

## Part B

5. Consider the following marriage market with four men and four women. Preferences are strict and given by

$$\begin{array}{ll}
 m_1 : w_1, w_2, w_3, w_4 & w_1 : m_4, m_3, m_2, m_1 \\
 m_2 : w_2, w_1, w_4, w_3 & w_2 : m_3, m_4, m_1, m_2 \\
 m_3 : w_3, w_4, w_1, w_2 & w_3 : m_2, m_1, m_4, m_3 \\
 m_4 : w_4, w_3, w_2, w_1 & w_4 : m_2, m_3.
 \end{array}$$

In the following, you may use without proof that

$$\mu_1 = \begin{array}{cccc}
 w_1 & w_2 & w_3 & w_4 \\
 m_3 & m_1 & m_4 & m_2
 \end{array}$$

is a stable matching.

- (a) Show that

$$\mu_2 = \begin{array}{cccc}
 w_1 & w_2 & w_3 & w_4 \\
 m_2 & m_4 & m_1 & m_3
 \end{array}$$

is a stable matching.

- (b) Find the men-optimal stable matching  $\mu_M$  and the women-optimal stable matching  $\mu_W$ .
- (c) Find two additional stable matchings that are different from  $\mu_1, \mu_2, \mu_M$  and  $\mu_W$ .
6. Consider a marriage market with strict preferences. A mechanism asks the agents to reveal their preferences and chooses the men-optimal stable allocation for the reported preferences. Suppose that (i) men report truthfully and (ii) each woman reports a preference list such that all reports form a Nash equilibrium.
- Show: The resulting matching is stable.
7. Show: In a marriage market with strict preferences, the set of people who are single is the same for all stable matchings.