Hand in written solutions of the exercises not covered in 
Wednesday’s tutorial before the tutorial on July 16th. 
You may work in groups of at most two students.

Exercises:

1. Consider a marriage market with strict preferences and suppose that $\mu$ and $\mu'$ are stable matchings. 
Show: $\mu \lor_M \mu'$ and $\mu \land_M \mu'$ are stable matchings.

2. Consider the following marriage market with four men and four women. 
Preferences are strict and given by

\[
\begin{align*}
m_1 &: w_1, w_2, w_3, w_4 \\
m_2 &: w_2, w_1, w_4, w_3 \\
m_3 &: w_3, w_4, w_1, w_2 \\
m_4 &: w_4, w_3, w_2, w_1 \\
\end{align*}
\]
\[
\begin{align*}
w_1 &: m_4, m_3, m_2, m_1 \\
w_2 &: m_3, m_4, m_1, m_2 \\
w_3 &: m_2, m_1, m_4, m_3 \\
w_4 &: m_2, m_3, m_1, m_4 \\
\end{align*}
\]

In the following, you may use without proof that $\mu_1 = w_1 w_2 w_3 w_4 m_3 m_1 m_4 m_2$ is a stable matching.

(a) Show that $\mu_2 = w_1 w_2 w_3 w_4 m_2 m_4 m_1 m_3$ is a stable matching.

(b) Find the men-optimal stable matching $\mu_M$ and the women-optimal stable matching $\mu_W$.

(c) Find two additional stable matchings that are different from $\mu_1, \mu_2, \mu_M$ and $\mu_W$.

3. Consider a marriage market with strict preferences. 
Show: The matching resulting from the men-proposing deferred acceptance algorithm is men-optimal.
4. Consider a marriage market with strict preferences. A mechanism asks the agents to reveal their preferences and applies the men-proposing deferred acceptance algorithm to the reported preferences. Suppose that (i) men report truthfully and (ii) each woman reports a preference list such that all reports form a Nash equilibrium (under complete information).

Show: The resulting matching is stable.

5. Consider a marriage market with strict preferences.

Show: The set of individuals that remain single is the same for all stable matchings.

6. Consider a marriage market with strict preferences.

Show: In the men-proposing deferred acceptance algorithm, truth-telling is a dominant strategy for men.