Microeconomics II, Summer 2016 Second Exam, September 16, 2016

You have 90 minutes to solve the following two exercises!

1. **(Dynamic) Roommate Problem**

   Consider a roommate problem in which each agent can be match with any other agent to form a group of size two. There are four agents $1, \ldots, 4$ in the economy.

   (a) Prove that there does not exist a stable matching if agents have the following (strict) ordinal preferences:

$$
\begin{aligned}
P(1) &= 2 \succ 3 \succ 4 \succ 1, \\
P(2) &= 3 \succ 1 \succ 4 \succ 2, \\
P(3) &= 1 \succ 2 \succ 4 \succ 3, \\
P(4) &= 1 \succ 2 \succ 3 \succ 4.
\end{aligned}
$$

   Now consider the following cardinal specification of the roommate problem. Each agent has a type. Agents 1 and 2 are low types, L, and agents 3 and 4 are high types, H. If two agents match, they generate a surplus $m(\cdot, \cdot)$ which depends on their types. We have $m(H, H) = 1$, $m(L, L) = 0$, and $0 < m(H, L) = k < \frac{1}{2}$. Each agent's utility is half of the surplus of his match. If an agent stays unmatched, his utility is $-\infty$.

   (b) Show that there exists a stable matching. Describe the stable matching that you found.

   We maintain the above cardinal specification but now there are two periods. In period 1 agents 1 and 2 arrive, and in period 2 agents 3 and 4 arrive. Types are not fixed any longer but drawn IID; with probability p an agents has type H. Consider a benevolent market maker who can simply assign agents to each other. In particular, he can let agents 1 and 2 wait to match them with agents 3 and 4 in the second period. However, if an agent waits, he incurs a waiting cost of c such that his utility is half of the match surplus minus c. The benevolent market maker wants to maximize the sum of agents' utilities.

   (c) Carefully describe the market maker's trade-off at the end of period 1, i.e., after agent 1 and 2 have arrived.

   There are three possible states at the end of period 1. Either two L-type agents arrived in period 1, or two H-type agents, or one H-type and one L-type agent.

   (d) Using the trade-off that you described in Part (c), argue that in two of these states it is optimal for the market maker to match agents immediately.

   (e) Consider the remaining state. Derive a condition on the waiting cost c such that it is optimal to wait with agents 1 and 2 if and only if this condition is satisfied. How does the condition change if k changes?

2. **Partnership dissolution**

   Two agents jointly own a private good: agent 1 initially owns a share $r_1 \in (0,1)$, agent 2 owns a share $r_2 = 1 - r_1$ (and this is common knowledge). The valuation for owning the whole good, θ_i, is agent i's private information and distributed independently and uniformly on $[0,1]$.

   The utility of agent i is given by $s_i\theta_i + t_i$, where s_i is the share agent i is awarded and t_i is the transfer agent i receives. If agent i refuses to participate in a mechanism he receives utility $r_i\theta_i$. Hence, the net utility agent i receives is $s_i\theta_i + t_i - r_i\theta_i$.

   To dissolve the joint ownership consider the following mechanism: Each agent i submits a bid b_i. If $b_i > b_j$, then agent i is allocated the whole good and pays an amount of b_i to agent j. (If $b_i = b_j$, then each agent receives the share he initially possessed and no money is transfered.)

   (a) Verify that both agents bidding according to the bid function

   $$b(\theta_i) = \frac{1}{3}\theta_i$$

   is a Bayes-Nash equilibrium.

   Is the corresponding allocation rule efficient?

   (b) Given this equilibrium behavior, calculate the interim expected net utility, i. e. the net utility an agent expects after being informed about his own type but before he is informed about the type of the other agent.

   (c) For a given initial ownership share r_1, which type θ_1 gets the lowest interim expected net utility?

   For what ownership shares r_1 of agent 1 is participating in this auction interim individually rational for all types θ_1 of agent 1?

   (d) For what initial ownership shares does an efficient, budget-balanced (i. e. $t_1 + t_2 \equiv 0$) and interim individually rational mechanism exist to dissolve the partnership?
   (Hint: Invoke the revenue equivalence theorem to determine expected net utilities in a direct mechanism. Use this result to determine a condition that is necessary and sufficient for individual rationality for both agents.)