Microeconomics II (PhD)
Tutorial 3, May 7th, 2018

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Hand in written solutions *before* the tutorial on may 7th.
You may work in groups of at most two students.

Exercises:

1. Consider the quasi-linear private values environment from the lecture. Let $\alpha_1, ..., \alpha_I \in \mathbb{R}_+ \setminus \{0\}$ and $\lambda_1, ..., \lambda_K \in \mathbb{R}$. A function $k : \Theta \to K$ is called an *affine maximizer* if

$$k(\theta) \in \arg\max_k \sum_{i=1}^{I} \alpha_i \cdot v_i(k, \theta_i) + \lambda_k.$$

Show: $k : \Theta \to K$ is truthfully implementable in dominant strategies if k is an affine maximizer.

2. Suppose there are two agents and the question whether a brigde should be built. The net valuation of agent i for having a bridge is θ_i. Utilities are quasi-linear: agent i gets utility $\theta_i + t_i$ if the bridge is built and t_i otherwise, where t_i denotes the transfer he receives.

   (a) Assume that $\Theta_i = \mathbb{R}$ for $i = 1, 2$. Show that there exists no VCG mechanism such that $\sum_i t_i(\theta) = 0$ for all $\theta \in \mathbb{R}^n$. *Hint: Consider distinct types $\theta_1,\ \theta_1',\ \theta_2,\ \theta_2'$ such that $\theta_1 + \theta_2 > 0,\ \theta_1' + \theta_2 < 0,\ \theta_1 + \theta_2' > 0,$ and $\theta_1' + \theta_2' < 0$.*

   In the following, assume the net valuation of agent i for having a bridge, θ_i, is independently and uniformly distributed on [-3,3].

   (b) Assume agents can either vote in favor or against the bridge and there are no transfers. The bridge will be built if and only if both agents vote for it. What is an equilibrium in dominant strategies? If agents follow these strategies, what is the expected aggregate utility (that is, the sum of the agents expected utilities)?

   (c) Suppose that agents' valuations were observed by a utilitarian social planner. Which decision rule should he implement and what is the resulting expected aggregate utility?

   (d) Assume that transfers are feasible. What is the expected aggregate utility if the Pivotal mechanism is implemented?

3. Consider an auction environment with symmetric independent private values, as introduced in the lecture. Assume there are $n \geq 2$ bidders, and bidder i's valuation θ_i for obtaining the object is uniformly distributed on the interval $[0, 1]$. An all-pay auction has the following rules:

   - each bidder submits a non-negative bid $b_i \geq 0$,
   - the bidder who submitted the highest bid wins the object (ties are broken at random),
   - each bidder pays his bid (irrespective of whether he wins the object).

   Derive a symmetric Bayesian Nash equilibrium with strictly increasing bidding strategy $\beta : [0, 1] \to \mathbb{R}_+$. *Hint: use revenue equivalence for finding an equilibrium candidate.*