Microeconomics II, Summer 2014

1. **Auction**

   Consider an auction setting: a seller has a single object for sale, which he does not value. There are 2 bidders, who are privately informed about their valuations θ_i. Valuations are drawn independently, where $\theta_1 \sim U[0,1]$ and $\theta_2 \sim U[0,2]$. Each buyer has a utility function $u_i = p_i \cdot \theta_i + t_i$, where p_i denotes the probability that buyer i gets the object and t_i denotes the transfer he receives.

   Any auction has to be Bayesian incentive compatible and give each bidder type an interim expected utility of at least 0.

   You may use all results from the lecture without proof.

   (a) Compute the allocation rule of the revenue maximizing auction. Illustrate graphically why this allocation rule is inefficient.

   (b) Compute the interim expected utility of bidder 1 as a function of his type.

   (c) Suppose the seller has a commonly known reservation value of $\frac{1}{2}$. His utility is therefore given by $u_S = (1 - p_1 - p_2) \cdot \frac{1}{2} - t_1 - t_2$. Compute the allocation rule of the auction that is optimal for the seller.

   (Hint: You derived in the lecture that the expected transfer for buyer i is given by $\mathbb{E}[t_i(\theta)] = -\mathbb{E}\left[p_i(\theta)\left(\theta_i - \frac{1-F_i(\theta_i)}{f_i(\theta_i)}\right)\right]$.)

   (d) Suppose that types are not drawn independently. Instead, types are distributed on $[0,1]^2$ according to the density $f(\theta_1, \theta_2) = 1$ if $\theta_1 = \theta_2$ and 0 else (i.e., types are perfectly correlated).

   How does an optimal auction look like? Check that incentive and participation constraints are fulfilled. Compute the interim expected utilities of the buyers. (Hint: The seller can achieve an expected revenue of $\frac{1}{2}$.)

2. **Voting**

Consider a social choice setting with K alternatives and N agents. Each agent is characterized by his type $\theta_i \in \Theta$ and is privately informed about his type. Types are drawn independently according to some density function f, which is the same for each agent. No monetary transfers are allowed, and the utility of agent i, $u_i(\theta_i, k)$, depends on his own type and the alternative that is chosen. The designer is interested in maximizing ex-ante utilitarian welfare (i.e. maximizing the aggregate expected utility).

Assume that preferences are single-peaked and consider the successive voting procedure you encountered in the lecture: agents vote successively on each alternative in the linear order that underlies their single-peaked preferences and thresholds are decreasing.

(a) Show that sincere voting is optimal given that all other agents use monotone strategies.

(b) Show by example that sincere voting is not a dominant strategy in the game induced by this procedure.

Suppose from now on that there are two alternatives, 1 and 2. Utility is given by $u_i(\theta_i, p) = p \cdot \theta_i$, where p denotes the probability that alternative 2 is chosen. Let $\Theta = [-2, 1] \setminus \{0\}$.

(c) Suppose there are 5 agents and that

$$f(\theta_i) = \begin{cases} \frac{1}{4} & \text{if } -2 \leq \theta_i < 0 \\ \frac{1}{2} & \text{if } 0 < \theta_i \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Compute the ex-ante expected utility of agent 1 in the successive voting procedure given that a threshold of 4 is imposed for alternative 1.

(d) Let a social choice function be a mapping $p : \Theta^N \to [0, 1]$, where $p(\theta)$ denotes the probability that alternative 2 is chosen.

Show: Any social choice function that is dominant strategy incentive compatible depends only on the signs of the reports (i.e., $p(\theta) = p(\theta')$ whenever $\text{sgn } \theta = \text{sgn } \theta'$).