

**Exercises:**

1. Solve Exercise 5 from Tutorial 2.
2. Solve Exercise 23.D.6 in MWG.

3. (Variant of Exercise 23.D.5 in MWG)

Consider a *sealed-bid all-pay auction* in which every buyer submits a bid, the highest bidder receives the good (with symmetric tie-breaking), and *every* buyer pays the seller the amount of his bid regardless of whether he wins. Suppose there are  $I$  symmetric buyers and each buyer's valuation is independently drawn from the interval  $[\underline{\theta}, \bar{\theta}] \subset \mathbb{R}_+$  according to a strictly positive density.

(a) Argue that any symmetric pure strategy equilibrium of this auction yields the seller the same expected revenue as the sealed-bid second-price auction.

(b) Show that this auction indeed has a symmetric equilibrium in pure strategies.

[*Hint:* Try to construct an explicit equilibrium using the payoff equivalence result.]

4. (*Variant of an old exam question*)

Consider an auction setting: a seller has a single object for sale, which he does not value. There are 2 bidders, who are privately informed about their valuations  $\theta_i$ . Valuations are drawn independently, where  $\theta_1 \sim U[0, 1]$  and  $\theta_2 \sim U[0, 2]$ . Each buyer has a utility function  $u_i = p_i \cdot \theta_i + t_i$ , where  $p_i$  denotes the probability that buyer  $i$  gets the object and  $t_i$  denotes the transfer he receives.

Any auction has to be Bayesian incentive compatible and has to give each bidder type an interim expected utility of at least 0.

You may use all results from the lecture without proof.

(a) Compute the allocation rule of the revenue maximizing auction. Illustrate graphically why this allocation rule is inefficient.

(b) Compute the interim expected utility of bidder 1 as a function of his type.

(c) Suppose the seller has a commonly known reservation value of  $\frac{1}{2}$ . His utility is therefore given by  $u_S = (1 - p_1 - p_2) \cdot \frac{1}{2} - t_1 - t_2$ . Compute the allocation rule of the auction that is optimal for the seller.

(d) Suppose that types are not drawn independently. Instead, types are distributed on  $[0, 1]^2$  according to the cdf  $F(\theta_1, \theta_2) = \min\{\theta_1, \theta_2\}$  for  $0 \leq \theta_1, \theta_2 \leq 1$  (i.e., types are perfectly correlated).

How does an optimal auction look like? Check that incentive and participation constraints are fulfilled. Compute the interim expected utilities of the buyers.

(e) Suppose now that each bidder has either a valuation of 1 or 2.

$$\begin{aligned} \text{Prob}(\theta_1 = 1, \theta_2 = 1) &= \text{Prob}(\theta_1 = 2, \theta_2 = 2) = \frac{1}{4} + \varepsilon \\ \text{Prob}(\theta_1 = 1, \theta_2 = 2) &= \text{Prob}(\theta_1 = 2, \theta_2 = 1) = \frac{1}{4} - \varepsilon \end{aligned}$$

for some  $\varepsilon \in (0, 1/4)$ . Compute the optimal auction and the corresponding revenue.

- (f) Reconsider part (e). Verbally argue what would happen if the bidders communicated with each other before placing their respective bids.
5. Solve Exercise 23.D.4. In addition, explicitly compute the bidding functions of a symmetric equilibrium.