winning the object, buyer i gets utility $v(x_i, x_{-i}) - p$, where p denotes the payment made, and he gets utility of 0 in case of not winning. Suppose that v is positive, strictly increasing in all signals, symmetric in the last $N-1$ signals, and denote by $\overline{v}(x_i, y)$ the expected valuation of agent i given he received signal x_i and the highest signal among all other signals has value y.

   (a) Show: In a second price auction, each agent bidding according to the bid function $\beta(x_i) = \overline{v}(x_i, x_i)$ is a Bayes-Nash equilibrium.
   Is it a dominant strategy to follow this bid function? Is it an ex-post equilibrium?

   (b) Consider an open English auction. A symmetric strategy in an English auction is a collection $\boldsymbol{\beta} = (\beta^N, \beta^{N-1}, ..., \beta^2)$ of $N-1$ functions $\beta^k : [0, \overline{X}] \times \mathbb{R}_+^{N-k} \to \mathbb{R}_+$. The interpretation is that $\beta^k(x, p_{k+1}, ..., p_N)$ is the price at which bidder 1 will drop out of the auction if the number of bidders who are still active is k, his own signal is x, and the prices at which the other $N-k$ bidders dropped out were $p_{k+1} \geq p_{k+2} \geq ... \geq p_N$.
   Describe a symmetric Bayes-Nash equilibrium of the open English auction and show that this strategy profile constitutes indeed an equilibrium.
   Is it an equilibrium in dominant strategies? Is it an ex-post equilibrium?

(c) Show that the symmetric bidding strategies $\beta(x) = \frac{1}{G(x)} \int_0^x v(y,y) dG(y)$ form a Bayes-Nash equilibrium of the first-price auction.

(d) Suppose $N = 2$, bidder i's valuation is $v_i(x_i, x_j) = \eta x_i + (1 - \eta) x_j$. For which η is the outcome of the second-price auction efficient?