

Part B

Note: The second tutorial will take place on Wednesday, May 8!

3. Show: In an assignment game, the set of stable outcomes equals the core.
4. Let (P, Q, α) be an assignment game. Suppose that $|P| \geq |Q|$ and $\alpha_{ij} > 0$ for all $i \in P, j \in Q$.
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Show: There exists a stable payoff vector (u, v) such that $u_i = 0$ for all buyers $i \in P$ if and only if there is an optimal assignment x such that for all $i \in P$ and all $j \in Q, x_{ij} = 1$ implies $\alpha_{ij} \geq \alpha_{kj}$ for all $k \in P$.

5. There is a set of n men, $M = \{m_1, \dots, m_n\}$, and a set of p women, $W = \{w_1, \dots, w_p\}$. If man m_i and woman w_j are paired, they create a monetary value of $v(m_i, w_j)$, single individuals do not create value. Utility is given by the monetary value an agent realizes.
- (a) Suppose that utility is transferable, $n = p = 2$ and match values are given as follows:

	w_1	w_2
m_1	10	18
m_2	1	10

Compute the core of the game and draw the set of payoffs for men that are part of core allocations.

- (b) Now one additional man arrives, corresponding match values are given as follows:

	w_1	w_2
m_1	10	18
m_2	1	10
m_3	3	5

Determine the payoff vector in the core that men prefer the least. Compare to the payoff vector men prefer the least in part (a).

- (c) Consider the general setting with n men and p women and arbitrary match values. Suppose that utility is not transferable across agents and each man that is matched receives a share $s \in (0, 1)$ of the match value, each woman receives a share $1 - s$. Assume that match values are such that individuals have strict preferences. Show that there is a unique stable matching.