Hand in written solutions before the tutorial on April 23rd. You may work in groups of at most two students.

Exercises:

1. Adapted from Börgers (2015). In the problem of pricing a single indivisible good (section 1.2 in the lecture slides), assume that the buyer’s type is drawn uniformly from the interval $[0, 1]$. Consider the following two-stage mechanism: In stage one the seller posts a price $p_1 = 3/8$. Then the buyer decides whether to buy or not to buy. If the buyer buys, the game is over. If he does not buy, then a third party draws a price $p_2$ randomly from the interval $[0, 1]$, using the uniform distribution. The buyer can then either buy or not buy at the random price. Find the buyer’s optimal strategy for this mechanism. Then find an equivalent direct mechanism in which truth telling is optimal for the buyer.

2. Reconsider the previous exercise. Assume that the seller chooses the price $p_2$, and not the third party. Holding the buyer’s strategy fixed, which price $p_2$ is optimal for the seller? How would the buyer react?

3. Reconsider the problem of pricing a single indivisible good. Assume the type space is binary: the buyer’s valuation is either low, $\theta_l > 0$, or high, $\theta_h > \theta_l$. Characterize all incentive compatible direct mechanisms. Show that revenue equivalence fails, i.e. that there are two mechanisms with the same allocation rule $q(\cdot)$ and the same transfer to the lowest type $\theta_l$, that give rise to different levels of revenue for the seller. Explain this finding, by relating it to its continuous type counterpart. In a setting with finitely many (but more than two) types, does revenue equivalence hold?

Literature: