Microeconomics II, Summer 2016

First Exam, July 25, 2016

You have 90 minutes to solve the following two exercises!

1. **Matching markets and assignment games**

   (a) Consider a many-to-one matching market. Show that if firms do not have substitutable preferences, then it might be that no stable matching exists.

   (b) There are 8 sellers each of one horse and 10 potential buyers each of one horse in a horse market. All agents' utility gains in the horse market can be identified with their monetary gains, and the horses are homogeneous goods. The reserve price c_i of the sellers and the maximal willingness-to-pay h_j of the buyers are known to be given as in the following tables:

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
10	11	15	17	20	21.5	25	26

   and

h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
30	28	26	24	22	21	20	18	17	15

   Determine the set of stable payoff vectors.

   (c) Consider a general assignment game and fix some buyer $i \in P$. Show that if there is an optimal assignment x such that buyer i is not assigned, then for any stable payoff vector (u, v) we must have $u_i = 0$.

   Consider an assignment game with two buyers and two sellers. The reservation values of the sellers are 0 and all valuations are integer.

   (d) Suppose the valuations of the buyers are given by the following table:

	Object 1	Object 2
Buyer 1	5	8
Buyer 2	2	9

   Characterize the set of competitive equilibria.

   (e) Suppose buyers can report any pair of integer numbers as valuations for the two objects and the auctioneer then uses the simultaneous ascending clock auction together with the reported valuations to determine the allocation and prices.

   Show explicitly that reporting the true valuations is a dominant strategy for the buyers.

2. **Partnership dissolution**

Two agents jointly own a private good: agent 1 initially owns a share $r_1 \in (0,1)$, agent 2 owns a share $r_2 = 1 - r_1$ (and this is common knowledge). The valuation for owning the whole good, θ_i, is agent i's private information and distributed independently and uniformly on $[0,1]$.

The utility of agent i is given by $s_i\theta_i + t_i$, where s_i is the share agent i is awarded and t_i is the transfer agent i receives. If agent i refuses to participate in a mechanism he receives utility $r_i\theta_i$.

(a) Is there an efficient mechanism that is dominant-strategy incentive compatible and ex-post budget-balanced? Provide an example of such a mechanism or prove that no such mechanism exists.

Let $k^*(\theta)$ denote the efficient allocation given type profile θ and let $v_i(k,\theta_i)$ denote agent i's valuation of allocation k if his type is θ_i. Define

$$t_1^*(\theta) = v_2(k^*(\theta),\theta_2) - v_2(k^*(r_1,\theta_2),\theta_2)$$
$$t_2^*(\theta) = v_1(k^*(\theta),\theta_1) - v_1(k^*(\theta_1,r_2),\theta_1).$$

(b) Show that the mechanism (k^*, t^*) is interim individually rational and that this constraint is binding for type $\theta_i = r_i$.

(c) Show: If $\mathbb{E}_\theta[t_1^*(\theta) + t_2^*(\theta)] > 0$, then there exists no efficient and Bayesian incentive compatible mechanism that is interim individually rational and ex-post budget-balanced.

(d) Suppose that r_1 does not lie in the interval $\left[\frac{1-\sqrt{\frac{1}{3}}}{2}, \frac{1+\sqrt{\frac{1}{3}}}{2}\right]$.

Conclude from the previous part that no efficient and Bayesian incentive compatible mechanism exists that is interim individually rational and ex-post budget-balanced.

(e) Show: If $\mathbb{E}_\theta[t_1^*(\theta) + t_2^*(\theta)] \leq 0$, then an efficient and Bayesian incentive compatible mechanism exists that is interim individually rational and ex-post budget-balanced.