Microeconomics II (PhD)
Tutorial 3, May 23

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Part A

1. Solve Exercises 23.D.6 in MWG.

2. (Variant of Exercise 23.D.5 in MWG)
Consider a *sealed-bid all-pay auction* in which every buyer submits a bid, the highest bidder receives the good (with symmetric tie-breaking), and *every* buyer pays the seller the amount of his bid regardless of whether he wins. Suppose there are I symmetric buyers and each buyer's valuation is independently drawn from the interval $[\underline{\theta}, \overline{\theta}] \subset \mathbb{R}_+$ according to a stricly positive density.
   (a) Argue that any symmetric pure strategy equilibrium of this auction yields the seller the same expected revenue as the sealed-bid second-price auction.
   (b) Show that this auction indeed has a symmetric equilibrium in pure strategies.
   [*Hint:* Try to construct an explicit equilibrium using the payoff equivalence result.]

3. *(Variant of an old exam question)*
Consider an auction setting: a seller has a single object for sale, which he does not value. There are 2 bidders, who are privately informed about their valuations θ_i. Valuations are drawn independently, where $\theta_1 \sim U[0,1]$ and $\theta_2 \sim U[0,2]$. Each buyer has a utility function $u_i = p_i \cdot \theta_i + t_i$, where p_i denotes the probability that buyer i gets the object and t_i denotes the transfer he receives.

   Any auction has to be Bayesian incentive compatible and give each bidder type an interim expected utility of at least 0.

   You may use all results from the lecture without proof.
   (a) Compute the allocation rule of the revenue maximizing auction. Illustrate graphically why this allocation rule is inefficient.
   (b) Compute the interim expected utility of bidder 1 as a function of his type.
   (c) Suppose the seller has a commonly known reservation value of $\frac{1}{2}$. His utility is therefore given by $u_S = (1 - p_1 - p_2) \cdot \frac{1}{2} - t_1 - t_2$. Compute the allocation rule of the auction that is optimal for the seller.
   (d) Suppose that types are not drawn independently. Instead, types are distributed on $[0,1]^2$ according to the cdf $F(\theta_1, \theta_2) = \min\{\theta_1, \theta_2\}$ for $0 \leq \theta_1, \theta_2 \leq 1$ (i.e., types are perfectly correlated).
   How does an optimal auction look like? Check that incentive and participation constraints are fulfilled. Compute the interim expected utilities of the buyers.
   (e) Suppose now that each bidder has either a valuation of 1 or 2.
$$Prob(\theta_1 = 1, \theta_2 = 1) = Prob(\theta_1 = 2, \theta_2 = 2) = \frac{1}{4} + \varepsilon$$
$$Prob(\theta_1 = 1, \theta_2 = 2) = Prob(\theta_1 = 2, \theta_2 = 1) = \frac{1}{4} - \varepsilon$$
   for some $\varepsilon > 0$. Compute the optimal auction and the corresponding revenue.

Part B

4. Solve Exercise 23.E.3 in MWG.

5. Solve Exercise 23.D.4, parts a and b. In addition, explicitly compute the bidding functions of a symmetric equilibrium.

6. Please look at the solution sketches for the exercises that we couldn't discuss in the last tutorial and send me any questions you might have until May 22.