Microeconomics II, Summer 2015

First Exam, July 20, 2015

1. **Matching and Assignment Games**

   (a) Consider an assignment game with four buyers and four sellers. In the following matrix, the entry in the i-th row and j-th column is buyer i's valuation for j's good:

   $$\alpha = \begin{pmatrix} 11 & 9 & 15 & 12 \\ 13 & 10 & 13 & 8 \\ 9 & 0 & 9 & 7 \\ 11 & 12 & 13 & 15 \end{pmatrix}$$

   All sellers have reservation values of 0.

   Calculate the buyer-optimal stable payoff vector.

   (b) Consider an assignment game with four buyers and three sellers. In the following matrix, the entry in the i-th row and j-th column is buyer i's valuation for j's good:

   $$\alpha = \begin{pmatrix} 15 & 7 & 13 \\ 14 & 5 & 14 \\ 8 & 9 & 11 \\ 7 & 1 & 7 \end{pmatrix}$$

   Seller 2 has a reservation value of 2, the other sellers have reservation values of 0.

   State the inequalities characterizing the set of all stable payoff vectors and calculate the minimal quasi-competitive price vector.

   (c) Consider a marriage market with strict preferences and denote by μ^M the men-optimal stable matching.

   Show that there does not exist an individually rational matching μ (stable or not), such that μ is strictly preferred to μ^M by all men.

2. **Regulating a Monopolist**

Consider a monopolist who faces a known demand function $q(p)$ which is positive, differentiable, and strictly decreasing. The monopolist privately observes his constant marginal cost of production $\theta \in [\underline{\theta}, \overline{\theta}]$. Assume that $0 < \underline{\theta} < \overline{\theta}$; hence, $\overline{\theta}$ is the "worst" type. θ is drawn according to a distribution function F with strictly positive density f on $[\underline{\theta}, \overline{\theta}]$ such that $\frac{F(\theta)}{f(\theta)}$ is non-decreasing.

The regulator can set the monopolist's price and make a transfer from or to the monopolist. The set of outcomes is therefore $X = \{(p,t) | p > 0 \text{ and } t \in \mathbb{R}\}$. The monopolist's outside option is to leave the market and get zero profits. If the regulator chooses the outcome (p,t) and the monopolist stays in the market, profits are $\pi(p,t,\theta) := (p-\theta) \cdot q(p) + t$ and consumer surplus is $\int_p^\infty q(s)ds - t$.

(a) Describe an incentive compatible mechanism that maximizes the sum of expected consumer surplus and expected profits (without proof).
(*Hint*: Think of the logic behind a VCG mechanism.)

(b) Suppose the direct mechanism $(p(\theta), t(\theta))$ is used. Show that the monopolist has an incentive to report his marginal cost truthfully if and only if

(i) $p(\theta)$ is non-decreasing and

(ii) $\Pi(\theta) = \int_\theta^{\overline{\theta}} q(p(s))ds + \Pi(\overline{\theta})$,

where $\Pi(\theta) = \pi(p(\theta), t(\theta), \theta)$.

Suppose the regulator wants to choose an incentive compatible and individually rational mechanism to maximize the following objective function:

$$\mathbb{E}\left[\int_{p(\theta)}^\infty q(s)ds - t(\theta) + \alpha \Pi(\theta)\right],$$

where $\alpha \in (0,1)$.

(c) Use the characterization of incentive compatibility to show that the objective function can be written as

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{p(\theta)}^\infty q(s)ds\right] + \left[p(\theta) - \theta - (1-\alpha)\frac{F(\theta)}{f(\theta)}\right] q(p(\theta))dF(\theta) - (1-\alpha)\Pi(\overline{\theta}).$$

(d) Derive the pricing rule $p(\theta)$ that maximizes the regulator's objective.