Microeconomics II (PhD)
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Exercises:

1. Solve Exercises 23.C.1 and 23.C.4 in MWG.

2. Solve Exercises 23.C.8 and 23.AA.1 in MWG.

3. Consider two agents, a sender S and a receiver R. S knows the state of the world $\theta \in \{1, 2\}$. R takes an action $a \in \mathbb{R}$. Payoffs are given as follows:

$$U_S(a, \theta) = -(a - \theta)^2, \quad U_R(a, \theta) = -(a - \theta + b(\theta))^2,$$

   where $b(1) = 1/2$ and $b(2) = -1/2$.

   Initially assume R can commit to her action in a mechanism, that is a mechanism $\Gamma = (M, g)$ consists of a set of strategies M (henceforth labeled as "messages") and a function $g : M \to \mathbb{R}$, mapping a message m into a receiver's action $g(m)$.

   (a) Consider the mechanism $\Gamma = (M, g)$ with $M = \{m_1, m_2, m_3\}$ and $g(\cdot)$ given as follows:

m	m_1	m_2	m_3
$g(m)$	$1/2$	$3/2$	$5/2$

   Determine all equilibrium strategies of the sender in the game induced by mechanism Γ.

   (b) For any strategy profile determined in part (a), state the social choice function that is implemented and find a direct mechanism that implements the same social choice function.

   Now assume that R cannot commit to an action, but chooses an action *after* observing the sender's message. Further assume the prior over states $\{1, 2\}$ is $(1/3, 2/3)$.

   (c) Which of the sender's stragies determined in (a) can be part of a Perfect Bayesian Equilibrium in which the receiver's strategy is given by $g(\cdot)$?

   (d) Fix some equilibrium determined in (c) where each message is used with strictly positive probability. Is it possible to implement the associated social choice function in a mechanism that uses only two messages?

4. Consider the quasi-linear private values environment from the lecture. Let $\alpha_1, ..., \alpha_I \in \mathbb{R}_+ \setminus \{0\}$ and $\lambda_1, ..., \lambda_K \in \mathbb{R}$. A function $k : \Theta \to K$ is called an *affine maximizer* if

$$k(\theta) \in \arg\max_k \sum_{i=1}^{I} \alpha_i \cdot v_i(k, \theta_i) + \lambda_k.$$

   Show: $k : \Theta \to K$ is truthfully implementable in dominant strategies if k is an affine maximizer.

5. Solve Exercise 23.C.10 in MWG.

   Assume throughout the exercise that (23.C.8) is a necessary condition for $(k^*, t_1, ..., t_I)$ to be truthfully implementable in dominant strategies. In part c insert "implementable" before "ex post efficient social choice function" and suppose that $V_i(\theta_{-i})$ is I times continuously differentiable for each i.

6. Suppose there are two agents and the question whether a brigde should be built. The net valuation of agent i for having a bridge is θ_i. Utilities are quasi-linear: agent i gets utility $\theta_i + t_i$ if the bridge is built and t_i otherwise, where t_i denotes the transfer he receives.

   Assume that $\Theta_i = \mathbb{R}$ for $i = 1, 2$.

   (a) Show that there exists no VCG mechanism such that $\sum_i t_i(\theta) = 0$ for all $\theta \in \mathbb{R}^n$.

   Hint: Consider distinct types $\theta_1,\ \theta_1',\ \theta_2,\ \theta_2'$ such that

$$\theta_1 + \theta_2 > 0,\ \theta_1' + \theta_2 < 0,\ \theta_1 + \theta_2' > 0,\ \theta_1' + \theta_2' < 0.$$

   (b) *(verbally)* The above result extends to n agents. What can you conclude from this for general private value settings (that is, not only binary public good settings) if all valuations are possible, i.e. $\{v_i(\cdot, \theta_i) | \theta_i \in \Theta_i\} = \mathcal{V}$?

7. Consider again the public good setting from the previous exercise. Suppose now that the net valuation of agent i for having a bridge, θ_i, is independently and uniformly distributed on [-3,3].

   (a) Assume agents can either vote in favor or against the bridge and there are no transfers. The bridge will be built if and only if both agents vote for it. What is an equilibrium in dominant strategies? If agents follow these strategies, what is the expected aggregate utility (that is, the sum of the agents expected utilities)?

   (b) Suppose that agents' valuations were observed by a utilitarian social planner. Which decision rule should he implement and what is the resulting expected aggregate utility?

   (c) Assume that transfers are feasible. What is the expected aggregate utility if the Pivotal mechanism is implemented?