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Part A

Exercises:

1. Consider a marriage market with strict preferences and suppose that μ and μ' are stable matchings.

   Show: $\mu \vee_M \mu'$ and $\mu \wedge_M \mu'$ are stable matchings.

2. Consider a marriage market with strict preferences.

   Show: The matching resulting from the men-proposing deferred acceptance algorithm is men-optimal.

3. Consider a marriage market with strict preferences.

   Show: The set of individuals that remain single is the same for all stable matchings.

Part B

4. Consider a marriage market with strict preferences. Consider a direct mechanism, where each agent submits a preference ranking. The designer then runs the men-proposing deferred acceptance algorithm on the reported preferences and implements the resulting matching.

   Show that truthtelling is a dominant strategy for men.

   Hint: Fix an agent i, reports by all other agents, and suppose that i has a best response that strictly improves over truthtelling. Denoting his match by j, show first that i gets a weakly preferred partner if he uses a cutoff strategy that ranks all agents he prefers to j truthful and all other agents as unacceptable.

5. Consider a marriage market with strict preferences. A mechanism asks the agents to reveal their preferences and applies the men-proposing deferred acceptance algorithm to the reported preferences. Suppose that (i) men report truthfully and (ii) each woman reports a preference list such that all reports form a Nash equilibrium (under complete information).

   Show: The resulting matching is stable.

6. There is a set of n men, $M = \{m_1, ..., m_n\}$, and a set of p women, $W = \{w_1, ..., w_p\}$. If man m_i and woman w_j are paired, they create a monetary value of $v(m_i, w_j)$, single individuals do not create value. Utility is given by the monetary value an agent realizes.

   (a) Suppose that utility is transferable, $n = p = 2$ and match values are given as follows:

	w_1	w_2
m_1	10	18
m_2	1	10

   Compute the core of the game and draw the set of payoffs for men that are part of core allocations.

   (b) Now one additional man arrives, corresponding match values are given as follows:

	w_1	w_2
m_1	10	18
m_2	1	10
m_3	3	5

   Determine the payoff vector in the core that men prefer the least. Compare to the payoff vector men prefer the least in part (a).

   (c) Consider the general setting with n men and p women and arbitrary match values. Suppose that utility is not transferable across agents and each men that is matched receives a share $s \in (0, 1)$ of the match value, each woman receives a share $1 - s$. Assume that match values are such that individuals have strict preferences. Show that there is a unique stable matching.