1. **Matching**

Consider a marriage market with a finite set of men M and a finite set of women W. Assume that (1) all men and women have strict preferences, and (2) each man (woman) finds at least one woman (man) acceptable. Consider the following matching algorithm.

Set $t = 1$, $M^1 = M$, and $W^1 = W$, and follow the procedure described below.

> **Round t:** Each man in M^t proposes to his most preferred acceptable woman in W^t. Each woman in W^t, who receives at least one proposal by an acceptable man, is matched to her most preferred proposer.
>
> Let W^{t+1} be the set of unmatched women who find at least one of the unmatched men acceptable. Let M^{t+1} be the set of unmatched men who find at least one of the women in W^{t+1} acceptable (set $M^{t+1} = \emptyset$ if $W^{t+1} = \emptyset$).
>
> If either $M^{t+1} = M^t$ or $M^{t+1} = \emptyset$, stop. Otherwise proceed to Round $t + 1$.

Let $f(R)$ denote the matching that is chosen by the above algorithm for the preference profile R.

(a) Briefly describe the differences between the above and the men proposing deferred acceptance algorithm. (Not more than two sentences!)

(b) Suppose all men are acceptable to all women and no woman is allowed to rank any man as unacceptable.

Show that f is strategy-proof for the women.

(c) Suppose all women report truthfully.

Show that all pure strategy Nash equilibrium outcomes of the revelation game among men induced by the above procedure are pairwise stable.

(d) Is f strategy-proof for the women if they are allowed to rank men as unacceptable? Justify your answer.

2. **Mechanism design**

   Suppose two agents decide collectively whether to implement a costless public project. Agent i gets utility

   $$u_i(\theta_i, k, t_i) \;=\; k \;\cdot\; \theta_i \;+\; t_i,$$

   where $k = 1$ if the public project is implemented, $k = 0$ otherwise, and t_i is the transfer agent i receives. The valuation of agent i, θ_i, is distributed uniformly and independently on $[-3, 3]$ and is privately observed by agent i.

   Participation in the decision procedure is compulsory, i.e., there are no participation constraints. The sum of transfers must be weakly negative, $t_1 + t_2 \leq 0$.

   Define the welfare W under a social choice function (k, t) to be the ex-ante aggregate expected utility, that is,

   $$W = \mathbb{E}\big[k(\theta_1, \theta_2) \cdot (\theta_1 + \theta_2) + t_1(\theta_1, \theta_2) + t_2(\theta_1, \theta_2)\big].$$

   (a) Suppose the decision is made by *unanimity voting*, i.e., the project is implemented if and only if all agents vote for it, and no payments are made. Assume that agents vote for the project if and only if they have a positive valuation.

   What is the welfare under this rule?

   (b) What would be the highest possible welfare if preferences were publicly observable?

   (c) What is the welfare under the pivotal (Clarke) mechanism?

   (d) What is the highest welfare that could be achieved in a Bayesian incentive compatible mechanism?