

Exercises:

- Suppose a single agent. The set of alternatives has three elements: $X = \{x_1, x_2, x_3\}$. The agent has three possible types $\Theta = \{\theta_1, \theta_2, \theta_3\}$. The agent has quasi-linear utility $u(a, \theta) + t$, where $t \in \mathbb{R}$ is the monetary transfer. The utility function $u(a, \theta)$ is given by

	θ_1	θ_2	θ_3
x_1	0	-1	1
x_2	1	0	-1
x_3	-1	1	0

Consider the choice rule $\bar{x} : \Theta \rightarrow X$, with $\bar{x}(\theta_i) = x_i$. Show that there is no transfer rule $\bar{t} : \Theta \rightarrow \mathbb{R}$ such that the social choice function (\bar{x}, \bar{t}) is implementable.

- Solve Exercises 23.B.2 and 23.B.4 in MWG.
- Consider the following problem: there are two agents, $i = 1, 2$, the set of alternatives is $X = \{a, b, c, d, e\}$, and the type set of agent i is $\Theta_i = \{\theta_i, \theta'_i\}$. Preferences are given in the following table (where $a - b$ means that alternatives a and b are indifferent):

$\succsim_1 (\theta_1)$	$\succsim_1 (\theta'_1)$	$\succsim_2 (\theta_2)$	$\succsim_2 (\theta'_2)$
$a - b$	a	$a - b$	a
c	b	c	b
d	d	d	d
e	c	e	c
	e		e

Consider the social choice function

$$f(\theta) = \begin{cases} b, & \text{if } \theta = (\theta_1, \theta_2), \\ a, & \text{else} \end{cases}$$

- Is f ex-post efficient?
- Examine the direct revelation mechanism that truthfully implements $f(\cdot)$. Is truth telling each agent's *unique* (weakly) dominant strategy? Show that if an agent chooses his untruthful (weakly) dominant strategy, then $f(\cdot)$ is not implemented.
- Exhibit a mechanism $\Gamma = (S_1, S_2, g(\cdot))$ that is not a direct revelation mechanism that truthfully implements $f(\cdot)$ in dominant strategies, and for which each agent has a *unique* (weakly) dominant strategy.

Literature:

Mas-Colell, Whinston & Green (1995). *Microeconomic Theory*. New York: Oxford University Press.