Microeconomics II (PhD)
Tutorial 7, July 24th, 2017

martin.pollrich@uni-bonn.de

Exercises:

1. To prove the statements below you may use, without proving it, the following Lemma:

   Lemma 1 (Gale and Sotomayor (1986), simplified)**.** *Let μ and μ' be stable matchings in a marriage market (M, W, P) with strict preferences. Let $M(\mu')$ be the set of men who prefer μ' to μ and let $W(\mu)$ be the set of women who prefer μ to μ'. Then μ' and μ map $M(\mu')$ onto $W(\mu)$.*

   Show:

   (a) The set of individuals that remain single is the same for all stable matchings.

   (b) $\mu \vee_M \mu'$ and $\mu \wedge_M \mu'$ are both matchings and stable.

2. Show that the matching resulting from the men-proposing deferred acceptance algorithm is men-optimal.

3. Consider the following marriage market with four men and four women.
   Preferences are strict and given by

$$\begin{array}{lllll} m_1: & w_1, & w_2, & w_3, & w_4 \\ m_2: & w_2, & w_1, & w_4, & w_3 \\ m_3: & w_3, & w_4, & w_1, & w_2 \\ m_4: & w_4, & w_3, & w_2, & w_1 \end{array} \qquad \begin{array}{lllll} w_1: & m_4, & m_3, & m_2, & m_1 \\ w_2: & m_3, & m_4, & m_1, & m_2 \\ w_3: & m_2, & m_1, & m_4, & m_3 \\ w_4: & m_2, & m_3. & & \end{array}$$

   In the following, you may use without proof that

$$\mu_1 = \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \\ m_3 & m_1 & m_4 & m_2 \end{array}$$

   is a stable matching.

   (a) Show that

$$\mu_2 = \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \\ m_2 & m_4 & m_1 & m_3 \end{array}$$

   is a stable matching.

   (b) Find the men-optimal stable matching μ_M and the women-optimal stable matching μ_W.

   (c) Find two additional stable matchings that are different from μ_1, μ_2, μ_M and μ_W.

4. *Strategic considerations*
   Consider a marriage market with strict preferences. A mechanism asks the agents to reveal their preferences and applies the men-proposing deferred acceptance algorithm to the reported preferences.
   (a) Show that truthtelling is a dominant strategy for men.
   (b) Suppose that (i) men report truthfully and (ii) each woman reports a preference list such that all reports form a Nash equilibrium (under complete information). Show: the resulting matching is stable.

   Hint for part (a): Fix an agent i, reports by all other agents, and suppose that i has a best response that strictly improves over truthtelling. Denoting his match by j, show first that i gets a weakly preferred partner if he uses a cutoff strategy that ranks all agents he prefers to j truthful and all other agents as unacceptable.

5. There is a set of n men, $M = \{m_1, ..., m_n\}$, and a set of p women, $W = \{w_1, ..., w_p\}$. If man m_i and woman w_j are paired, they create a monetary value of $v(m_i, w_j)$, single individuals do not create value. Utility is given by the monetary value an agent realizes.
   (a) Suppose that utility is transferable, $n = p = 2$ and match values are given as follows:

	w_1	w_2
m_1	10	18
m_2	1	10

   Compute the core of the game and draw the set of payoffs for men that are part of core allocations.
   (b) Now one additional man arrives, corresponding match values are given as follows:

	w_1	w_2
m_1	10	18
m_2	1	10
m_3	3	5

   Determine the payoff vector in the core that men prefer the least. Compare to the payoff vector men prefer the least in part (a).
   (c) Consider the general setting with n men and p women and arbitrary match values. Suppose that utility is not transferable across agents and each men that is matched receives a share $s \in (0, 1)$ of the match value, each woman receives a share $1 - s$. Assume that match values are such that individuals have strict preferences. Show that there is a unique stable matching.