1. **Mechanism design**

There are I agents deciding over the implementation of a public project. The cost of implementing the public project is 0. Let $k = 1$ if the public project is implemented and $k = 0$ else. Utility of agent i is

$$u_i(\theta_i) = k \cdot \theta_i + t_i,$$

where θ_i is distributed independently and uniformly on $[-2, 1]$, and θ_i is agent i's private information. $t_i \in \mathbb{R}$ denotes the transfer to agent i.

In a *voting mechanism*, transfers are equal to 0 and agents can either vote in favour of the project or against it. The decision is made by *majority voting*, if the project is implemented if and only if at least half of the agents vote in favour of it. The decision is made by *unanimity voting*, if the project is implemented if and only if all agents vote for it.

Voting behaviour is called *sincere*, if an agent votes for the project if and only if his private type is positive.

(a) Show that it is a dominant strategy to vote sincerely if the decision is made by majority voting and if the decision is made by unanimity voting.

(b) Suppose that $I = 3$. Does majority voting provide a higher expected welfare compared to unanimity voting if agents follow their dominant strategies?

(c) Suppose that $I = 2$. Describe a mechanism that is Bayesian incentive compatible (BIC), implements the efficient decision rule and is budget balanced, i.e. $t_1(\theta) + t_2(\theta) = 0$ for all θ.

Suppose the outside option of an agent is not to participate in the mechanism, in which case he receives a transfer of 0 and the public project is implemented if and only if the other agent has a positive type. Is there a mechanism which fulfills the required properties and which is (ex post) individually rational?

(d) Suppose now that $I = 2$ and agents have interdependent valuations, i.e. utility of agent i is given by

$$u_i(\theta) = k \cdot (\theta_i + \alpha\theta_{-i}) + t_i,$$

with $\alpha \in \mathbb{R}$.

Consider the direct revelation mechanism where the project is implemented if and only if both reports lie above some threshold $r \in (-2, 1)$. Transfers are equal to 0 for all agents. Determine the threshold such that this mechanism is BIC.

2. **Matching**

There is a set of n men, $M = \{m_1, ..., m_n\}$, and a set of p women, $W = \{w_1, ..., w_p\}$. If man m_i and woman w_j are paired, they create a monetary value of $v(m_i, w_j)$, single individuals do not create value. Utility is given by the monetary value an agent realizes.

(a) Suppose that utility is transferable, $n = p = 2$ and match values are given as follows:

	w_1	w_2
m_1	10	18
m_2	1	10

Compute the core of the game and draw the set of payoffs for men that are part of core allocations. Compute the Shapley value of this game. Is the Shapley value in the core?

(b) Now one additional man arrives, corresponding match values are given as follows:

	w_1	w_2
m_1	10	18
m_2	1	10
m_3	3	5

Determine the payoff vector in the core that men prefer the least. Compare to the payoff vector men prefer the least in part (a).

(c) Consider the general setting with n men and p women and arbitrary match values. Suppose that utility is not transferable across agents and each men that is matched receives a share $s \in (0, 1)$ of the match value, each woman receives a share $1 - s$. Assume that match values are such that individuals have strict preferences. Show that there is a unique stable matching.