Microeconomics II (PhD)
Tutorial 2, May 2

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Part A

1. Solve Exercises 23.C.3 and 23.C.4 in MWG.

2. Solve Exercise 23.AA.1 in MWG.

3. Consider a bilateral trade setting, where the seller values the object at 0 and the buyer is privately informed about his valuation v. There are two periods, and both agents have a common discount rate $0 < \delta \leq 1$. In each period, the seller posts a price p_t and the buyer decides whether to buy at the proposed price or whether to reject the current offer.

   (a) Assume that $v \sim U[0,1]$ and suppose the seller can commit to a price schedule (p_1, p_2) at the start of period 1. Which prices does he propose?

   Assume for the remainder of the exercise that the seller cannot commit to the second-period price when proposing the first-period price.

   (b) Assume that $v \sim U[0,1]$ and suppose that $\delta = 1$. Find a perfect Bayesian equilibrium of the game. Is it an equilibrium if $\delta < 1$?

   (c) Suppose now that $\delta < 1$, and that $v = v_H$ with probability μ_H and $v = v_L$ with probability $1 - \mu_H$, where $v_H > v_L > 0$. Assume that $\mu_H < \frac{v_L}{v_H}$. Find the perfect Bayesian equilibrium.

   (d) *(verbally)* Suppose now that the seller can propose in each period a general mechanism. Is it without loss of generality for the seller to propose in each period a direct and incentive-compatible mechanism (that is, does the revelation principle apply in this situation)?

Part B

4. Consider the quasi-linear private values environment from the lecture. Let $\alpha_1, ..., \alpha_I \in \mathbb{R}_+ \setminus \{0\}$ and $\lambda_1, ..., \lambda_K \in \mathbb{R}$. A function $k : \Theta \to K$ is called an *affine maximizer* if

$$k(\theta) \in \arg\max_k \sum_{i=1}^{I} \alpha_i \cdot v_i(k, \theta_i) + \lambda_k.$$

   Show: $k : \Theta \to K$ is truthfully implementable in dominant strategies if k is an affine maximizer.

5. Solve Exercise 23.C.10 in MWG.

   Assume throughout the exercise that (23.C.8) is a necessary condition for $(k^*, t_1, ..., t_I)$ to be truthfully implementable in dominant strategies. In part c insert "implementable" before "ex post efficient social choice function" and suppose that $V_i(\theta_{-i})$ is I times continuously differentiable for each i.

6. Suppose there are two agents and the question whether a brigde should be built. The net valuation of agent i for having a bridge is θ_i. Utilities are quasi-linear: agent i gets utility $\theta_i + t_i$ if the bridge is built and t_i otherwise, where t_i denotes the transfer he receives.

   Assume that $\Theta_i = \mathbb{R}$ for $i = 1, 2$.

   (a) Show that there exists no VCG mechanism such that $\sum_i t_i(\theta) = 0$ for all $\theta \in \mathbb{R}^n$.

   Hint: Consider distinct types $\theta_1,\ \theta_1',\ \theta_2,\ \theta_2'$ such that

$$\theta_1 + \theta_2 > 0,\ \theta_1' + \theta_2 < 0,\ \theta_1 + \theta_2' > 0,\ \theta_1' + \theta_2' < 0.$$

   (b) *(verbally)* The above result extends to n agents. What can you conclude from this for general private value settings (that is, not only binary public good settings) if all valuations are possible, i.e. $\{v_i(\cdot, \theta_i) | \theta_i \in \Theta_i\} = \mathcal{V}$?

7. Consider again the public good setting from the previous exercise. Suppose now that the net valuation of agent i for having a bridge, θ_i, is independently and uniformly distributed on [-3,3].

   (a) Assume agents can either vote in favor or against the bridge and there are no transfers. The bridge will be built if and only if both agents vote for it. What is an equilibrium in dominant strategies? If agents follow these strategies, what is the expected aggregate utility (that is, the sum of the agents expected utilities)?

   (b) Suppose that agents' valuations were observed by a utilitarian social planner. Which decision rule should he implement and what is the resulting expected aggregate utility?

   (c) Assume that transfers are feasible. What is the expected aggregate utility if the Pivotal mechanism is implemented?