Microeconomics II, Summer 2017

September 19^{th}, 2017

You have 90 minutes to solve the following two exercises!

1. **Bilateral Trade**

   Suppose there is a seller with privately known cost $c \in [0, 1]$ of producing a single indivisible good. Suppose also there is a buyer with a privately known value $v \in [0, 1]$ of consuming the good. The cost c and the valuation v are independently and uniformly distributed. All this is commonly known. The seller's utility from trading the good while receiving monetary transfer t_S is $t_S - c_S$. The buyer's utility from trading the good and paying the monetary transfer t_B is $v_B - t_B$. Each agent's utility is normalized to zero when no trade takes place.

   (a) Describe the socially efficient allocation rule, both formally and in a suitable diagram.

   Now consider the *double auction* mechanism, in which the seller submits a bid $b_S \geq 0$ and the buyer submits a bid $b_B \geq 0$. If $b_S \geq b_B$, the seller keeps the good and no monetars transfers are made. If $b_S < b_B$, the buyer gets the good and pays the seller and amount $\frac{1}{2}(b_S + b_B)$.

   (b) Verify that the following bidding strategies form a Bayes-Nash equilibrium.

   $$\beta_S(c) = \frac{1}{4} + \frac{2}{3}c, \quad \text{and} \quad \beta_B(v) = \frac{1}{12} + \frac{2}{3}v.$$

   (c) Which allocation rule is implemented by this Bayes-Nash equilibrium? Depict it also in the diagram you used for part (a).

   A VCG-mechanism in this setting is described by the following procedure: the seller announces a type $\widetilde{c}$ and the buyer announces a type $\widetilde{v}$. Now, if $\widetilde{v} \leq \widetilde{c}$ no trade takes place and no payments are made. If $\widetilde{v} > \widetilde{c}$ the good is exchanged, the buyer pays $\widetilde{c}$ and the seller receives $\widetilde{v}$.

   (d) Verify that it is a weakly dominant strategy for the seller to announce his true costs, and for the buyer to announce her true valuation. Is the VCG-mechanism individually rational?

   (e) Show that the VCG-mechanism runs an expected deficit.

   (f) Briefly argue, why there is no incentive compatible and individually rational mechanism that is ex-post efficient. (*Hint: No calculations required. Use the findings in this exercise and your knowledge from the lecture.*)

2. **Matching**

Consider a marriage market with a finite set of men M and a finite set of women W. Assume that (1) all men and women have strict preferences, and (2) each man (woman) finds at least one woman (man) acceptable. Consider the following matching algorithm.

Set $t = 1$, $M^1 = M$, and $W^1 = W$, and follow the procedure described below.

> **Round t:** Each man in M^t proposes to his most preferred acceptable woman in W^t. Each woman in W^t, who receives at least one proposal by an acceptable man, is matched to her most preferred proposer.
>
> Let W^{t+1} be the set of unmatched women who find at least one of the unmatched men acceptable. Let M^{t+1} be the set of unmatched men who find at least one of the women in W^{t+1} acceptable (set $M^{t+1} = \emptyset$ if $W^{t+1} = \emptyset$).
>
> If either $M^{t+1} = M^t$ or $M^{t+1} = \emptyset$, stop. Otherwise proceed to Round $t + 1$.

For the preference profile R let $f(R)$ denote the matching that is chosen by the above algorithm.

(a) Briefly describe the differences between the above and the men proposing deferred acceptance algorithm. (Not more than two sentences!)

(b) Suppose all men are acceptable to all women and no woman is allowed to rank any man as unacceptable.

Show that f is strategy-proof for the women.

(c) Suppose all women report truthfully.

Show that all pure strategy Nash equilibrium outcomes of the revelation game among men induced by the above procedure are pairwise stable.

(d) Is f strategy-proof for the women if they are allowed to rank men as unacceptable? Justify your answer.