Microeconomics II (PhD)
Tutorial 8, July 11th and 16th, 2018

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Hand in written solutions of the exercises not covered in wednesday's tutorial *before* the tutorial on july 16th. You may work in groups of at most two students.

Exercises:

1. Consider a marriage market with strict preferences and suppose that μ and μ' are stable matchings.

   Show: $\mu \vee_M \mu'$ and $\mu \wedge_M \mu'$ are stable matchings.

2. Consider the following marriage market with four men and four women. Preferences are strict and given by

$$\begin{array}{llllll} m_1: & w_1, & w_2, & w_3, & w_4 \\ m_2: & w_2, & w_1, & w_4, & w_3 \\ m_3: & w_3, & w_4, & w_1, & w_2 \\ m_4: & w_4, & w_3, & w_2, & w_1 \end{array} \qquad \begin{array}{llllll} w_1: & m_4, & m_3, & m_2, & m_1 \\ w_2: & m_3, & m_4, & m_1, & m_2 \\ w_3: & m_2, & m_1, & m_4, & m_3 \\ w_4: & m_2, & m_3. \end{array}$$

   In the following, you may use without proof that

$$\mu_1 = \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \\ m_3 & m_1 & m_4 & m_2 \end{array}$$

   is a stable matching.

   (a) Show that

$$\mu_2 = \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \\ m_2 & m_4 & m_1 & m_3 \end{array}$$

   is a stable matching.

   (b) Find the men-optimal stable matching μ_M and the women-optimal stable matching μ_W.

   (c) Find two additional stable matchings that are different from μ_1, μ_2, μ_M and μ_W.

3. Consider a marriage market with strict preferences.

   Show: The matching resulting from the men-proposing deferred acceptance algorithm is men-optimal.

4. Consider a marriage market with strict preferences. A mechanism asks the agents to reveal their preferences and applies the men-proposing deferred acceptance algorithm to the reported preferences. Suppose that (i) men report truthfully and (ii) each woman reports a preference list such that all reports form a Nash equilibrium (under complete information).

   Show: The resulting matching is stable.

5. Consider a marriage market with strict preferences.

   Show: The set of individuals that remain single is the same for all stable matchings.

6. Consider a marriage market with strict preferences.

   Show: In the men-proposing deferred acceptance algorithm, truthtelling is a dominant strategy for men.